# Question NO 1

State difference and Similarity between gradient and divergence Providing relevent example.

## Ans

(i) The main difference betwen gradient and divergence is "The gradient is the directional rate of change of Scalar Function in Rn."

Whereas the divergence measure the amount of output vs input for unit volume of vector valued "Flow" in Rn.

(2)

Gradient means the slope of a Line or such things as mountain sides.

Divergence means how much a series of numbers moves from a given number

(3) A glib answer is that "gradient" is a vector and "divergence" is a scaler.

(4) The divergence (of a vector field) Provides a measure of how much "flux" (or flow) is passing through a surface surrounding a point in the field (positive for flow away from that point, negative for flow toward, zero for no net flow).

Similarly difference between gradient and divergence.

The gradient is a vactor Field with the Part derivatives of a Scalar Field. While the divergence is a Scalar Field with the Sum of derivatives of a vactor Field.

The result of a gradient of is a vactor Field.

The result of a divergence is Scalar Field.

For example :-

The gradient of the distance from a given point is a vactor field of unit length vactors pointing away from the givent.

Next for example :-

The divergence of a flow with no source or sink is 0. If there is a net source. The divergence is positive and if there is a net sink the divergence is nagative.

## Question No 2

Find gradient of function F at Point (1, 1, 2) for $F = (x^3 + y^3 z)$

**Solution**

Point (1, 1, 2)

$$F = x^3 + y^3 z$$

$$\vec{\nabla} F = \frac{\partial F}{\partial x} i + \frac{\partial F}{\partial y} j + \frac{\partial F}{\partial z} K$$

$$\vec{\nabla} F = \frac{\partial (x^3 + y^3 z)}{\partial x} i + \frac{\partial (x^3 + y^3 z)}{\partial y} j \; \frac{(x^3 + y^3 z)}{\partial z} K$$

$$\text{Grad}(F) = \vec{\nabla} F = 3y^2 z i + 3x^2 z j + 3y^2 z K$$

At Point. (1, 1, 2)

$$\text{Grad } F = \vec{\nabla} F = 6i + 6j + 6K$$

# Question No 3

## Solution

Let's Compute the divergence and there isn't much to do other than run through the formula.

$$div \vec{F} = \nabla \cdot \vec{F} = \frac{\partial}{\partial x}(x^2 y) + \frac{\partial}{\partial y}(3x - z^3) + \frac{\partial}{\partial z}(4y^2) = \boxed{2xy}$$

Be Careful to watch for minus Signs in front of any of the vector Components (2nd Component in this Case) it is easy to get in a hurry and miss them.

## Step 2

The Curl is a little more work but still just formula work so here is the Curl.

Solution



$$\vec{F} = \nabla \times \vec{F} \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial y}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & 3x - z^3 & 4y^2 \end{vmatrix}$$

$$= \frac{\partial}{\partial y}(4y^2)\vec{i} + \frac{\partial}{\partial z}(x^2 y)\vec{j} + \frac{\partial}{\partial x}(3x - z^3)\vec{k}$$

$$- \frac{\partial}{\partial y}(x^2 y)\vec{k} - \frac{\partial}{\partial x}(4y^2)\vec{j} - \frac{\partial}{\partial z}(3x - z^3)\vec{i}$$

$$= 8y\vec{i} + 3\vec{k} - x^2\vec{k} + 3z^2\vec{i}$$

$$= \boxed{(8y + 3x^2)\vec{i} + (3 - x^2)\vec{k}}$$

# Question No '4'

The relationship between electric potential and Potential difference with example.

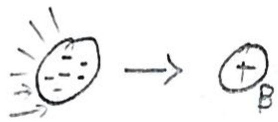
**Ans**

(i) The electrical potential is defined as the amount of work done to carrying a unit charge from one Point to another in an electric field.

In other words, the Potential difference is defined as the difference in the electric potential of two charged bodies.

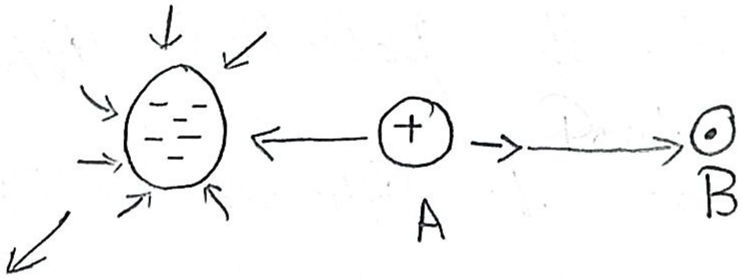
First we explain Electric potential and potential Difference.

