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Subject :- Numerical Analysis

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:- Mid term Exam

Qno1

(i) Find a root of the Equation $x^3 + 3.993 \times 10^{-4} = 0.165x^2$

Use newton Raphson method with $x_0 = 0.02$

Solution:-

Reavranging the Equation

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.02 - \frac{0.00033}{-0.0054}$$

$f(x_0) = 0.00033$
$f'(x_0) = -0.0054$

$$\boxed{x_1 = 0.081}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.081 - \frac{(-0.00016)}{-0.0070}$$

$f(x_1) = -0.00016$
$f'(x_1) = 0.0070$

$$\boxed{x_2 \quad 0.058}$$

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$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.058 - \frac{0.000030}{(-0.0090)}$$

$$\boxed{x_3 = 0.061}$$

$f(x_2) = 0.000030$

$f'(x_2) = 0.0090$

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Qno2

Use the numbers $x_0 = 2$, $x_1 = 2.75$, $x_2 = 4$ to find the lagrange interpolation polynomial for $f(x) = \frac{1}{x}$ at $x = 3$

Solution:-

Putting value in the function

$y_0 = 0.5$, $y_1 = 0.364$, $y_2 = 0.25$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(3-2.75)(3-4)}{(2-2.75)(2-4} (0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} \times 0.364$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75} (0.25)$$

$$= 0.083 + 0.388 + 0.025$$

$$\Rightarrow \boxed{y = 0.33}$$

Qno3
Complete the divided difference table for the given data and construct the interpolating polynomial that uses all this data.

$X = 1.0 \quad 1.3 \quad 1.6 \quad 1.9 \quad 2.2$

$Y = 0.7651977 \quad 0.6200860 \quad 0.4554022 \quad 0.2818186 \quad 0.1103623$

x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}, x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}, x_{i-2}, x_{i-1}, x_i)$
x_0 1	0.7651977				
		-0.4837056			
x_1 1.3	0.6200860		-0.108734		
		-0.548946		0.0658785	
x_2 1.6	0.4554022		-0.0494433		-0.0028049
		-0.578612		0.06251255	
x_3 1.9	0.2818186		0.006818		
		-0.571521			
x_4 2.2	0.1103623				

(i) $f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{0.6200860 - 0.7651977}{1.3 - 1}$

$\boxed{f(x_0, x_1) = 0.4837056}$

2) $f(x_1 x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \frac{0.4554022 - 0.6200860}{1.6 - 1.3}$

$\boxed{f(x_1, x_2) = -0.548946}$

3) $f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \frac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$\boxed{f(x_2, x_3) = -0.578612}$

④ $f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3}$

$= \frac{0.1103623 - 0.2818186}{2.2 - 1.9}$

$\boxed{f(x_3, x_4) = -0.571521}$

Second Divided difference

$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

$$= -\frac{0.548946 - (0.4837056)}{1.6 - 1}$$

$$= \boxed{0.108734}$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$\boxed{f(x_1, x_2, x_3) = 0.0494433}$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$= \boxed{0.006818}$$

Third Divided Difference.

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{0.0494433 - (0.108734)}{1.9 - 1}$$

$$= \boxed{0.0658785}$$

$$f(x_1,x_2,x_3,x_4) = \frac{f(x_2,x_3,x_4) - f(x_1,x_2,x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-0.049443)}{2.2 - 1.3}$$

$$= \boxed{0.06251255}$$

4th divided diffvence

$$f(x_0,x_1,x_2,x_3,x_4) = \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$= \boxed{-0.0028049}$$

$$f(x) = f(x_0) + (x-x_0) f(x_0,x_1) + (x-x_0)(x-x_1) f(x_0,x_1,x_2) + (x-x_0)(x+x_1)(x-x_2) f(x_0,x_1,x_2,x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3) f(x_0,x_1,x_2,x_3,x_4)$$

$$= 0.7651977 + (x-1) - 0.4837656 + (x-1)(x-1.3)(0.108734) + (x-1)(x-1.3)(x-1.6)(0.0658785) + (x-1)(x-1.3)(x-1.6)(x-1.9)(-0.0028049)$$