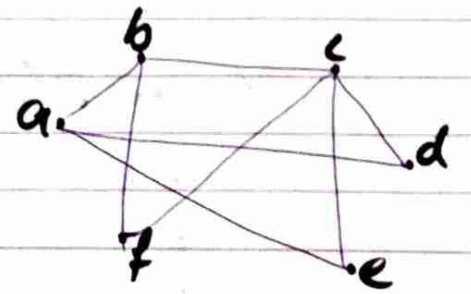
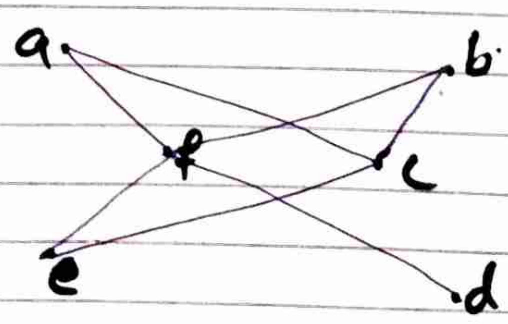
Name :: Shahid Abbas
ID # ~~16694~~ 16694
Semester :: 2nd (spring) BS(CS)
Subject :: Discrete structure
Instructor :: Mr. Saifullah Jan
Final Assignment Spring 2020
Date :: 24/06/2020
Class Time :: TUE: 08:00 to 11:00

Q1:: Determine whether the Graphs are bipartite.

(i)



(ii)

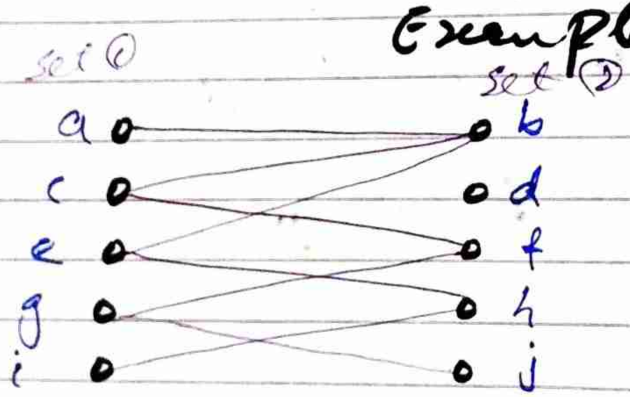


P.T.O

Bipertite Graphs:

Def: A Graph is said to be a bipertite graph, when vertices of that graph can be divided into two independent sets such that every edge in the graph is either start from the first set and ended in the second set, or starts from the second set, conected to the first set, in ~~other~~ other words, we can say that no edge can found in same set.

Example.



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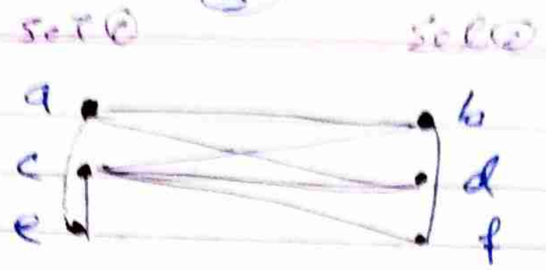
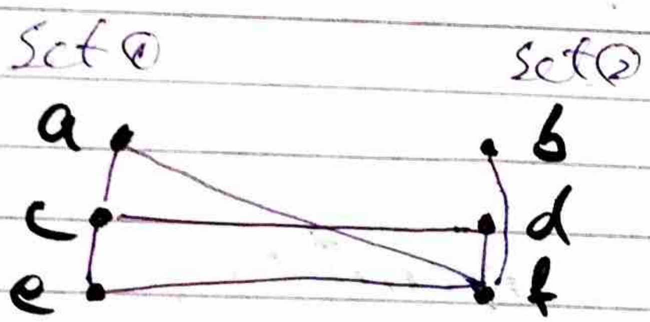
(i)



part (i) is not a bipartite graph because the element of set ① a is connected to e and e is connected to c. and on the other side the element of set ② b is connected directly with f. That is why it is not a bipartite graph.