Those the imaging a larg nagetive Charged Fixed in Space and the positive Charged neared by the nagetive.

The nagetive Charge we Know attreacts the positive Charge 



Then The nagetive Charge producing the electric Field arronding ~~atrac~~ nagetive Charge



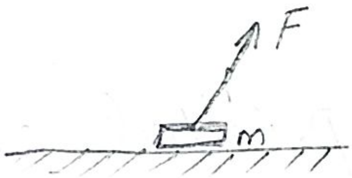


Electric Field.

If we move the point B and
if we push exert force So
the work done

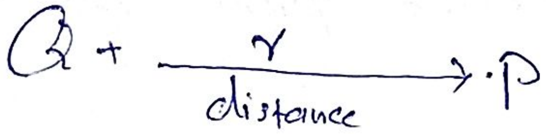
Work = F . X

we expand energy so the energy
goes the potential energy.
if we Supose the Surface of
earth.




Electric potential

$V = K \frac{Q}{r}$ Q+ Single Charge

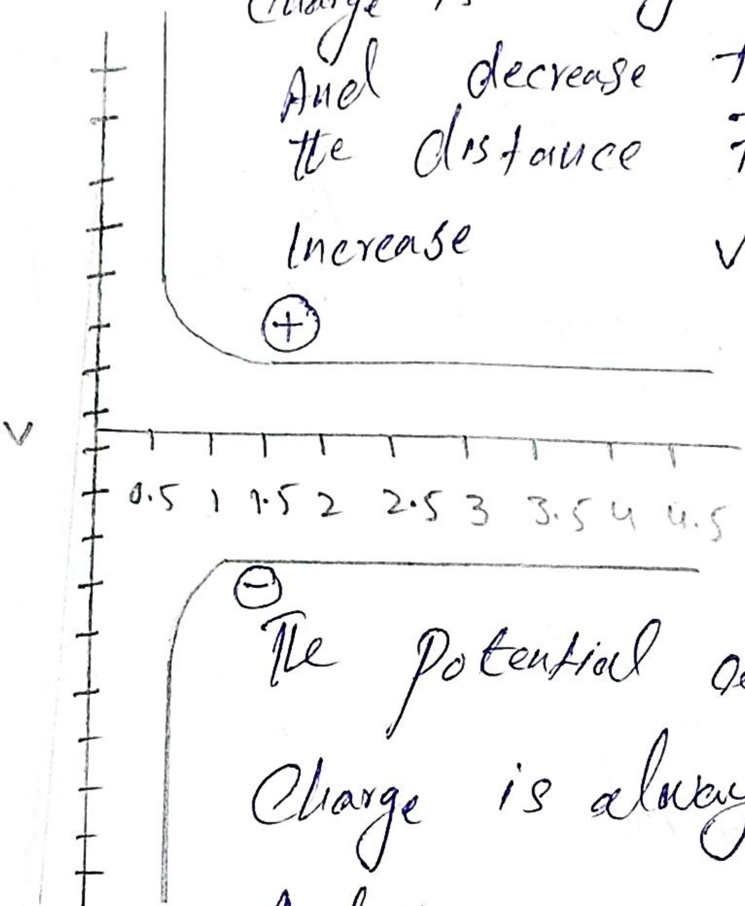



electric Potential energy Per Unit of Charge
V = Potential

$V = \frac{U_e}{q} = \frac{J}{C}$ = Volts

$V = \frac{KQ\cancel{x}/r}{\cancel{x}}$ $\quad V = K\frac{Q}{r}$

The potential around Postive Charge is always positive. And decrease to Zero as the distance From the Charge Increase $V = K\frac{Q}{r}$




The potential around Negative Charge is always nagetive. And increase to Zero as the distance From the Charge Increase.

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# Question No 5

**Ans**

The integral expression for the work done in moving a point Charg $Q$ from one position to another. without using vactor analysis we should have to write.

$$W = -Q \int_{\cancel{\text{limit}}\ \text{initial}}^{\text{final}} E_L\, dL$$

where $E_L$ = Component of E along $dL$

**Now**

Let Suppose the initial position B to a final A and uniform electric field selected for simplicity

Now the path is divided into six segment. $\Delta L_1, \Delta L_2 \ldots\ldots \Delta L_6$ and the involved in moving a charge Q B to A is then approximately

$$\Rightarrow w = -Q(E_{L_1} \cdot \Delta L_1 + E_{L_2} \cdot \Delta L_2 \ldots\ldots + E_{L_6} \cdot \Delta L_6)$$

Next if we assumed a uniform field.

