Q49) Homogenous :- The differential equation of any order is homogenous if once all the terms involving the unknown function are collected together on one side of the equation & the other side is Identically zero.

Example $y'' - 2y' = 0$

NON-Homogenous :- The non homogenous differential equation has terms on both sides, This type of equation is in the form of :- $y'' + py' + qy^* = f(x)$.

where p, q are real no & can be Real & complex.

QNO 1(b)(i) :- $4y'' - 6y' + 7y = 0$

Sol :- $7y(x) - 6\frac{d}{dx}y(x) + 4\frac{d^2}{dx^2}y(x) = 0$

÷ing both sides by x of derivative of y^4

we get $7y(x) - \frac{3\,d/dx\,y(x)}{2} + \frac{d^2}{dx^2}y(x) = 0$

$y'' + py' + qy = 0,$

where $p = -3/2$

$q = 7/y$

It is called linear homogenous.

$y'' + py' + qy = 0$

Trying the Roots.

$q + (K^2 + Kp) = 0$

The caracteristic equation will be

$K^2 - 3K/2 + 7/4 = 0$

This is a simple quadratic equation.

The Roots of the equation.

$$K_1 = 3/4 - \frac{\sqrt{19i}}{4}$$

$$K_2 = 3/4 + \frac{\sqrt{19i}}{4}$$

$$y(x) = e^{K_1 x} c_1 + e^{K_2 x} c_2$$

The Final Answer

$$y(x) = C_1 e^x\left(3/4 - \frac{\sqrt{19i}}{4}\right) + C_2 e^x\left(3/4 + \frac{\sqrt{19i}}{4}\right)$$

Q1(b)(ii) $y'' - 4y' - 12y = 3e^{5n}$

Sol:-

$$\frac{d^2}{dn^2} y(n) - 4\frac{d}{dn} y(n) - 12y(n)$$

The differential equation has the form of

$$y'' + py + qy = s$$

where $p = -4$

$q = -12$

$s = 3e^{5n}$

It is called linear homogenous.

Now

Roots of the characteristic equation:-

$$q + (K^2 + Kp) = 0$$

In this case the equation will be

$$K^2 - 4K - 12 = 0$$

$$K_1 = -2$$

$$K_2 = 6$$

Roots are Not complex,

then $K_2 = 6$

$$y(n) = C_1e^{K_1 n} + C_2e^{K_2 n}$$

$$y(n) = C_1e^{-2n} + C_2e^{6n}$$

Now Solving the Inhomogenous equation

$$y'' + py + qy = s$$

Suppose C_1 & C_2, it is functions.

The general solution is

$$y(n) = C_1(n)e^{-2n} + C_2(n)e^{6n}$$

$$y_1(x)\frac{d}{dx}c_1(x)+y_2(x)\frac{d}{dx}c_2(x)=0$$

$$\frac{d}{dx}c_1(x)\frac{d}{dx}y_1(x)+\frac{d}{dx}c_2(x)\frac{d}{dx}y_2(x)=0$$

where $y_1(x)$ & $y_2(x)$ - linearly Independand particular sol of linear ODE

$$y_1(x)=\exp(-2x)\,(c_1=1,c_2=0)$$
$$y_2(x)=\exp(6^x)\,c_1=0,c_2=4)$$

$$f(x)=3e^{5x}$$

So,

$$e^{6x}\,d/_{dx}\,c_2(x)+e^{-2x}\,d/_{dx}\,c_1(x)=0$$

$$d/_{dx}\,c_1(x)\,d/_{dx}\,e^{-2x}+d/_{dx}\,c_2(x)\,d/_{dx}\,e^{6x}=3e^{5x}$$

or

$$e^{6x}\,d/_{dx}\,c_2(x)+e^{-2x}\,d/_{dx}\,c_1(x)=0$$

$$6e^{6x}\,d/_{dx}\,c_2(x)-2e^{-2x}\,d/_{dx}\,c_1(x)=3e^{5x}$$

Now soluy the system.

$$d/_{dx}\,c_1(x)=\frac{3e^{7x}}{8}$$

$$d/_{dx}\,c_2(x)=\frac{3e^{-x}}{8}$$

it is simple differential equation, soluing these equations

$$C_1(x)=C_3+\int\left(-\frac{3e^{7x}}{8}\right)dx$$

$$C_2(x)=C_4+\int\frac{3e^{-x}}{8}\,dx$$

OR

$$C_1(x)=C_3-\frac{3e^{7x}}{56}$$

$$C_2(x)=C_4-\frac{3}{8}e^{-x}$$

$$y(x)=C_1(x)e^{-2x}+C_2(x)e^{6x}$$

The final Answer.

$$y(x)=C_3e^{-2x}+C_4e^{6x}-\frac{3e^{5x}}{7}$$ where C_3 & C_4 are constants.

