NAME : UMAIR KHAN

I.D : 14596

SECTION : B

SEMESTER : 4th BS (SE)

CLASS TIMING : 11:00 - 2:00 PM FRIDAY

SUBJECT : LINEAR ALGERA

INSTRUCTER : Muhammad Shakeel

QUESTION : 1

Determine the following is consistant or not - - - - ?

SOLUTION :-

My I.D = 14596

$$x_1 - 5x_2 + x_3 = 0 \longrightarrow (1)$$

$$2x_2 - 8x_3 = 8 \longrightarrow (2)$$

$$5x_1 - 5x_3 = 10 \longrightarrow (3)$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -5 & 1 & 0 \\ 0 & 2 & 8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -5 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & -25 & 10 & -10 \end{array}\right] R_3 - 5R_1$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -5 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 0 & 90 & 90 \end{array}\right] R_3 + 25/2\, R_2$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -5 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 0 & 1 & 1 \end{array}\right] \frac{1}{90} R_3$$

From R_3 : $x_3 = 1$

From R_2: $2x_2 - 8x_3 = 8$

$2x_2 - 8 = 8 \Rightarrow 2x_2 = 8+8$

$\Rightarrow 2x_2 = 16$

$\boxed{x_2 = 8}$

From R_1 : $x_1 - 5x_2 + x_3 = 0$

$x_1 - 5(8) + 1 = 0$

$x_1 - 40 + 1 = 0$

$x_1 - 39 = 0$

$\boxed{x_1 = 39}$

Hense $\{x_1 = 39, x_2 = 8, x_3 = 1\}$

As the solution of the given system exists, therefore it is consistent.

QUESTION : 2

Find the inverse of

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4th\text{-}ID \\ 5 & -2 & 7 \end{bmatrix}$$ by adjoint method.

SOLUTION:-

My I.D is = 14596

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 9 \\ 5 & -2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{Adj(A)}{|A|} \longrightarrow (*)$$

$$Adj\, A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$= \begin{bmatrix} 11 & 31 & 1 \\ -38 & -4 & 26 \\ 41 & -17 & -11 \end{bmatrix}^t$$

$$\text{adj } A = \begin{bmatrix} 11 & -38 & 41 \\ 31 & -4 & -17 \\ 1 & 26 & -11 \end{bmatrix} \rightarrow (1)$$

$$|A| = a_{i1}A_{i1} + a_{i2}A_{i2} - - - - - -$$

$$|A| = 3(11) + 4(31) + 5(1)$$

$$= 33 + 124 + 5 = 162 \rightarrow (2)$$

By putting (1) and (2) in (*)

We get: $A^{-1} = \frac{1}{162}\begin{bmatrix} 11 & -38 & 41 \\ 31 & -4 & -17 \\ 1 & 26 & -11 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} 11/162 & -38/162 & 41/162 \\ 31/162 & -4/162 & -17/162 \\ 1/162 & 26/162 & -11/162 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 11/162 & -19/162 & 41/162 \\ 31/162 & -2/81 & -17/162 \\ 1/162 & 13/81 & -11/162 \end{bmatrix}$$ ANS

QUESTION: 3

Solve the following equation by Gauss - Jordan method.

SOLUTION:-

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix} \xrightarrow{R_1 = 1/2 R_1} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix}$$

$$\xrightarrow[R_3 - 3R_1]{R_2 - R_1} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \xrightarrow{R_2 \to 1/2 R_2} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -3 & -13 \end{bmatrix}$$

$$\xrightarrow{R_3 + 2R_2} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} \xrightarrow[R_3 = -1/3 R_3]{R_1 \to R_1 - R_2} \begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

$$R_1 - 2R_3 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \Rightarrow \begin{array}{l} x = 1 \\ y = 2 \\ z = 3 \end{array}$$

QUESTION : 4

Show that this matrix is diagnosible.

