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Subject : Differential Equation.

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Q1. $4y'' - 20y' + 25y = 0$

Soln:

Here $4y'' - 20y' + 25y = 0$.

$a = -20$, $b = 25$.

So,

$4\lambda^2 - 20\lambda + 25 = 0$.

$4\lambda^2 - 10\lambda - 10\lambda + 25 = 0$

$2\lambda(2\lambda - 5) - 5(2\lambda - 5) = 0$.

$(2\lambda - 5)(2\lambda - 5) = 0$.

$2\lambda - 5 = 0$.

$\Rightarrow \boxed{\lambda = 5/2}$

So, roots are real and equal

$y = (c_1 + c_2 x) e^{\lambda x}$.

$\boxed{y = (c_1 + c_2 x) e^{5/2 x}}$ Ans

P-2

(Q3) $y'' + 2y' + y$

$y(0) = 4, y'(0) = -6.$

Soln:

$y'' + 2y' + y = 0.$

$\lambda^2 + 2\lambda + 1 = 0.$

$\lambda^2 + \lambda + \lambda + 1 = 0.$

$\lambda(\lambda + 1) + 1(\lambda + 1) = 0.$

$\lambda_1 = -1, \quad \lambda_2 = -1.$

So roots are same

$y = (C_1 + C_2 x)e^{\lambda x}.$

$y = (C_1 + C_2 x)e^{-x}$ — (1).

when $x = 0, \ y = 4.$

$4 = (C_1 + C_2(0))e^{-0}.$

$4 = C_1 e^{-0} + C_2(0)e^{-0}.$

$\boxed{4 = C_1}$

Scanned with CamScanner

Now

$x = 0, \quad y = -6.$

$-6 = (C_1 + C_2 x)e^{-x}.$

$-6 = C_1 e^{-x} + C_2 x e^{-x}.$

$-6 = C_1 e^{-0} + C_2 (0) e^{-0}.$

$-6 = -C_1 + C_2.$

$\Rightarrow$

$C_1 = 4$

$C_2 = -6 + C_1$

$C_2 = -6 + 4$

$\boxed{C_2 = -2}$

So

$C_1 = 4$ and $C_2 = -2.$

$y(x) = (C_1 + C_2 x)e^{-x}.$

$\boxed{y(x) = -2e^{-x}(x-2)}$

Ans

P-4

(Q#2 (b))

Soln:

$$x^2y'' + 3xy' + y = 0$$

$$x^2y'' + 3xy' + y = 0$$

$a = 3, \quad b = 1.$

$m^2 + (a-1)m + b = 0$

$m^2 + (3-1)m + 1 = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$m = -1, \quad m = -1$

roots are real and equal

so

$$y = (C_1 + C_2 \ln x)\, x^m$$

$P-1$

Q3 $y'' + y' - 6y = 6x^3 - 3x^2 + 12x$.

Soln:- $y'' + y' - 6y = 6x^3 - 3x^2 + 12x$ — (1)

for Homogenous Equation.

$a = 1$, $b = -6$.

$\lambda^2 + \lambda - 6$

$\lambda^2 + 3\lambda - 2\lambda - 6$.

$\lambda(\lambda + 3) - 2(\lambda + 3)$.

$(\lambda - 2)(\lambda + 3)$.

$\lambda - 2 = 0$, $\lambda + 3 = 0$.

$\lambda = 2$, $\lambda = -3$

So,

$y = C_1 e^{2x} + C_2 e^{-3x}$.

for choice.

$y_p = K_3 x^3 + K_2 x^2 + K_1 x + K_0$.

$y_p' = 3K_3 x^2 + 2K_2 x + K_1$

$y_p'' = 6K_3 x + 2K_2$

Put in (1).

$\underbrace{6K_3 x + 2K_2}_{y''} + 3K_3 x^2 + 2K_2 x + K_1$

$- 6K_3 x^3 - 6K_2 x^2 - 6K_1 x - 6K_0$

$= 6K_3 x^3 - 3K_2 x^2 + 12x$ / y

$-6k_3x^2 + (3k_3 - 6k_2)x^2 + (6k_3 + 2k_2) - 6k_1)x$
$+ 2k_2 + k_1 - 6k_0 = 6x^3 - 3x^2 + 12x.$

$-6k_3 = 6.$

$\Rightarrow \boxed{k_3 = -1}$

$3k_3 - 6k_2 = -3.$

$3(-1) - 6k_2 = -3$

$-k_2 = 0. \Rightarrow \boxed{k_2 = 0}$

$6k_3 + 2k_2 - 6k_1 = 12.$

$6(-1) + 2(0) - 6k_1 = 12.$

$-6 + 0 - 6k_1 = 12.$

$-6k_1 = 12 + 6.$

$k_1 = -18/6.$

$\boxed{k_1 = -3}.$

$2k_2 + k_1 - 6k_0 = 0$

$0 + (-3) - 6k_0 = 0$

$-6k_0 = 3$

$k_0 = -3/6.$

$\boxed{k_0 = -1/2}$

So,

$y_{p} = -x^3 + 0x^2 - 3x - 1/2 = -x^3 - 3x - 1/2$

Ans

(Q 4) P-7

$$y'' - 4y' + 4y = x^2 e^{2x}$$

Soln:

$$y'' - 4y' + 4y = x^2 e^{2x}$$

for equation.

$$y'' - 4y' + 4y = 0.$$

$$\lambda^2 + 4\lambda + 4 = 0.$$

$$\lambda^2 - 2\lambda - 2\lambda + 4 = 0.$$

$$\lambda(\lambda - 2) - 2(\lambda - 2) = 0.$$

$$(\lambda - 2)(\lambda - 2) = 0.$$

$$\lambda = +2 \quad \lambda_2 = 2$$

=) Roots are real and equal,

$$y = (c_1 + c_2 x) e^{\lambda x}.$$

$$y = c_1 e^{2x} + c_2 x e^{2x}.$$

$$y_1 = e^{2x}, \quad y_2 = x e^{2x}.$$

$$y_1' = 2e^{2x}, \quad y_2' = e^{2x} + 2x e^{2x}$$

Ans.

Q5

ODE $y'' + ay' + by = 0$ for the basis $1, e^{-3x}$.

Soln:

$$y_1 = e^{0x}, \quad y_2 = e^{-3x}$$

$$y = c_1 e^{0x} + c_2 e^{-3x}$$

So roots are real and distinct.

$$y = (c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}).$$

So, $\lambda_1 = 0, \quad \lambda_2 = -3.$

$$(\lambda)(\lambda + 3) = 0.$$

$$\lambda^2 + 3\lambda = 0.$$

$$\lambda^2 + a\lambda + b = 0.$$

So $y'' + ay' + by = 0$ or $y'' + 3y' = 0$.

Ans

THE END