(ii)



part (ii) is also not a bipartite graph because the elements of set ① is connected with eachother and the elements of set ② b is also connect with eachother.

From the definition part (i) and (ii) both are not ~~a~~ bipartite graphs.

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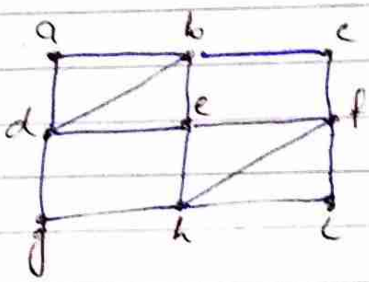
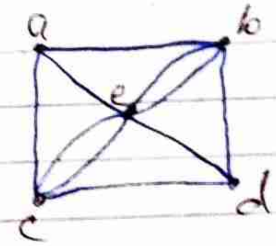
Q4:. Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

(i)



(ii)



Euler Circuit:.

An Euler circuit in a graph G ~~has~~ is a simple circuit containing every edge of G.

Euler Path:. A Euler path in G is a simple path containing every edge of G.

Theorem 1:. A connected multigraph ~~has an~~ with at least two vertices has an Euler circuit if and only if each of its vertices has an ~~[illegible]~~ even degree.

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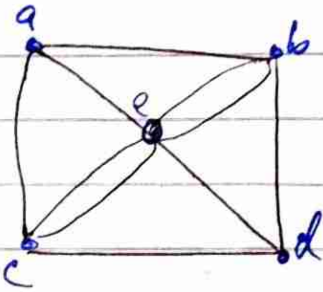
Theorem 2: A connected multigraph has an Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree.

Ans: part (ii)



By Theorem 1. This graph does not have an Euler circuit because we have two vertices with odd degrees (a and d). This graph does have an Euler path by Theorem 2. The path is as follows:

a, e, c, e, b, e, d, b, a, c, d.

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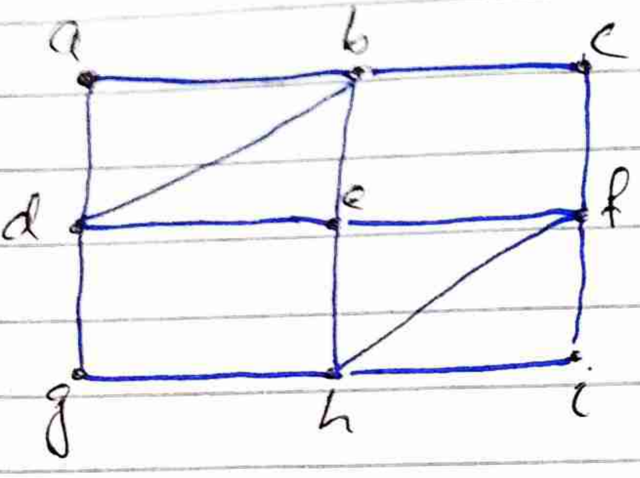
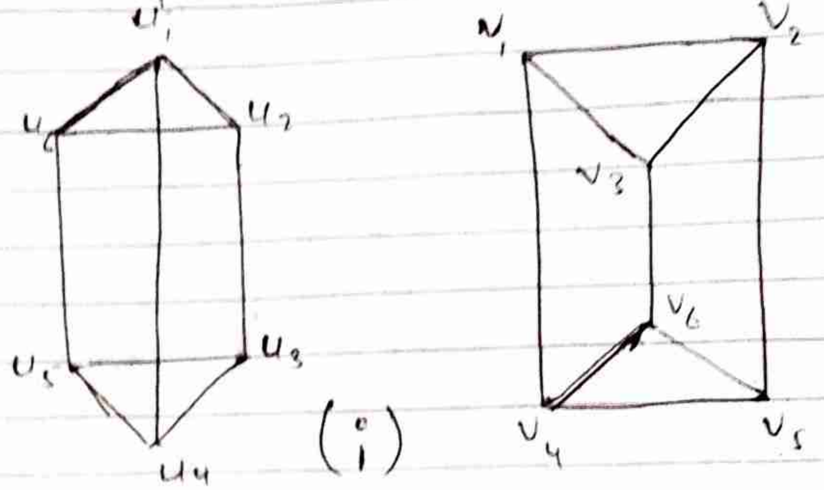
Ans:- Part (i)



