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7684

Section

B

Department

BE (Civil)

Subject

Introduction

to earth quake

Date

29/06/2020

Semester

8th

Instructor:

Engr

Yaseen

Important

In

Q1

given Data:

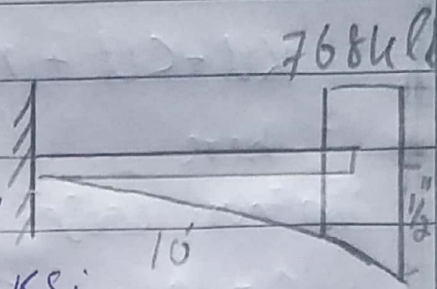
$$\text{distance} = 1/2''$$

$$E = 29000 \text{ KSI}$$

$$I = 158 \text{ in}^4$$

δ = deflection due to 7684 lb

$$L = 10 \text{ ft}$$

required:

natural time period = ?

equation of motion = ?

Amplitude = ?

graph of vibration of displacement with time = ?

graph of vibration of equivalent static force with time = ?

Solution:

as we know that from general equation of motion for SDOF self degree of freedom system

important

$$kU + c\dot{U} + m\ddot{U} = P(t)$$

here

in our case system is undamped

$c = 0$ so under going free
vibration $P(t) = 0$

Hence general EOM become

$$kU + m\ddot{U} = 0 \quad \text{--- (1)}$$

Now we know that

$$k = \frac{3EF}{L^3} = 2$$

put value

$$= \frac{3 \times 2900 \times 150}{(10 \times 12)^3}$$

$$= \boxed{7.55 \text{ K/in}}$$

$$= \boxed{9.0625 \text{ lb/in}}$$

Important

Now we mass (m)

$$m = \frac{F/a}{g} = \frac{7684}{32.2} = \boxed{238.64 \text{ slug}}$$

Now we find ω_n

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{96025}{238.64}} = \boxed{20.1 \text{ rad/sec}}$$

Now put value in
equation

$$Ku + m\ddot{u} = 0$$

$$\boxed{90625u + 31.06\ddot{u} = 0}$$

Now find Time period

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{20.1} = \boxed{0.313 \text{ sec}}$$

Now we also know that
from Central solution
to the free EOM for
undamped free vibration

$$u(t) = u(0) \cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n} \sin(\omega_n t)$$

