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SECTION # "B"

SUBJECT # DIFFERENTAIL EQUATION.

SEMESTER # 4.

Q_1

QUESTION # 1

Objectives.

(i) The order of matrix A is $m \times p$ and the order of B is $p \times n$. then the order of matrix AB is?

Solution $AB = mn$

(ii) The number of non-zero rows in an Echelon form?

Solution Rank of the matrix

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a Singular matrix than $a = ?$

Solution $a = 8$.

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Solution $|A| = 3$.

(v) The matrix $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ is ?

Solution: Scalar matrix

(vi) Solution of $\frac{dy}{dx} + 2xy = y$?

Solution: $y = Ce^{x(1-x^2)}$.

(vii) The order & degree of differential Equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ is?}$$

Solution: order $= 1$
degree $= 3$.

(viii) The order & degree of differentail equation

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is?}$$

Solution: order $= 2$
degree $=$ undefined.

(ix) The differential Equation $2\frac{dy}{dx} + x^2y = 2x + 3, y(0) = 5$ is ?

Solution:- $2y + x^3/3 = x^2 + 3x + 10.$

(x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Solution:- $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a).$

Q2 (i) Express the determinat

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

As the product of factors which are linear in a, b, c?

Solution:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2b^3c - a^3b^2c$$

$\Rightarrow$ Taking Common (abc)

$$= abc\,(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$= abc\,[bc(c-b) - ac(c-a) + ab(b-a)]$$

ANSWER.

Q2 (ii) Find the Eigen value.

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:-

$$\begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix}$$

characteristic equ $|A-\lambda I|=0$.

$$\begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinant,

$|A-\lambda I|=0$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0.$$

Expand by R_1, (6)

$$2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - 1(-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \text{ -(i)}$$

Again,

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expond by R_1,

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\left[(3-\lambda)(2-\lambda) - (-1)(-1)\right] + 1 \left[(-1)(2-\lambda) - (-1)(-1)\right] - 1\left[(-1)(-1) - (-1)(3-\lambda)\right]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + 1(-2+\lambda-1) - 1(1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= -\lambda^3 + 8\lambda^2 - 18\lambda + 8 \rightarrow (a)$$

Now,

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$ Expand By C_1.

$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)$

$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$

$= -\lambda^2+6\lambda-8 \rightarrow$ (b)

$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$

Expand by C_1

$\Rightarrow -1\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$

$\Rightarrow -[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)$

$= -(3-\lambda+\lambda^2-5\lambda+5)$

$= -\lambda^2+5\lambda-5-3+\lambda$

$= -\lambda^2+6\lambda-8$ — (c)

Now,

putting all the equs

in equ (i)

0

1-

λ

equ ① $\Rightarrow$

$\Rightarrow (2-\lambda)(-\lambda^3+8\lambda^2-18\lambda+8)-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8=0$

$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda-8-$
$\lambda^2+6\lambda-8=0$

$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+18\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda+6\lambda+6\lambda$
$+\cancel{16}-\cancel{8}-\cancel{8}=0$

$\Rightarrow \quad \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

Now,
solve by upper equation By
Synthetic Equation

	1	-10	32	-32
2			-16	32
	1	-8	16	0

So we get

$(\lambda-2)(\lambda^3-8\lambda+16\lambda)=0$

$\Rightarrow \lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$\lambda=0 \quad | \quad \lambda-2=0$
$\lambda=2$

$\lambda^2-8\lambda+16=0$
$\lambda^2-4\lambda-4\lambda+16=0$
$\lambda(\lambda-4)-4(\lambda-4)=0$
$(\lambda-4)=0 \quad (\lambda-4)=0$
$\lambda=4 \quad \lambda=4$

So $\lambda_1 = 0$ (9)

$\lambda_2 = 2$

$\lambda_3 = 4$

$\lambda_4 = 4$

ANS.

Q3:- The rate of change in the form of differential equation is given by

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

Find the general solution at

$x=2$, $y=6$

lutions:-

Given that

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$x=2$, $y=6$

$$(x^2+3y^2)\,dx = 2xy\,dy$$

$\div$ by $2xy\,dx$

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow ①$$

let $y = Vx$

Diff

$dy = V\,dx + x\,dv$

$\div$ by dx,

$$\frac{dy}{dx} = V + \frac{x\,dx}{dx} \longrightarrow Ⓐ.$$

put Ⓐ in ①

$$V + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{V\cancel{x}} + \frac{3V\cancel{x}}{\cancel{x}}\right]$$

$$V + \frac{x\,dv}{dx} = \frac{1}{2}\left[1/V + 3V\right]$$

$\times$ing by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{V} + 3V.$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$\frac{2x\,dv}{dx} = \frac{1+v^2}{v}$$

$\times$ing By $\frac{dx}{dv}$

$$2x\,dx = \frac{1+v^2}{v}\,dx$$

$\div$ing by $v/x\,(1+v^2)$

$$\frac{2v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

"$\int$" on B.S.

$$\int \frac{2v}{1+v^2}\,dv = \int 1/x\,dx + C$$

$$\ln|1+v^2| = \ln x + \ln C$$

"e" on B.S

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

put $v = y/x$.

$$1+(y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3C \text{ --- (ii)}$$

putting $x=2$ $y=6$

in equ (ii)

eq (ii) $\Rightarrow$

$$(2)^2+(6)^2 = (2)^3 c$$

$$4+36 = 8C$$

$$\frac{40}{8} = \frac{8c}{8}$$

$$\boxed{c=5}$$

put $c=5$ in equ (ii)

equ (ii) $\Rightarrow$ $x^2+y^2 = 5x^3$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

"$\sqrt{}$" on B.S (13)

$\sqrt{y^2} = -\sqrt{x^2} \times \sqrt{(5x-1)}$

$\sqrt{y^2} = -\sqrt{x^2} \times \sqrt{(5x-1)}$

$$\boxed{y = \pm x\sqrt{5x-1}}$$

ANSWER