Q2(i):- $16y'' - 40y' + 25y = 0$ $y(0) = 3$ $y'(0) = 9/4$ (5)

Sol:- $25y(x) - 40\frac{d}{dx}y(x) + 16\frac{d^2}{dx^2}y(x) = 0$

Divide both sides of the equation by the Multiplier of derative of y'' 16

we get $\frac{25y(x)}{16} - \frac{5\, d/dx\, y(x)}{2} + \frac{d^2}{dx^2}y(x) = 0$

The diff eq has the form

$y'' + py' + qy = 0$

$p = -5/2$

$q = 25/16$

Finding Roots

$q + (k^2 + kp) = 0$

Characteristic eq will be :-

~~$q + (k^2 + kp) = 0$~~

$k^2 - \frac{5k}{2} + \frac{25}{16} = 0$

Roots of eq will be

$k_1 = 5/4$

Now,

$y(x) = e^{k_1 x} c_1 + e^{k_1 x} c_2 x$

Substitute, $k_1 = 5/4$

$y(x) = c_1 e^{5x/4} + c_2 x e^{5x/4}$ Ans.

Q2(ii):- $y'' + 14y' + 49y = 0 \quad y(-4) = -1 \quad y'(-4) = 5$

$$49y(n) + 14\, d/dn\, y(n) + \frac{d^2}{dn^2} y(n) = 0$$

$$y'' + py' + qy = 0$$

where $p = 14$
$q = 49$

$$y'' + py' + qy = 0$$

Finding the Roots

$$q + (K^2 + Kp) = 0$$

characteristic eq :-

$$K^2 + 14K + 49 = 0$$

$$K_1 = -7$$

$$y(n) = e^{K_1 n} c_1 + e^{K_1 n} c_2 n$$

Substitute $= K_1 = -7$

$$C_1 = \frac{9}{e^{28}}$$

$$C_2 = -2/e^{28}$$

$$y(n) = (C_1 + C_2 n) e^{-7n}$$

$$y(n) = C_1 e^{-7n} + C_2 n e^{-7n}$$

Now

$$y(-4) = -1$$

$$\left[\begin{cases} -4 & \text{for } 0 = 1 \\ 1 & \text{for } 1 = 1 \\ 0 & \text{otherwise} \end{cases}\right) \left. d/dn\, y(n) \right|_{n=-4} = 5$$

$$d/dn\, y(n) = C_2 e^{-7n} - 7(C_1 + C_2 n) e^{-7n}$$

$$y(n) = (C_1 + C_2 n) e^{-7n}$$

$$5 = C_2 e^{--28} - 7(C_1 + (-4) C_2) e^{-28}$$

$$-1 = (C_1 + (-4) C_2) e^{--28}$$

$$\Rightarrow \boxed{C_1 = -9/e^{28}}$$

Q2 (iii) $y'' - 4y' + 9y = 0 \quad y(0) = 0 \quad y'(0) = -8$

$$9y(x) - 4\frac{d}{dx}y(x) + \frac{d^2}{dx^2}y(x) = 0$$

The differential equation has the form

$$y'' + py' + qy = 0$$

where, $p = -4$

$q = 9$ $\Longrightarrow$

$q + (K^2 + Kp) = 0$

$K^2 - 4K + 9 = 0$

$K_1 = 2 - \sqrt{5}i$

$K_2 = 2 + \sqrt{5}i$

$y(x) = e^{K_1 x} c_1 + e^{K_2 x} c_2$

$y(0) = 0$

$$\left(\begin{cases} 0 & \text{for } 0 = 1 \\ 1 & \text{for } 1 = 1 \\ 0 & \text{otherwise} \end{cases}\right) \left.\frac{d}{dx}y(x)\right|_{x=0} = -8$$

$$\frac{d}{dx}y(x) = 2\left(c_1 \sin(\sqrt{5}x) + c_2 \cos(\sqrt{5}x)\right)e^{2x}$$

$$y(x) = \left(c_1 \sin(\sqrt{5}x) + c_2 \cos(\sqrt{5}x)\right)e^{2x}$$

$$-8 = 2\left(c_1 \sin(0\sqrt{5}) + c_2 \cos(0\sqrt{5})\right)e^{0\cdot 2} + ($$

$$0 = \left(c_1 \sin(0\sqrt{5}) + c_2 \cos(0\sqrt{5})\right)e^{0\cdot 2}$$

$$c_2 = 0$$

$$c_1 = -\frac{8\sqrt{5}}{5}$$

$$y(x) = -\frac{8\sqrt{5}e^{2x}\sin(\sqrt{5}x)}{5}$$

(iv) $y'' - 8y' + 17y = 0,\ y(0) - 4\ y'(0) = -1$ ⑧

$r^2 e^{rx} - 8re^{rx} + 17e^{rx} = 0$

$e^{rx}(r^2 - 8r + 17) = 0$

$r^2 - 8r + 17 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{(8)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{16 - 68}}{2}$$

