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Subject # Mechanics of solid(II)

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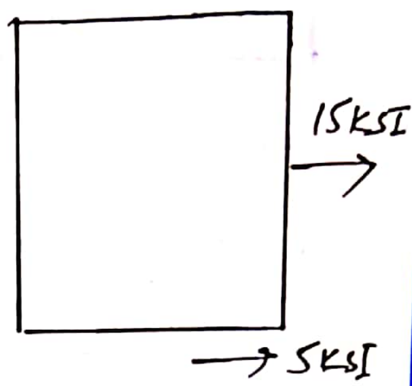
Q#01 (a)

Ans

Given Data



$\sigma_x = 15\,ksi$

$\sigma_y = 0$

$\tau_{xy} = -5\,ksi$

Required Data:-

ⓐ principal stress

ⓑ Max-plan shear stress

ⓒ Average Normal stress

Solution

ⓐ principal stress

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{15+0}{2} \pm \sqrt{\left(\frac{15-0}{2}\right)^2 + (-5)^2}$$

$$\sigma_{1,2} = 7.5 \pm 9.01$$

$$\Rightarrow \boxed{\sigma_1 = 16.51\,KSI}$$

$\sigma_2 = 7.5 - 9.01$

$$\boxed{\sigma_2 = -1.51 \text{ ksi}}$$

Now we find orientation

:- we know that

$$2\theta_2 = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$2\theta_2 = \frac{-5}{(15-0)/2}$$

$$\boxed{\theta_2 = -0.33}$$

⇒ Now we check angles which goes with which principle stress -

we know that

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{15+0}{2} + \frac{15-0}{2} \cos 2(-0.33) + (-5) \sin 2(-0.33)$$

$$= \frac{15}{2} + \frac{15}{2}(0.99) + (-5)(-0.72)$$

$$= 14.925 + 0.6$$

$$\boxed{= 15.525}$$

(b) Max - plan shear stress:-

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \tau_{max} = \sqrt{\left(\frac{15-0}{2}\right)^2 + (-5)^2}$$

$$\boxed{= \tau_{max} = 9.01 \text{ KSI}}$$

(c) Average Normal stresses

$\sigma_x = 15$ KSI

$\tau_{xy} = -5$KSI

$\sigma_y = 0$

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{15+0}{2}$$

$C = 7.5$ksi

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{15-0}{2}\right)^2 + (-5)^2}$$

$R = 9.01$

Scale = 1 small box = 0.5 KSI

Now we find orientation

we know that

$$\tan 2\theta = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$= \frac{(15-0)/2}{-5}$$

$\tan 2\theta = +1.5$

$2\theta = \tan^{-1}(+1.5)$

$2\theta = 56$

$\boxed{\theta = 28}$

we know that

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{15-0}{2} \sin 2(28) + (-5) \cos 2(28)$$

$$= -7.5(0.82) - 2.8$$

$$\boxed{= -8.95}$$

Q#01 Part (b)

Average Normal stresses

$\sigma_x = 15 \text{ KSI}$

$\tau_{xy} = -5 \text{ KSI}$

$\sigma_y = 0$

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{15+0}{2}$$

$$\boxed{C = 7.5 \text{ KSI}}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{15-0}{2}\right)^2 + (-5)^2}$$

$$R = 9.01$$

Scale = 1 small box = 0.5 KSI

Now we find orientation

we know that

$$\tan 2\theta = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\theta = \frac{(15-0)/2}{-5}$$