$$u(0) = \frac{1}{2}'' = \frac{1}{24}ft$$

$$\dot{u}(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(20 \cdot 1T)$$

Equivalent static force
at any time "t" is

$$F_s(t) = k \cdot u(t) = \frac{90625 \times \cos(54.9T)}{24}$$

$$= 3776.042t$$

$$= 3776 \cos t$$

$$= \boxed{3776 \cdot 042 \cos 54.9t}$$

Important

Now we find Amplitude of dynamic displacement U_0 for undamped free vibration

$$U_0 = \sqrt{(U(0))^2 + (U(0)/\omega_n)^2}$$

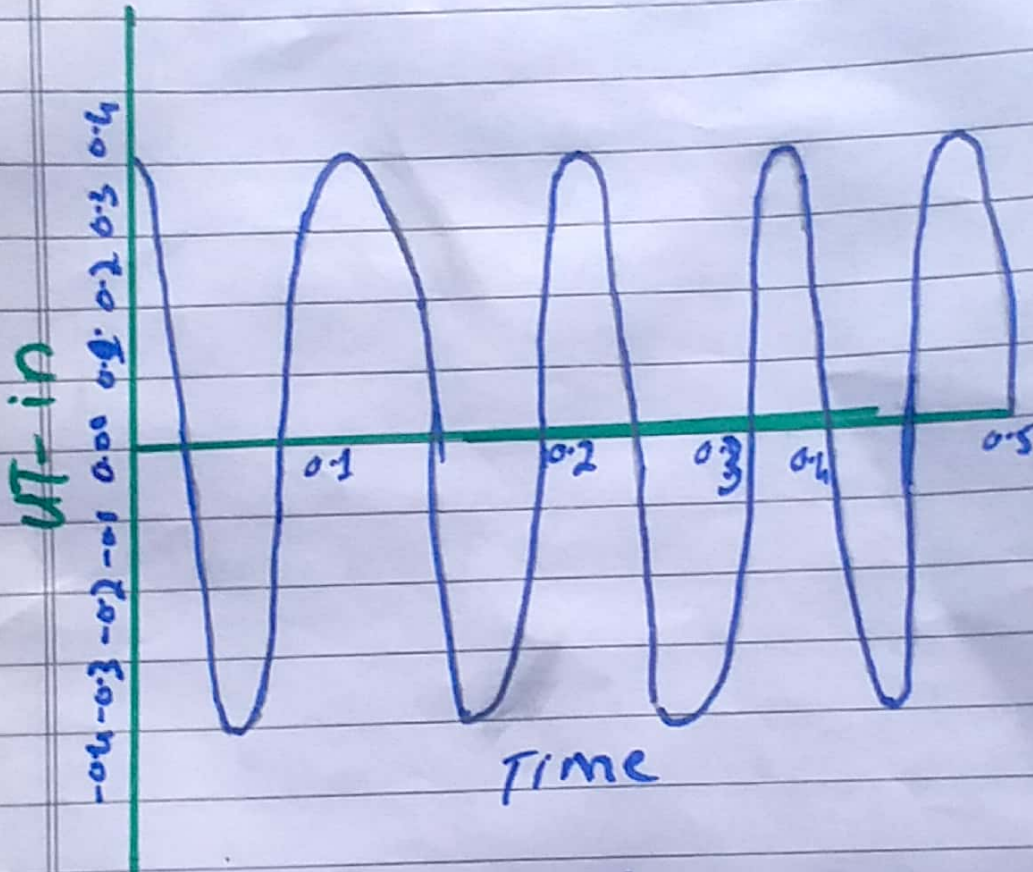
$$= \sqrt{(\frac{1}{2}u)^2 + 0} = \sqrt{(\frac{1}{2}u)^2}$$

$$= \frac{1}{2}u = 0.042 \text{ fr}$$

Now find Amplitude of equivalent static force

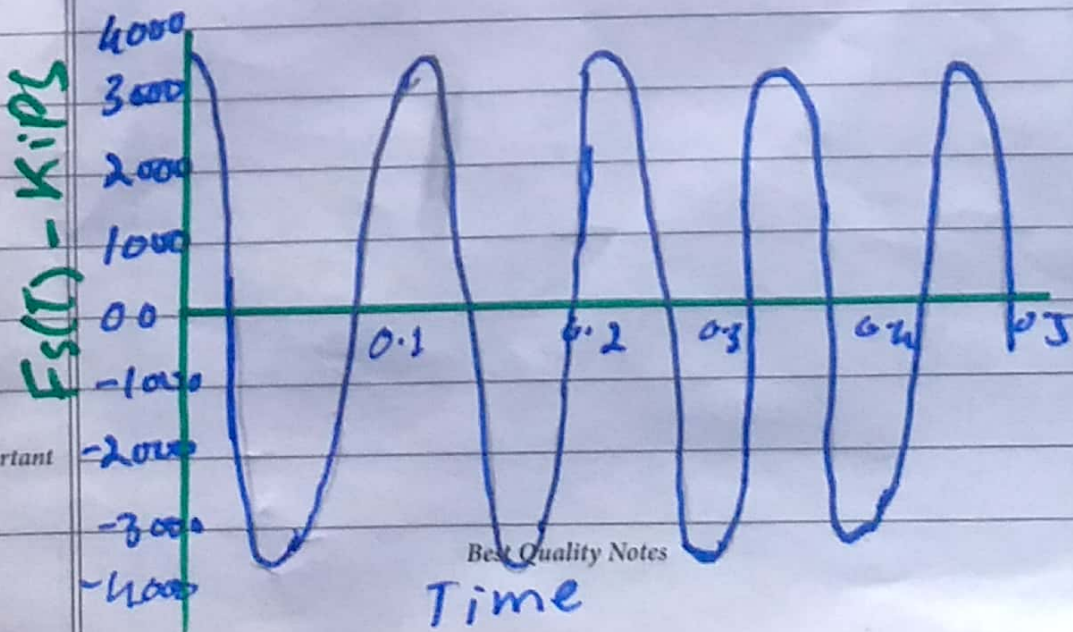
$$KU = 90625 \times \frac{1}{24} = \boxed{3776.042 \text{ N}}$$

undamped free vibration



variation of displacement with time.