Solution :-

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ 2 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 1 \\ -5 & 3 & 2 \\ 4 & 2 & -2 \end{bmatrix} \quad R_1 \longleftrightarrow R_3$$

$$= \begin{bmatrix} 1 & 2 & 1/2 \\ -5 & 3 & 2 \\ 4 & 2 & -2 \end{bmatrix} \quad 1/2\, R_1$$

$$= \begin{bmatrix} 1 & 2 & 1/2 \\ 0 & 13 & 9/2 \\ 0 & 6 & 4 \end{bmatrix} \quad \begin{matrix} R_2 + 5R_1 \\ R_3 - 4R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 2 & 1/2 \\ 0 & 13 & 9/2 \\ 0 & 0 & -25/13 \end{bmatrix} \quad R_3 - \tfrac{6}{13} R_2$$

$$= \begin{bmatrix} 1 & 2 & 1/2 \\ 0 & 13 & 9/2 \\ 0 & 0 & 1 \end{bmatrix} \; -13/25 \, R_3$$

$$= \begin{bmatrix} -1 & -2 & 0 \\ 0 & -13 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; \begin{matrix} R_2 - 9/2 \, R_3 \\ R_1 - 1/2 \, R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -13 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; R_1 - 2/13 \, R_2$$

QUESTION: 5

Determine if the following s- - -
- - - - - set?

SOLUTION:-

$$3x_1 + 5x_2 - 4x_3 = 0$$

$$-3x_1 - 25x_2 + 4x_3 = 0$$

$$6x_1 + x_2 - 8x_3 = 0$$

$$\underbrace{\begin{bmatrix} 3 & 5 & -4 \\ -3 & 25 & 4 \\ 6 & 1 & -8 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{B}$$

First we find $|A|$.

$$|A| = \begin{bmatrix} 3 & 5 & -4 \\ -3 & 25 & 4 \\ 6 & 1 & 8 \end{bmatrix} = 3\begin{vmatrix} 25 & 4 \\ 1 & -8 \end{vmatrix} - 5\begin{vmatrix} -3 & 4 \\ 6 & -8 \end{vmatrix}$$

$$+ (-4)\begin{vmatrix} -3 & 25 \\ 6 & 1 \end{vmatrix}$$

$$|A| = 3(-200-4) - 5(24-24) - 4(-3-150)$$

$$|A| = 3(-204) - 5(0) - 4(-153)$$

$$|A| = \cancel{-612} - 0 + \cancel{612}$$

$$|A| = 0$$

Hence the solution is non-devisible we have to find the angmented matrix will be.

$$[A/B] = \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & 25 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right]$$

$$[A/B] = \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 20 & -8 & 0 \\ 0 & 9 & 0 & 0 \end{array}\right] \begin{matrix} R_2 - R_1 \\ R_3 - 2R_1 \end{matrix}$$

$$[A/B] = \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 20 & -8 & 0 \\ 0 & 0 & -18/5 & 0 \end{array}\right] R_3 - 9/20 R_2$$

$$[A/B] = \left[\begin{array}{ccc|c} 3 & 5 & -6 & 0 \\ 0 & 20 & -8 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] -5/18 R_3$$

$$[A/B] = \left[\begin{array}{ccc|c} 3 & 5 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_2 + 8R_3 \\ R_1 + 4R_3 \end{array}$$

$$[A/B] = \left[\begin{array}{ccc|c} -3 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] R_1 - 1/4 R_2$$

R_3 : $x_3 = 0$, $20x_2 = 0 \Rightarrow x_2 = 0$

$-3x_1 = 0 = x_1 = 0$

Solution set = $\{0, 2, 4\}$

(10) 14196

QUESTION # 6

Reduce the matrix to Normal form and find its rank.
$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

SOLUTION:-

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$$R_3 - R_1 \Rightarrow \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1-1 & 3-3 & 4-4 & 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$R_2 - 3R_1 = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3-3 & 9-9 & 12-12 & 3-9 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$R_3 + R_1 \Rightarrow \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ ANS}$$

$$\boxed{\text{RANK} = 2}$$