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Sec :- A

Subject :- Applied Calculus

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FINAL TERM EXAM

Q1) Find PQ where P is the point in three-dimensional space with coordinates (4,1,3) & the point Q with co-ordinates (1,2,4). Find the distance b/w P & Q, find the position vector of the point dividing PQ in the ratio 1:3.

Sol:-

Coordinate of $P = (4,1,3)$

$$OP = 4i + 1j + 3k$$

or

$$OQ = \vec{OQ} - \vec{OP}$$

$$= (i+2j+4k) - (4i+1j+3k)$$

$$= -3i + 1j + 1k \rightarrow (1)$$

Now distance between P & Q = |PQ|

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9+1+1}$$

$$= \sqrt{11} \rightarrow (2)$$

Let M be the point which divided PQ in ratio 1:3, then by ratio theorem

Position vector of $M = \vec{OM}$

$$= \frac{3(4i+1j+3k) + (1)(1i+2j+4k)}{1+3}$$

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$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \quad -③$$

Hence eq ①, ② & ③ are the required sol-

Q2) Evaluate

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x} dx$$

Sol:-

$$\begin{array}{r|l} & 2x - 1 \\ 2x^2 + x & 4x^3 + 10x + 4 \\ & \cancel{\pm 4x^3} \quad \pm 2x^2 \\ \hline & \cancel{-2x^2} + 10x + 4 \\ & \cancel{\mp 2x^2} \mp x \\ \hline & 11x + 4 \end{array}$$

So $2x - 1 + \frac{11x+4}{2x^2+x} = \frac{4x^3 + 10x + 4}{2x^2 + x}$

$$\Rightarrow \int \frac{4x^3 + 10x + 4}{2x^2 + x} = \int 2x - 1 + \int \frac{11x + 4}{2x^2 + x} \quad -①$$

$$= 2\int x\, dx - \int 1\, dx + \int \frac{11x + 4}{2x^2 + x} dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \quad — ②$$

Now find

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \quad — Ⓐ$$

$$\cancel{\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)}}$$

$$\frac{11x+4}{x(2x+1)} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$11x+4 = A(2x+1)+Bx \quad — ③$$

put $x=0$ in eq ③

$$\boxed{4 = A}$$

Now put $x = -\frac{1}{2}$ in eq ③

$$11\left(-\frac{1}{2}\right) + 4 = B\left(-\frac{1}{2}\right)$$

$$-\frac{11}{2} + 4 = -\frac{B}{2}$$

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$$\frac{-11+8}{2} = \frac{-B}{2}$$

$$-3 = -B$$

$$\boxed{\Rightarrow B = 3}$$

putting value of A & B in (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

putting these values in (2)

$$= x^2 - x + 4\ln|x| + 3/2\ln|2x+1|$$

put value in (1)

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C$$

Ans-

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Q3
a) $\int_0^2 x^2 e^x dx$

Sol:- Now find integration first

$= \int x^2 e^x dx$

$= x^2 \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x^2\right) dx$

$= x^2 e^x - \int e^x (2x) dx$

$= x^2 e^x - 2 \int x e^x dx$

$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x\right) dx\right]$

$= x^2 e^x - 2x e^x + 2e^x$

put limits

$= \left(x^2 e^x - 2x e^x + 2e^x\right)\Big|_0^2$

$= 2^2 e^2 - 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0)$

$(= 4e^2 - 4e^2 + 2e^2 - 2)$

$= \boxed{2e^2 - 2}$ Ans

b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ 7826 6

Sol:-

First find integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = ? \quad -① $$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$2\,dy = \frac{1}{\sqrt{x}}\,dy \quad \text{put in eq ①}$$

$$\int \sin(y)\,(2dy) = 2\int \sin(y)\,dy$$

$$= 2(-\cos y)$$

$$= -2\cos y \quad \text{put } y = \sqrt{x}$$

$$= -2\cos\sqrt{x}$$

Put Limit,

$$\Rightarrow -2\left|\cos\sqrt{x}\right|_1^2 = -2(\cos\sqrt{x} - \cos 1)$$

$$= -2\cos\sqrt{2} + 2\cos(1) \text{ Ans-}$$

Q4) Verify that 7820 (7)

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the three dimensional Laplace equation.

Sol:- The laplace eq in 3dis

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad \text{—— (A)}$$

so $u(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial^2 u}{\partial x^2} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x\left(-\frac{3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \quad \text{—— (1)}$$

Now

$$\frac{\partial u}{\partial y} = \frac{-1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}y)$$

$$\frac{\partial u}{\partial y} = -y\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y\left(-\frac{3}{\cancel{2}}\right)\left(x^2+y^2+z^2\right)^{-5/2}(\cancel{2}y) + \left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2\left(x^2+y^2+z^2\right)^{-5/2} + \left(x^2+y^2+z^2\right)^{-3/2} \quad — (2)$$

$$\frac{\partial u}{\partial z} = \frac{-1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2} \quad — (3)$$

putting eq (1), (2) & (3) in eq (A)

$$3x^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2} + 3y^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}$$

$$+ 3z^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2)+3z^2-(x^2+y^2+z^2)\right]$$

$$=(x^2+y^2+z^2)^{-5/2}\left[3x^2-x^2-y^2-z^2+3y^2-x^2-y^2-z^2+3z^2-x^2-y^2-z^2\right]$$

$$=(x^2+y^2+z)^{-5/2}(0) = 0$$

So the given $u(x,y,z)$ is solution of Laplace equation