undamped free vibration



variation of equivalent force with time



given Data:

$$\text{distance} = 1/2''$$

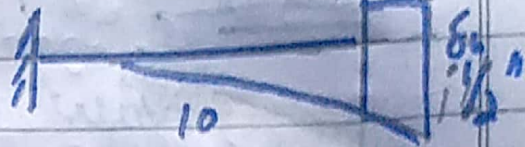
$$E = 29000 \text{ KSI}$$

$$I = 150 \text{ in}^2$$

$$S = 7684 \text{ lb}$$

$$L = 10 \text{ ft}$$

$$\text{Damping ratio} = \zeta = 4\%$$



required:

① develop and solve the equation of motion for vibration at free end = ?

② also develop equation showing vibration in the equivalent static force with time = ?

③ draw graph the variation of displacement with time and the variation of equivalent static force with time = ?

Important

Solution: for Part 1 and 2

Now we know that
from Equation of motion
for damped vibration

$$Ku + Cu + m\ddot{u} = 0 \quad \text{--- (1)}$$

we know that

$$K = 3EI/L^3 = 3 \times 29000 \times 150$$

$$(12 \times 12)^3$$

$$= 7.55 \text{ K/in}$$

$$= 90625 \text{ lb/ft}$$

Now mass

$$m = F/a = \frac{7684}{32.2} = 238 \quad \boxed{238.64 \text{ slugs}}$$

Now we find C

We know that

$$C = \frac{4}{100} \times 2m\omega_n \quad \text{--- (11)}$$

$$= \frac{4}{100} \times 2 \times 238.64 \times \omega_n$$

First we find ω_n

$$\omega_n = \sqrt{K/m} = \sqrt{\frac{96025}{238.64}}$$

$$= \boxed{20.1 \text{ rad/sec}}$$

Now put in eq (11)

$$C = \frac{4}{100} \times 2 \times 238.64 \times 20.1$$

$$= \boxed{38374 \text{ lb} \cdot \text{sec/ft}}$$

Now put these value in (1)

$$kU + cU' + m\ddot{U} = 0$$

$$90625U + 383.74\dot{U} + 238.64\ddot{U} = 0$$

Now solution to the E.O.M for damped free vibration

$$U(t) = e^{-\delta\omega_n t} \left[U(0) \cos(\omega_d t) + \frac{1}{\omega_d} (U'(0) + \delta\omega_n U(0)) \sin(\omega_d t) \right]$$

$$\omega_d = 20.1 \text{ rad/sec}$$

$$U(t) = e^{-0.04 \times 20.1 t} \left[\frac{1}{24} \times \cos(20.1 t) + \frac{1}{20.1} \left[0 + \frac{1}{24} \times 0.04 \times 20.1 \times \sin(20.1 t) \right] \right]$$

$$e^{-0.804 t} \left[0.042 \cos(20.1 t) + 0.0017 \sin(20.1 t) \right]$$

$$= e^{-0.804 t} \left[0.042 \cos(20.1 t) + 0.0017 \sin(20.1 t) \right]$$

Now we know that

$$f(s) = K \cdot U(t)$$

$$K = 90625$$

$$f(s) = 90625 \left(e^{-0.804t} \times 0.042 \cos(20.1t) + 0.0017 \sin(20.1t) \right)$$

