

Q#1 Consider the following vectors \mathbb{R}^3 .

$$v_1 = \begin{bmatrix} 101 \\ 102 \\ 103 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 102 \\ 103 \\ 104 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 103 \\ 104 \\ 105 \end{bmatrix}$$

Solution:

$$a_1 v_1 + a_2 v_2 + a_3 v_3 = 0$$

$$a_1 (6, 8, 4) + a_2 (8, 4, 9) + a_3 (4, 9, 1) = 0$$

$$(6a_1 + 8a_2 + 4a_3, 8a_1 + 4a_2 + 9a_3, 4a_1 + 9a_2 + a_3) = 0$$

$$6a_1 + 8a_2 + 4a_3 = 0$$

$$8a_1 + 4a_2 + 9a_3 = 0$$

$$4a_1 + 9a_2 + a_3 = 0$$

$$\text{Let } A = \begin{bmatrix} 6 & 8 & 4 \\ 8 & 4 & 9 \\ 4 & 9 & 1 \end{bmatrix}$$

$$|A| = 6 \begin{vmatrix} 4 & 9 \\ 9 & 1 \end{vmatrix} - 8 \begin{vmatrix} 8 & 9 \\ 4 & 1 \end{vmatrix} + 4 \begin{vmatrix} 8 & 4 \\ 4 & 9 \end{vmatrix}$$

$$= 6(4 - 81) - 8(8 - 36) + 4(72 - 16)$$

$$= 6(-77) - 8(-28) + 4(56)$$

$$= -462 + 224 + 224$$

$$|A| \neq 0$$

The system is linearly independent.

$$Q\#2 \quad \begin{bmatrix} 450 & 400 \\ 250 & 350 \\ 150 & 150 \end{bmatrix} \quad \begin{bmatrix} 1000 \\ 500 \end{bmatrix}$$

Solution:-

$$(a) = \begin{bmatrix} 450 \times 1000 + 400 \times 500 \\ 250 \times 1000 + 350 \times 500 \\ 150 \times 1000 + 150 \times 500 \end{bmatrix}$$

$$= \begin{bmatrix} 450000 + 200000 \\ 250000 + 175000 \\ 150000 + 75000 \end{bmatrix}$$

$$= \begin{bmatrix} 650000 \\ 425000 \\ 225000 \end{bmatrix}$$

Now Total of material = 650000

Total cost of labour = 425000

Total cost of overhead = 225000

(b) Linear Transformation

Let $U(F)$ and $V(F)$ are vector spaces: A mapping F such that $f: U \rightarrow V$ is a linear transformation of U into V

$$\text{if } \left. \begin{array}{l} \text{(i) } f(x+y) = f(x) + f(y) \\ \text{(ii) } f(ax) = a f(x) \end{array} \right\} \begin{array}{l} x, y \in U \\ a \in F \end{array}$$

- Properties of L-T $T: U \rightarrow F$
- (i) $f(0) = 0'$, when 0 is identity
 - (ii) $f(-\alpha) = -f(\alpha) \quad \forall \alpha \in U$
 - (iii) $f(\alpha - \beta) = f(\alpha) - f(\beta) \quad \forall \alpha, \beta \in U$

example:

$$\begin{aligned}
 a \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) &\rightarrow \begin{bmatrix} n-y \\ 2xy \\ 2m \end{bmatrix} \\
 T\left(\begin{bmatrix} 6 \\ 8 \end{bmatrix}\right) &\rightarrow \begin{bmatrix} 6-8 \\ 6+8 \\ 2(6) \end{bmatrix} = \begin{bmatrix} -2 \\ 14 \\ 12 \end{bmatrix} \\
 T\left(\begin{bmatrix} 6 \\ 8 \end{bmatrix}\right) &\rightarrow \begin{bmatrix} -2 \\ 14 \\ 12 \end{bmatrix} \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \rightarrow \begin{bmatrix} n-y \\ n-y \\ y \end{bmatrix}
 \end{aligned}$$