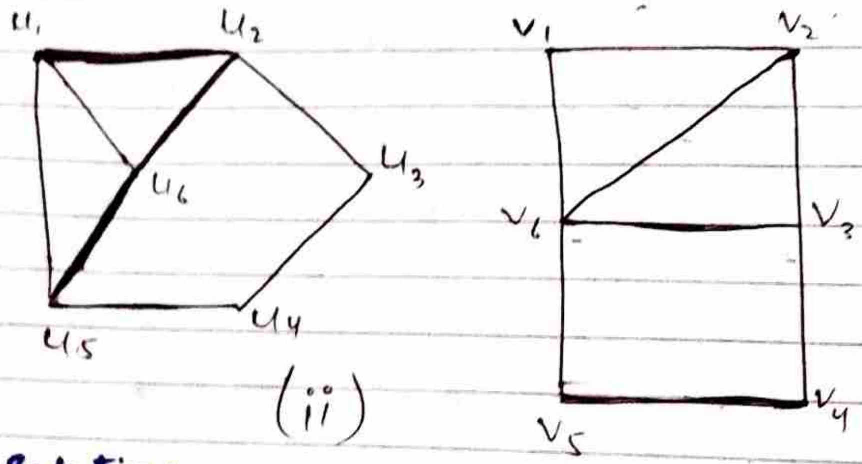
By Theorem 1. There is an Euler circuit because every vertex has an even ~~of~~ ~~adg~~ degree. The circuit is as~~fo~~ follows:

a, b, c, d, e, f, i, g, h, i
d~~e~~, a, b, e b, h e, f.

Q#2: Determine whether the given pair of graphs is isomorphic

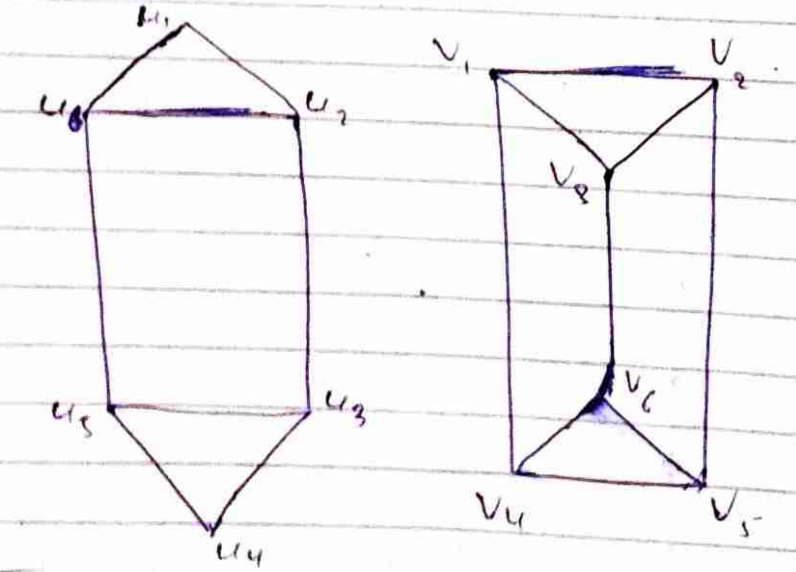


(i)

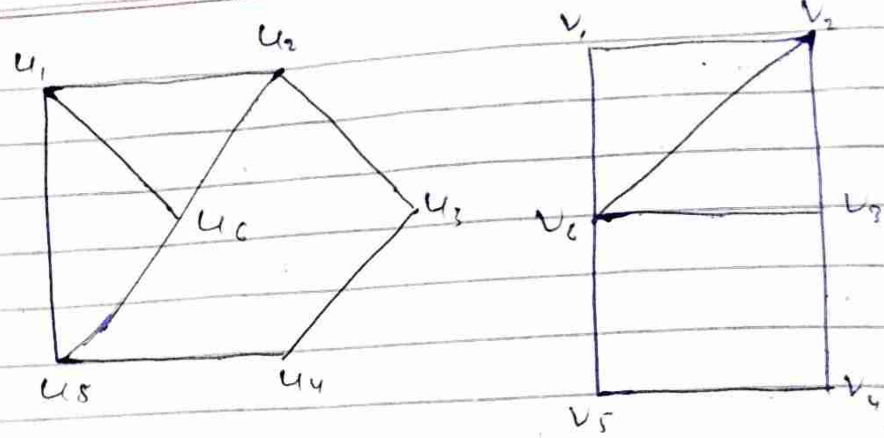


(ii)