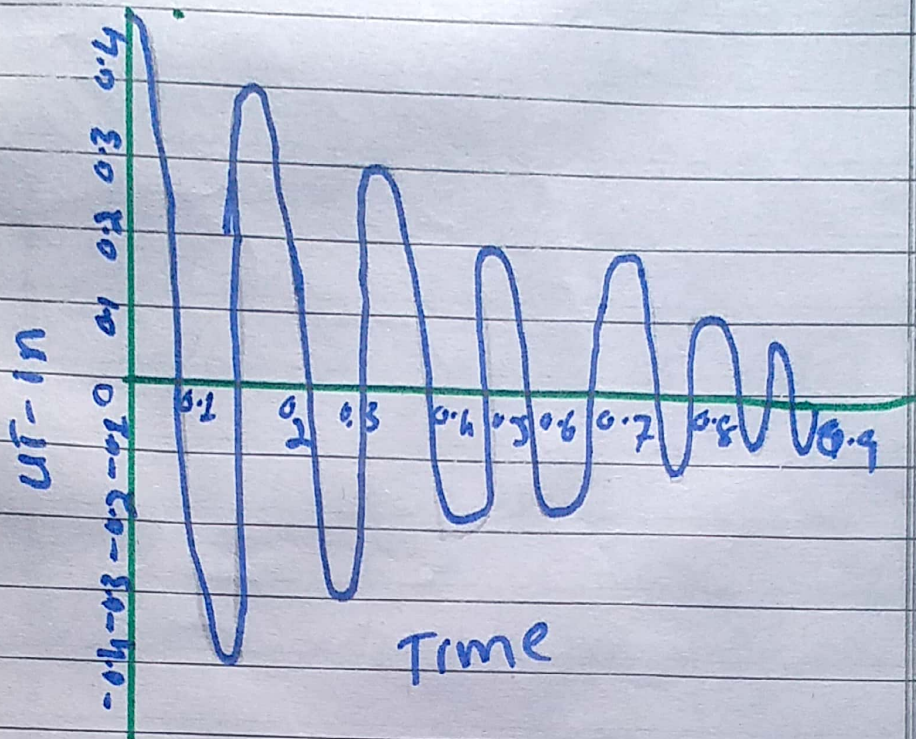
$$= e^{-0.804t} \left(90625 \times 0.042 \cos(20.1t) + 90625 \times 0.0017 \sin(20.1t) \right)$$

$$f_s = e^{-0.804t} \left(3806.25 \cos(20.1t) + 154.0625 \sin(20.1t) \right)$$

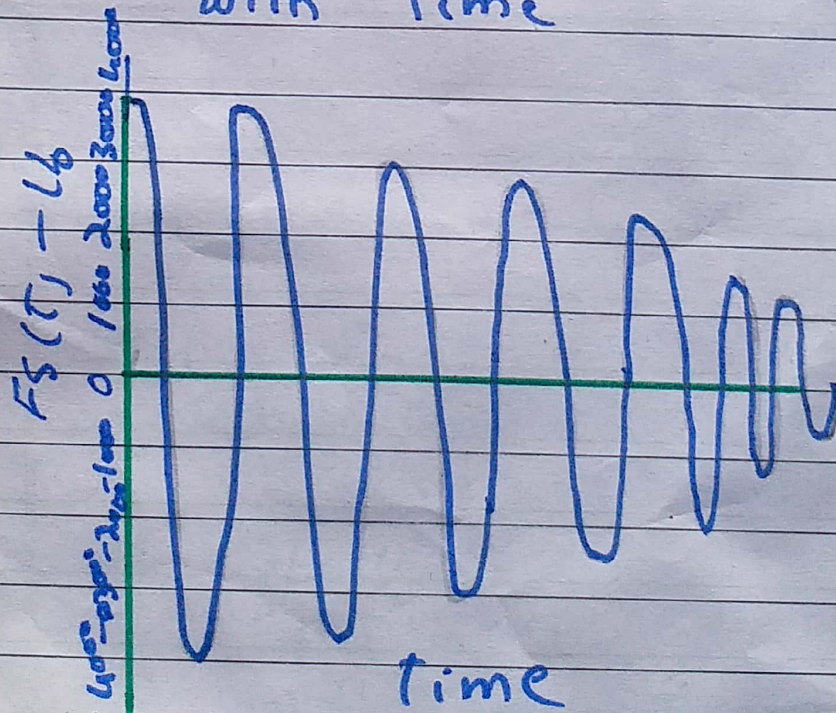
$$f_s = e^{-0.804t} \left(3806.25 \cos(20.1t) + 154.0625 \sin(20.1t) \right)$$