eg

$$T(U+V) = T(U) + T(V)$$

$$U = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad V = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right) = T\left(\begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} a_1 + b_1 - (a_2 + b_2) \\ a_1 + b_1 + (a_2 + b_2) \\ 2(a_1 + b_1) \end{bmatrix} = \begin{bmatrix} a_1 + b_1 - a_2 - b_2 \\ a_1 + b_1 + a_2 + b_2 \\ 2a_1 + 2b_1 \end{bmatrix}$$

example:

$$T(U+V) = T(U) + T(V)$$

$$U = \begin{bmatrix} 6 \\ 8 \end{bmatrix}, \quad V = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 6 \\ 8 \end{bmatrix} + \begin{bmatrix} 4 \\ 9 \end{bmatrix}\right) = T\left(\begin{bmatrix} 6+4 \\ 8+9 \end{bmatrix}\right)$$

$$= T\left(\begin{bmatrix} 10 \\ 17 \end{bmatrix}\right) = T\left(\begin{bmatrix} 10-17 \\ 10+17 \\ 2(10) \end{bmatrix}\right)$$

$$= T\left(\begin{bmatrix} -7 \\ 27 \\ 20 \end{bmatrix}\right)$$

2nd condition:

$$T(cU) = cT(U)$$

$$\text{let } U = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = T\left[c \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right] = T\left[\begin{bmatrix} c \cdot a_1 \\ c \cdot a_2 \end{bmatrix}\right]$$

$$= \begin{bmatrix} ca_1 - ca_2 \\ ca_1 + ca_2 \\ 2ca_1 \end{bmatrix}$$

Q3 ~~Part~~ Part

Let V be a set where elements are called vectors and F be a field, where elements are called scalars. The set V is called vector space or linear space or linear vector space over field F if $V(F)$ satisfy the following condition.

- (i) $(V, +)$ is a abelian group w.r.t internal composition.
 (ii) $V(F)$ is closed under scalar multiplication w.r.t " \cdot ".
 i.e $\forall a \in F, \forall \alpha \in V \Rightarrow a \cdot \alpha = \alpha \cdot a \in V$

(iii) $(V, +, \cdot)$ satisfy

$$\text{(i) } a \cdot (\alpha + \beta) = a\alpha + a\beta \quad \forall a \in F \text{ \& } \alpha, \beta \in V$$

$$\text{(ii) } (a+b) \alpha = a\alpha + b\alpha \quad \forall a, b \in F \text{ \& } \alpha \in V$$

$$\text{(iii) } \left. \begin{array}{l} a(b\alpha) = (ab)\alpha \\ 1 \cdot \alpha = \alpha \end{array} \right\} \begin{array}{l} \forall 1, a, b \in F \\ \alpha \in V. \end{array}$$

(a) \cdot is a vector space because it's satisfy all the condition of a vector space.

(b) It is not a vector space because it's not satisfy the closure property of a vector space.

Determinants Let $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a 2×2 matrix.

$$A = \begin{pmatrix} ID1 & ID1 & ID2 \\ ID2 & ID3 & ID2 \\ ID4 & ID1 & ID5 \end{pmatrix}$$

Question:-

part a

The following matrix have inverse exist

$$M_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

part b

$$M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

part c

$$M_1 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

part d

$$A = \begin{bmatrix} 6 & 6 & 6 \\ 8 & 4 & 9 \\ 9 & 6 & 0 \end{bmatrix}$$

$$|A| = 6 \begin{vmatrix} 4 & 8 \\ 6 & 0 \end{vmatrix} - 6 \begin{vmatrix} 8 & 8 \\ 9 & 0 \end{vmatrix} + 6 \begin{vmatrix} 8 & 4 \\ 9 & 6 \end{vmatrix}$$

$$= 6(0 - 48) - 6(0 - 72) + 6(48 - 36)$$

$$= -288 + 432 + 72 = \underline{216} \text{ Ans}$$