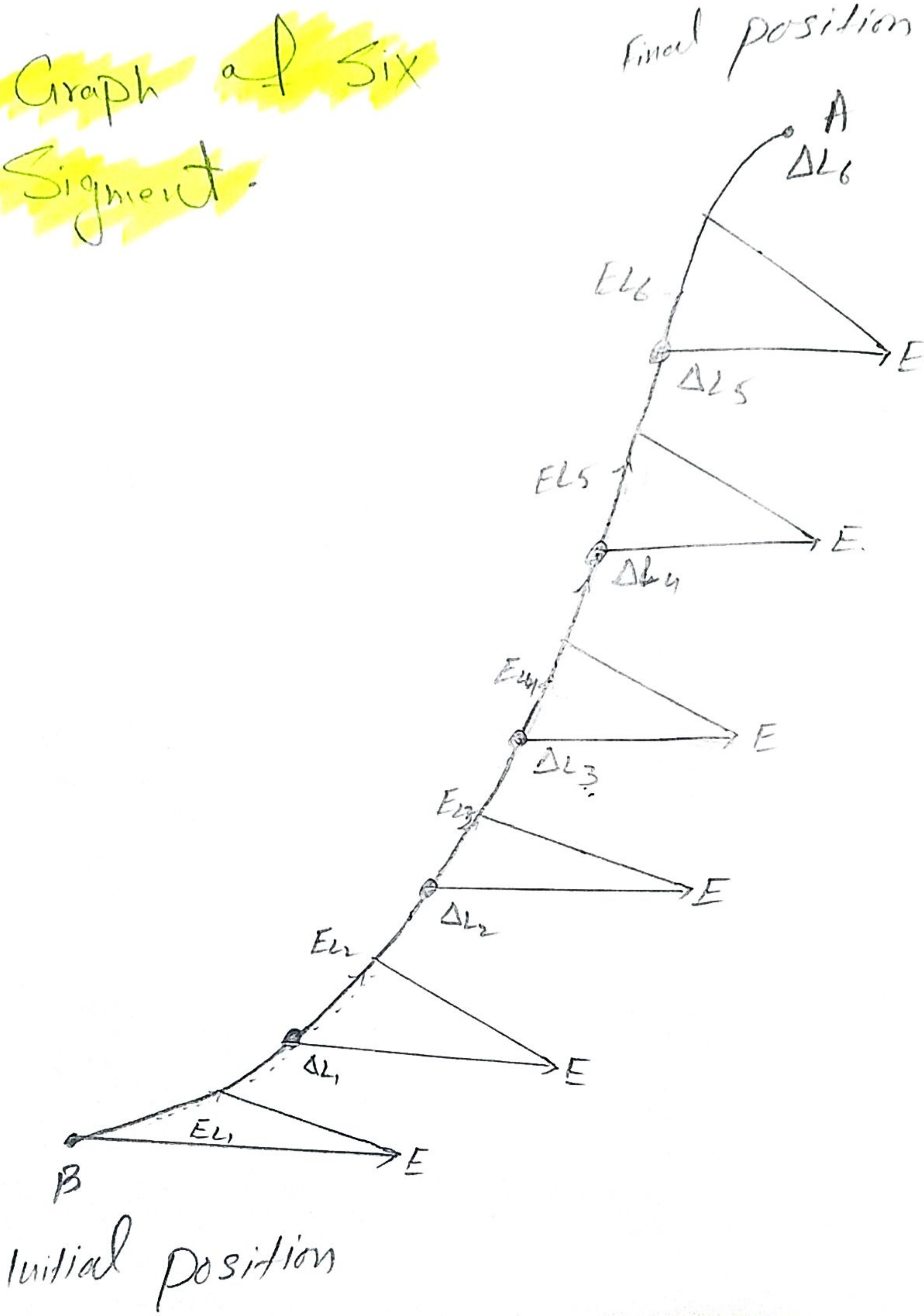
$$\Rightarrow \quad E_1 = E_2 = E_3 = E_4 = E_5 = E_6$$

$$\Rightarrow \quad W = -QE \cdot (\Delta L_1 + \Delta L_2 + \Delta L_3 + \ldots\ldots \Delta L_6)$$

Now the sum is just the vector directed from the initial point B to the final point A

$L_{BA}$ $\Rightarrow$ $W = -QE \cdot L_{BA}$ (Uniform E)

# Graph of Six Signment.

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