$$\tan 2\theta = +1.5$$

$$2\theta = \tan^{-1}(+1.5)$$

$$2\theta = 56$$

$$\boxed{\theta = 28}$$

We know that

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

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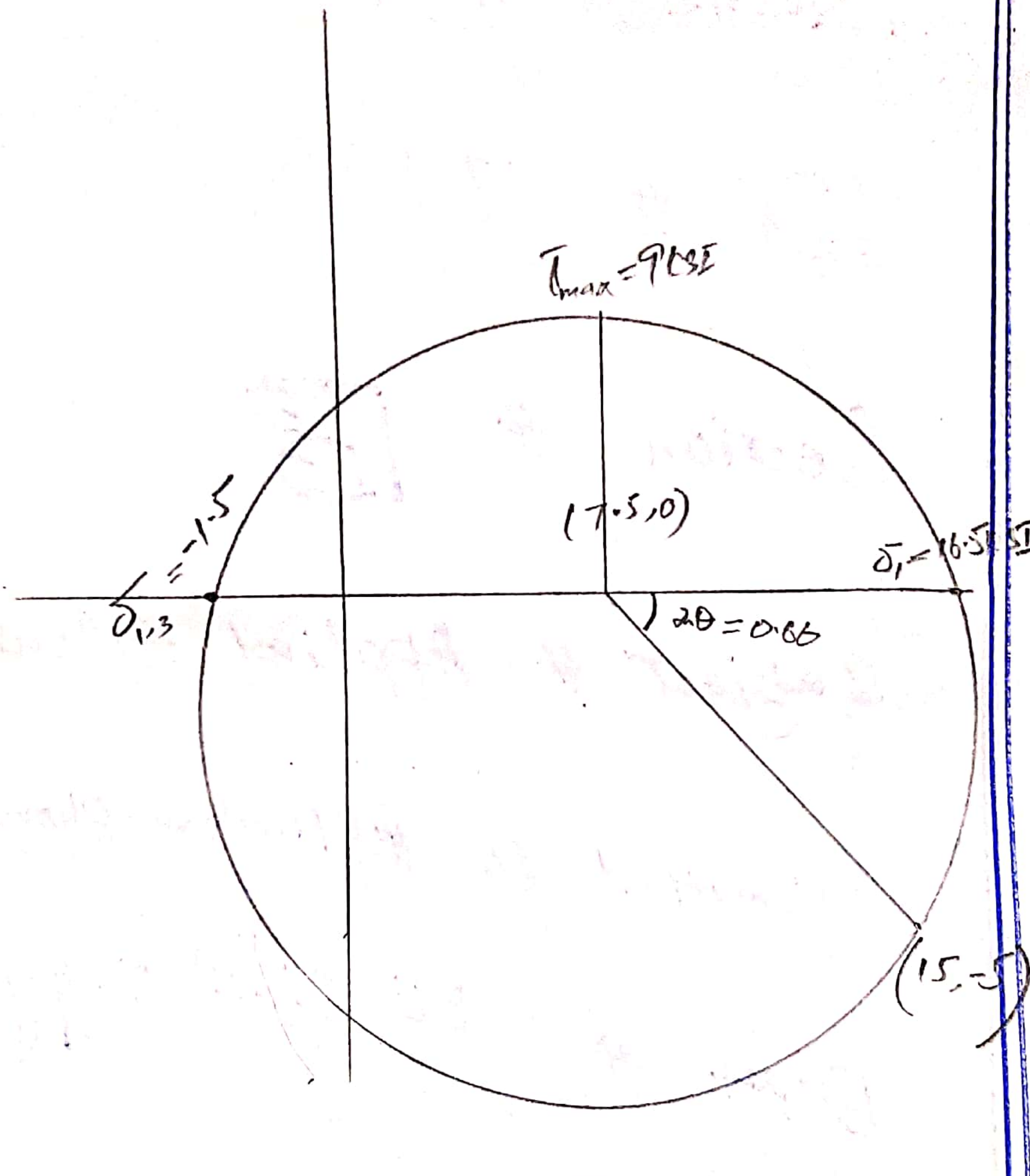