Important

Damped free vibration



variation of displacement with Time



variation of Equivalent static force with Time

Q3Given Data:

7684/1000 inches

load $z = f = 60 \text{ kips}$ displace the tank
by force

$$= \frac{7684}{1000} = 7.684''$$

figure

Time Period = 3.57 sec

cycle = 7

Amplitude of displacement = 2.286 cm

$$= \frac{2.286}{2.54}$$

$$= 0.9''$$

$$= 0.9''$$

required:

- ① Damping ratio = ?
- ② natural period of undamped vibration = ?
- ③ Stiffness of structure = ?
- ④ weight of tank = ?
- ⑤ Damping coefficient = ?

⑧ number of cycles to reduce
the displacement amplitude
to 0.5"

Solution:

① Damping ratio = $\zeta =$

as we know that

$$\zeta = \frac{1}{2\pi n} \ln \left[\frac{U_i}{U_{i+1}} \right]$$

$$\zeta = \frac{1}{2\pi n} \ln \left(\frac{U_i}{U_{i+1}} \right)$$

$$U_i = 7.8 \quad U = 7.684''$$

after $n = 7$

$$U_{i+1} = 7+1 = 8$$

now put these value
in eqn (1)

$$\zeta = \frac{1}{2\pi \times 7} \ln \left(\frac{7.684}{0.9} \right)$$

$$\zeta = 0.0487$$

Best Quality Notes

$$= 0.0487 \times 100$$

$\frac{100}{100}$

$$= 4.87\%$$

⑥ natural period of undamped vibration = $T_n =$

as we know that

7 cycle of vibration are complete in 3.57 sec

Time required to complete one cycle = $3.57/7$

$$T_D = 0.51 \text{ sec}$$

Now we know that

$$W_D = W_n (\sqrt{1 - \zeta^2})$$

$$2\pi/W_D = 2\pi/W_n (\sqrt{1 - \zeta^2})$$

$$T_D = T_n / \sqrt{1 - \zeta^2}$$

$$T_n = T_D / \sqrt{1 - \zeta^2} \quad \text{--- (1)}$$

Important

Now put in eq(1)

$$T_n = 0.51 \sqrt{2 (0.5087)^2}$$

$$T_n = 0.5087 = \boxed{0.51 \text{ sec}}$$

③ Stiffness of Structure

K

as we know that

$$K = \frac{F \cos \theta}{2} = \frac{60 \times \cos 60}{2}$$

$$= \boxed{15 \text{ kips}} = \boxed{180000 \text{ lb/ft}}$$

④ weight of the tank

as we know that

$$W_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{W/g}} = \sqrt{\frac{K \cdot g}{W}}$$

important

NOW

$$W = kg / Wn^2$$

$$Wn = 2\pi / Tn$$

$$W = kg / \frac{Wn^2}{Tn^2} = \frac{kg \cdot Tn^2}{WT^2}$$

$$= \frac{180000 \times 0.51^2 \times 32.2}{4 \times (3.14)^2}$$

$$= \frac{1507539.6}{39.4384}$$

$$= 38225.2 \text{ N}$$

$$= 38.3 \text{ K}$$

⑤ Damping Co-efficient

as we know that

$$\zeta = \frac{C}{2mWn}$$

$$C = \zeta \times 2mWn = \zeta \times 2m \left(\frac{2\pi}{Tn} \right)$$

PUT value

$$= \frac{0.0487 \times 2 \times 38225}{32.2}$$

Important

$$\omega = 115.63 \text{ lb. sec / ft}$$

⑥ No. of cycle To reduce displacement Amplitude from 7.684" to 0.9"

$$j = \frac{1}{2\pi\xi} \ln \left[\frac{u_1}{u_{i+1}} \right]$$

$$j = \frac{1}{2 \times \pi \times 0.0487} \times \ln \left(\frac{7.684}{0.9} \right)$$

$$j = 7.00 \text{ Cycles}$$