Solution:-



This graph is isomorphic. One isomorphic is $f(u_1) = v_5$, $f(u_2) = v_2$, $f(u_3) = v_3$, $f(u_4) = v_6$, $f(u_5) = v_4$ & $f(u_6) = v$



These both graphs 6 vertices, 8 edges and 1 component. but these are non-isomorphic.

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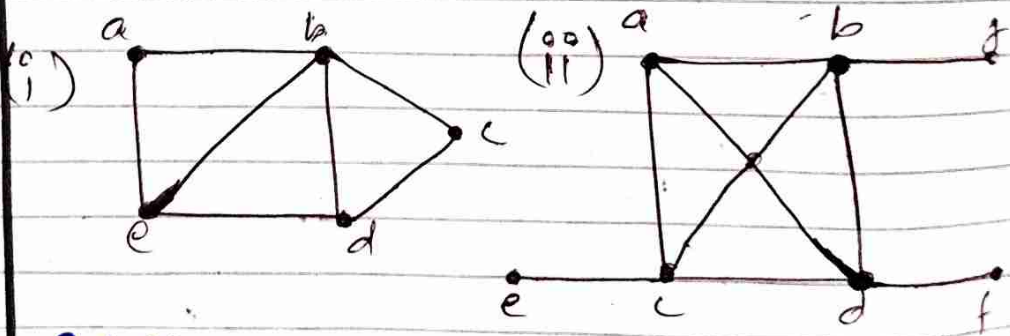
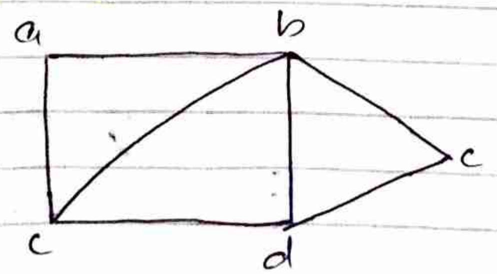
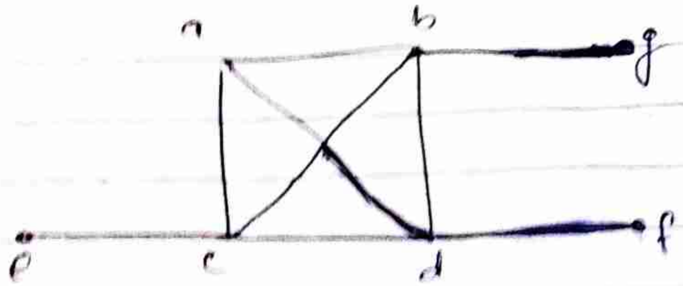
Q#5:- Determine whether the given graph has a Hamilton Circuit If it does, find such a circuit If it does not give an argument to show why no such circuit exist.



Solution:



This graph has a hamilton circuit a, b, c, d, e, a is a circuit.



there is no hamilton circuit because there are vertices of degree 1 (pendants) in the graph

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Q#3: Are the simple graphs with the following adjacency matrices isomorphic?

(i)

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Solution:

As isomorphisms have to preserve degree, there are only 2 possible ones, the first given by

$\phi_1 : 1 \to 3,\ 2 \to 4,\ 3 \to 1,\ 4 \to 2$

and

$\phi_2 : 1 \to 4,\ 2 \to 3,\ 3 \to 1,\ 4 \to 2$

ϕ_1 maps the first matrix to

(That means the following matrix has $a_{\phi_1^{-1}(i), \phi_1^{-1}(j)}$ as (i, j)-th entry.

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

and so ϕ_1 is an isomorphism

α —— α ——