$= 4 \pm \sqrt{-52}$

$= 4 \pm \sqrt{-1 \times 52}$

$= 4 \pm i\sqrt{04 \cdot 13}$

$= 4 \pm 2i\sqrt{13}$

$y = 4+2i\sqrt{13},\ y = 4-2i\sqrt{13}$

$y = c_1 e^{4+2i\sqrt{13}},\ y = c_1 e^{4 2i\sqrt{13}}$

$y(0) = -4,\ y(0) = c_1 e^{4+2i\sqrt{3}(0)} + c_2 e^0$

$= c_1 + c_2 = -4$

$y' = (0) = -1,\ y'(0) = 4+2i\sqrt{3}c_1 e^{4+2i\sqrt{3}(0)} + c_2 e^0$

$= 4+2i\sqrt{3}c + c_2$

$c_1 + c_2$

$c_2 = -4 - c_1$

Now $4+2i\sqrt{3}c_1 + c_2$

$= 4+2i\sqrt{3}c_1 + (-4-4)$

$= 4+2i\sqrt{3}c_1 - 4 + c_1$

$0 = 4+2i\sqrt{3}\,2c_1 - 4$

$4 = 2c$

$\boxed{c_1 = 2}$

$c_2 = -4 - 2$

$c_2 = -6$

$y = 2e^{4+2i\sqrt{13}} - 6e^{4-2i\sqrt{13}}$ Ans

Q3:- Define laplace transform along with example:-

laplace transformation is a technique for solving differential equations. Hence differential equation of time domain form is first transformed to algebric equation of frequency domain form. After solving the algebric equation in frequency domain, the result then is finally transformed to time domain form to achieve the ultimate solution of the differential equation. In other words it can be said that the laplace transformation is nothing but a shortcut method of solving differential equation.

Example:-

$f''(t) + 3f'(t) + 2f(t) = 0$ where $f(0) = 1$ & $f'(0) = 0$,

(3)) $h(t) = e^{3t} + \cos 6t - e^{3t} \cos 6t$

$$g(s) = \frac{1}{s-3} + \frac{s}{s^2+(6)^2} - \frac{s-3}{(s-3)^2+(6)^2}$$

$$= \frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36}$$

(1) $f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$

Sol:-

$$f(s) = 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}.$$

(2) $g(t) = 4\cos 4t - 9\sin 4t + 2\cos(10t)$

$$\Rightarrow g(s) = 4\frac{s}{s^2+4^2} - 9\frac{4}{s^2+4^2} + 2\frac{s}{s^2+(10)^2}$$

$$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$$

Q4:- (i) $y'' - 10y' + 9y = 5t$, $y(0) = -1$, $y'(0) = 2$ (11)

Applying the laplace transform to both sides, we find

$(s^2 - 10s + 9)Y + s - 2 - 10 = \frac{5}{s^2} \Rightarrow Y(s) = \frac{5 + 12s^2 - s^3}{s^2(s-9)(s-1)}$

To find, we need to simplify the expression on the $Y(s)$ using the Partial fraction decomposition.

$$\frac{5 + 12s^2 - s^3}{s^2(s-9)(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}$$

$B = \frac{5}{9} \cdot D = -2, C = \frac{31}{81}, A = \frac{50}{81}.$

There fore, using the linearity of the Inverse laplace transform

$$y(t) = \frac{50}{81} + \frac{5t}{9} + \frac{31}{81}e^{9t} - 2e^t .$$

Q4(ii) :- $y'' - 6y' + 15y = 2\sin 3t \cdot y(0) = -1, y'(0) = -4$

we have

$(s^2 - 6s + 15)Y + s - 2 = \frac{6}{s^2+9} \Rightarrow Y(s) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2+9)(s^2 - 6s + 15)} = \frac{As + B}{s^2+9} + \frac{Cs + D}{s^2 + 6s + 15}$

To find the constants, we need to find the common ~~desactive~~ denominator

$s^3 : A + C = -1$

$s^2 : -6A + B + D = 2$

$s^1 : 15A - 6B + 9C = -9$

The sol is: $s^0 : 15B + 9D = 24$

$A = \frac{1}{10} \cdot B = \frac{1}{10} \cdot C = -\frac{11}{10} \cdot D = \frac{5}{2}.$

we got :-

$$Y(s) = \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11s + 25}{s^2 - 6s + 15}\right)$$

Finding Inverse laplace

$$\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+9}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+9} + \frac{1}{s^2+9}\right\} = \mathcal{L}\left\{\frac{s}{s^2+9}\right\} + \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$$

$= \cos 3t + \frac{1}{3}\sin 3t .$

Reranging the Expression

$$\frac{-11s+25}{s^2-6s+15} = \frac{-11s+25}{(s-3)^2+6}$$

$$= \frac{-11(s-3)-8}{(s-3)^2+6}$$

$$= -11\frac{(s-3)}{(s-3)^2+6} - \frac{8}{\sqrt{6}}\frac{\sqrt{6}}{(s-3)^2+6}$$

$$\mathcal{L}\left\{\frac{-11.s+25}{s^2-6s+15}\right\} = -11e^{3t}\cos\sqrt{6}t - \frac{8}{\sqrt{6}}e^{3t}\sin\sqrt{6}t$$

Hence the Ans is

$$y(t) = \mathcal{L}^{-1}\{Y\} = \frac{1}{10}\left(\cos 3t + \frac{1}{3}\sin 3t - 11e^{3t}\cos\sqrt{6}t - \frac{8}{\sqrt{6}}e^{3t}\sin\sqrt{6}t\right)$$