Iqra National University, Peshawar
Department of Electrical Engineering

IQRA NATIONAL UNIVERSITY

Mid – Term Examination summer2020
Date:22/8/2020

Course Code: MTH 203 **Course Title:** Differential equation
Prerequisite: CALCULUS **Instructor:** HIMAYATULLAH
Module: 3 **Program:** BEE **Total Marks:** 30 **Time Allowed:** _____

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	(a)	Estimate the general solution of $y' = (x+2)y^2$.	Marks 5 PLO1 C2
	(b)	Estimate the general solution of $y' = (y+9x)^2$	Marks 5 PLO1 C2
Q2	(a)	Estimate the general solution of $x^3dx + y^3dy = 0$	Marks 10 PLO1 C2
Q 3	(a)	Find the general solution $4y'' - 20y' + 25y = 0$	Marks 5 PLO1 C2
	(b)	Estimate general solution of $4y'' - 6y' - 7y = 0$.	Marks 5 PLO1 C2

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Subject Differential equation

Q2 Estime the general solution of

$$y' = (x+2)y^2$$

Sol:

Sol:- $y' = (x+2)\,y^2$

using seperate variable method as

$$y' = dy/dx$$

So,

$$dy/dx = (x+2)y^2$$

Now seperate variable

$$dy/y^2 = (x+2)\,dx$$

Taking $\int$ both side.

$$\int dy/y^2 = \int (x+2)\,dx$$

$$\int y^{-2}\,dy = \int x\,dx + \int 2\,dx$$

$$y^{-2+1}/_{-2+1} = x^2/_2 + 2x + c$$

$$y^{-1}/_{-1} = x^2/_2 + 2c + c$$

$$-1/y = x^2/_2 + 2x + c$$

multiply both by 2.

$$-2/y = x^2 + 4x + c$$

$$-2 = (x^2 + 4x)\,y + c$$

$$y = -2/x^2 + 4x + c$$

$$y = -1/x^2/_2 + 4x/_2 + c$$

$$\boxed{y = -1/x^2/_2 + 2x + c}$$

Ans.

Q1

(b) Estimate the general solution of $y' = (y+9x)^2$

Sol:- $y' = (y+9x)^2$

Let $y+9x = u$. So $dy/dx = u^2$.

Taking derivate

$$d/dx\,(y+9x) = d/dx\,(u)$$

$$dy/dx + d/dx\,(9x) = du/dx$$

$$dy/dx + 9 = du/dx.$$

$$dy/dx = du/dx - 9$$

putting $dy/dx = u^2$

we get

$$u^2 = du/dx - 9$$

$$u^2 + 9 = du/dx$$

seperate.

$du = (u^2 + 9)\, dx$

Taking $\int$ on both side

$$\int du/u^2+9 = \int dx$$

$$\int 1/3^2 + u^2$$

$$du = \int dx$$

$$1/3 \tan^{-1}(u/3) = x + c$$

$$\tan^{-1}(u/3) = 3x + 3c$$

$$u/3 = \tan(3x + c)$$

$$u = 3\tan(3x + c)$$

$$y + 9x = 3\tan + c$$

Ans

Q2 Estimate the general solution of $x^3\,dx + y^3\,dy = 0$.

Sol:- $x^3\,dx + y^3\,dy = 0$

$M\,dx + N\,dy = 0$

$M = x^3$, $N = y^3$

$\frac{\partial M}{\partial y} = 0$, $\frac{\partial N}{\partial x} = 0$

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So exact

$u = \int M\,dx + k(y)$

$u = \int x^3\,dx + k(y)$

$\Rightarrow \frac{x^4}{4} + \frac{x^4}{4} + k(y) - (1)$

$\frac{\partial u}{\partial y} = 0 + d/dy\ k(y)$

$\frac{\partial u}{\partial y} = d/dy\ k(y)$

Since $\frac{\partial u}{\partial y} = N = y^3$

$\Rightarrow y^3 = d/dy\ k.$

Hence exact

$$u = \int m\,dx + k(y)$$

$$u = \int (x-y)\,dx + k(y)$$

$$u = \frac{x^2}{2} - xy + k(y) \quad \text{—①}$$

$$\frac{\partial u}{\partial y} = 0 - x + d/dy\ k(y)$$

$$\frac{\partial u}{\partial y} = -x\ d/dy\ k(y)$$

Since $\frac{\partial u}{\partial u} = N = -(x-y)$

So,

$$-(x-y) = -x + d/dy\ k(y)$$

$$y = d/dy\ k(y)$$

$$\int d(cy) = \int y\,dy$$

$$c(y) = y^2/2 + c_1 \quad \text{— ii}$$

Put eq (ii) in eq (i)
we get

$$u = \frac{x^2}{2} - xy + y^2/2 + c_1$$

$$\frac{x^2}{2} - xy + y^2/2 = c_2 - c_1$$

$$\boxed{x^2/2 + y^2/2 - xy = c}$$

Ans

— x — x — x — x —

Q3 (a) Find the general solution of $4y'' - 20y' + 25y = 0$

Sol:- $4y'' - 20y' + 25y = 0$

First we find root through auxilary equation.

So, $4m^2 - 20m + 25 = 0$ auxilary equation.

Where coffiecent

$a = 4 \quad b = 20$

$c = 25$

using Quadratic formula

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)}$$

$$m = \frac{20 \pm \sqrt{400 - 400}}{8}$$

$$m = \frac{20 \pm \sqrt{0}}{8}$$

$$m = \frac{20}{8} \pm 0$$

$$m1 = \frac{20 + 0}{8} = 20/8 = 2.5$$

$$m2 = \frac{20 - 0}{8}$$

$m2 = 2.5$

So root are repeted.

$y = k_1 e^{mx} + k_2 x e$

$y_1 = k_1 e^{2.5x} + k_2 e^{2.5x}$.

$\boxed{e^{2.5x}(k_1 + k_2 x)}$

Ans.

— x — x — x — x —

Q3

(b) Estimate general solution of $4y'' - 6y' - 7y = 0$.

Sols. $4y'' - 6y' - 7y = 0$

so auxilary equation

$4m^2 - 6m - 7 = 0$

solve it.

$a = 4 \quad b = -6 \quad c = -7.$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(-7)}}{2(4)}$

$$= -6 \pm \frac{\sqrt{36+112}}{8}$$

$$= \frac{6 \pm \sqrt{148}}{8}$$

$$m_1 = \frac{6+12}{8} \qquad m_2 = \frac{6-12}{8}$$

$$m_1 = 2.25 \qquad m_2 = 1.5$$

So root are real & dx

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}.$$

$$\boxed{y = c_1 e^{2.25x} + c_2 e^{1.5x}}$$

— x — x — x —