Q#02

Ans

$$\tau_{\substack{max\\abs}} = \frac{\sigma_1}{2} = \frac{32}{2} = 16\ MPa$$

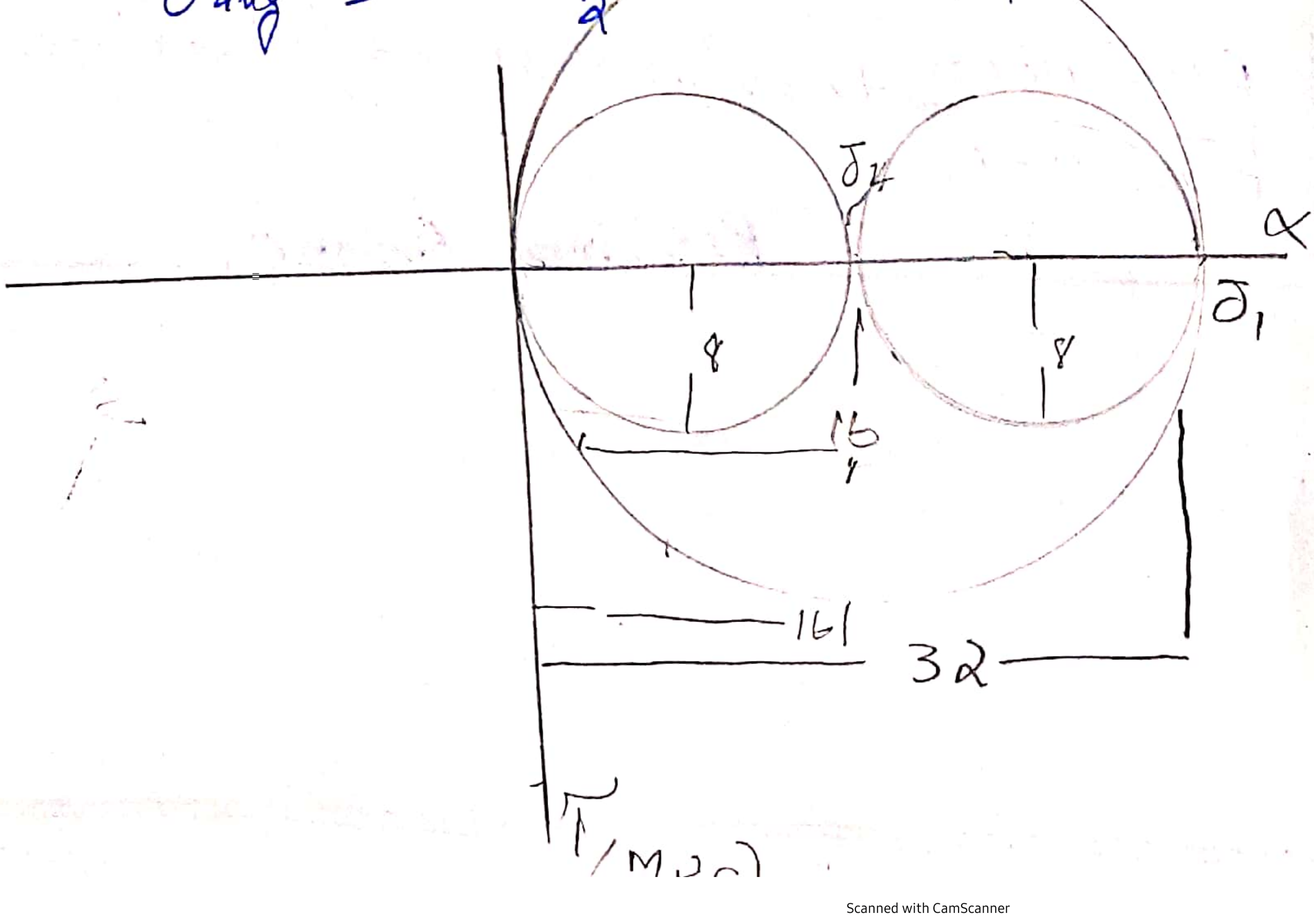
$$\sigma_{avg} = \frac{32+0}{2} = 16\ MPa$$

By Compression The maximum in plane shear stress can be determined from The Mohr's circle drawn b/w $\sigma_1 = 32\ MPa$ and $\sigma_2 = 16\ MPa$

This gives a value of

$$\tau_{max\,plane} = \frac{32-16}{2} = 8\ MPa.$$

$$\sigma_{avg} = \frac{32+16}{2} = 24\ MPa$$



Q#3 ⇒ What are the main stresses responsible for failure of Ductile & brittle materials? Name of 2 failure theories for ductile materials and 2 for Brittle materials-

Ans ⇒ **Stresses are responsible for failure of Ductile and Brittle Materials:-**

⇒ Ductile materials are limited by their shear strength - Ductile materials usually fails because the shear stresses exceeds the strength of ductile material-

⇒ Brittle materials are limited by their tensile strengths - Brittle materials fails when the tensile stresses exceeds the strength of material-

Two Failure Theories for Ductile Materials

1 :- Maximum Shear Stress Theory

⇒ According to this theory
" Failure in Ductile materials
occurs when the maximum shear
stress in the part exceeds the
shear stress in a tensile test
specimen (of the same material) at yield-
The maximum shear stress can be
determined by observing Mohr's
circle for the element-
The result Indicate that

$$\tau_{max} = \sigma_Y / 2$$

Maximum - Distortion - Energy Theory

⇒ It was stated in Sec 3.5 that
an external loading will deform
a material, causing it to
store energy Internally throughout
its volume - The energy per unit
volume of material is called
the strain energy density -

and if The material is subjected to a uniaxial stress The strain energy density becomes

$$u = \frac{1}{2} \sigma \epsilon$$

⇒ **Two Failure Theory For Brittle Materials**

① **Maximum Normal Stress Theory**

⇒ According To This Theory " A Brittle material will fail when maximum tensile stresses in The material reaches a value That is equal to ultimate normal stress The material can sustain when it is subjected to simple Tension -

Mohr's Failure ~~Cri~~ Criterion

:- In some brittle materials tension and compression properties are different - when This occurs a criterion based on use The use of Mohr's circle may be used to predict Failure -

Failure criterion

:- To apply it, one first perform Three Test on The material - A uniaxial tensile test and uniaxial compressive Test are used to determine The ultimate tensile $(\sigma_{ult})_t$ and compressive stresses, and $(\sigma_{ult})_c$ respectively. Also a torsion test is performed to determine the material's ultimate shear stress (τ_{ult}). Mohr's circle for each of These stress conditions is plotted as shown in Figure - These Three circle are contained in a "Failure envelop" Indicated by extrapolated colored curve That is drawn Tangent to all circles-