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SECTION A

MODULE 4TH SEMESTER

SUBJECT

DIFFRENTIAL EQUATION.

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بسم الله الرحمن الرحیم

ANSWER TO QUESTION NO 1: WHICH IS OBJECTIVE TYPE QUESTION.

① The order of Matrix AB is $m \times n$ beause if we consider the order of matrix 'A' is $m \times p$ and the order of B is $p \times n$.

x————x————x.

② The nomber of non-zero rows in an Echelon form is one. The non-zero rows is also called Ramk.

x————x————x

③ if B is a singular matrix then

$a = 4$.

x————x————x

④ Solution: If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

then

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= -2i^2 - i^2 \qquad \text{beause } i^2 = -1$$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$= 3.$$

⑤ The matrix A is $\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a Scalar matrix (P2)

because we know that when diagnal elamed are Same and non-diagnal are zero than its called Scalar matrix.

X ———— Y ———— Y ———— Y.

⑥ Solution: $\frac{dy}{dx} + 2xy = y$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$

$$\frac{dy}{dx} = y(1-2x)$$

$$\frac{dy}{dx} = (1-2x)\,dx$$

Taking Integration.

$$\int \frac{1}{y}\,dy = \int (1-2x)\,dx$$

$$\ln y = \int 1\,dx - \int 2x\,dx.$$

$$\ln y = x - \frac{2x^2}{2} + c$$

$$\ln y = x - x^2 + c$$

$$\ln y = x - x^2 + c$$

$$e^{\ln y} = e^{x - x^2 + c} \Rightarrow \boxed{y = e^{x(1-x)+c}}\ \text{Ans.}$$

7) The order & degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$ is

order is 1 &

degree is 3.

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(8) The order & degree of differential equation

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$$ is

order Two

Degree one.

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(9) Solution: $2y' + x^2 y = x^2+3$, $y(0)=5$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{x^2+3}{2}$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{1}{2}(x^2+3).$$

$$u = \frac{x^2}{2}$$

$$e^{\int \frac{x^2}{2}dx} = e^{x^3/6}$$

$$e^{x^3/6}y' + e^{x^3/6}\left(\frac{x^2}{2}\right)y = \frac{1}{2}e^{x^3/6}(x^2+3)$$

$$y(x) = \frac{e^{x^3/6}x^2 + 3e^{x^3/6} + c}{2e^{x^3/6}}$$

$$y(0) = \frac{0+3}{2} = 3/2$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + \frac{3}{2} \text{ Ans}$$

x ———— x ———— x

(10)

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expanding by C_1.

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$$\Rightarrow 1(bc^2 - cb^2) - 1(ac^2 + a^2c) + 1(ab^2 - a^2b)$$

$$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b$$

$$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$$

$$= a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2$$

$$= a^2(c-b) + b^2(a-c) + c^2(b-a) \text{ Ans}$$

x ———— x ———— x.

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ANSWER TO PART A IN QUESTION NO 2:

Solution:
$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2).$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

Take abc common.

$$abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b).$$

$$\Rightarrow abc\left[bc(c-b) - ac(c+a) + ab(b-a)\right]$$

Ans.

Q NO 2 PART B :

(P6)

Sol:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic eq $\rightarrow |A - \lambda I| = 0 \rightarrow$ (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinant.

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & +2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$- 1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow \text{(B)}$$

Again Expand by R_2

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda)\Big[\big((3-\lambda)(2-\lambda)-(-1)(-1)\big) + 1\big((-1)(2-\lambda) - (-1)(-1)\big)$$
$$-\big((-1)(-1)-(-1)(3-\lambda)\big)\Big]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2+1)+(-2+\lambda-1)-(+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

$$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= -\lambda^3+8\lambda^2-18\lambda+8 \rightarrow (a).$$

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand By C_1.

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$\Rightarrow -\lambda^2+3\lambda-5-3+\lambda$

$= -\lambda^2+6\lambda-8 \rightarrow b$

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} +0\right]$$

$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right.$

$= -(3-\lambda+\lambda^2-5\lambda+5)$

$-\lambda^2+5\lambda-5-3+\lambda$

$-\lambda^2+6\lambda-8 \rightarrow c$

Put (a) & b, c in (B).

$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8.$

$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda-8$

$-\lambda^2+16\lambda-8$

$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda+6\lambda+6\lambda$

$+16-16$

$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0.$

By Synthatic division.
we get.

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0$$

$$\boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0.$$

By factorization method

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$(\lambda-4)(\lambda-4).$$

$$\lambda=4,\ \lambda=4.$$

$$\boxed{\lambda_1=0,\ \lambda_2=2,\ \lambda_3=4,\ \lambda_4=4}$$

Ans

x ———— x ———— x.

ANSWER TO QUESTION NO 3:

$(x^2+3y^2)\,dx - 2xy\,dy = 0$.

$x=2,\; y=6$.

<u>Solution</u>, $(x^2+3y^2)\,dx - 2xy\,dy = 0$

$(x^2+3y^2)\,dx = 2xy\,dy$

Dividing B.S by $2xy\,dx$, we get.

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{y} + \frac{3y}{x}\right) \rightarrow (A)$$

Let $y = vx$

Diff: $dy = v\,dx + x\,dv$

Dividing by dx.

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \rightarrow (a)$$

Put (a) in A.

$$v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{x}{xv} + 3\frac{vx}{x}\right).$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1}{v} + 3v\right)$$

Multplying B.S by 2

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplyng B S by $\frac{dx}{dv}$, we get.

$$2\,x\,dv = \frac{1+v^2}{v}\,dx.$$

Multiplyng B.S by $\frac{v}{x(1+v^2)}$.

we get.

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx.$$

Take "$\int$" on B.S

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c.$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on B.S

$$e^{\ln|1+v^2|} = e^{\ln(xc)}$$

$$1+v^2 = xc.$$

P12

$1+v^2 = xc$

Put $v = y/x$

$1+(y/x)^2 = xc$

$\frac{x^2+y^2}{x^2} = xc$

$x^2+y^2 = x^3c \rightarrow$ B

Put $x=2, y=6$ in eq B.

$(4)+(36) = 8c$

$\boxed{c = 5}$

So Put $c=5$ in B, we get.

$x^2+y^2 = 5x^3$.

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x-1)$

Taking $\sqrt{\ }$ on B.S, we get.

$y = +x\sqrt{5x-1}$, $y = -x\sqrt{5x-1}$

Ans.