Department of Electrical Engineering

Course Title: Signals & Systems

Module: 4th semester

Student Detail

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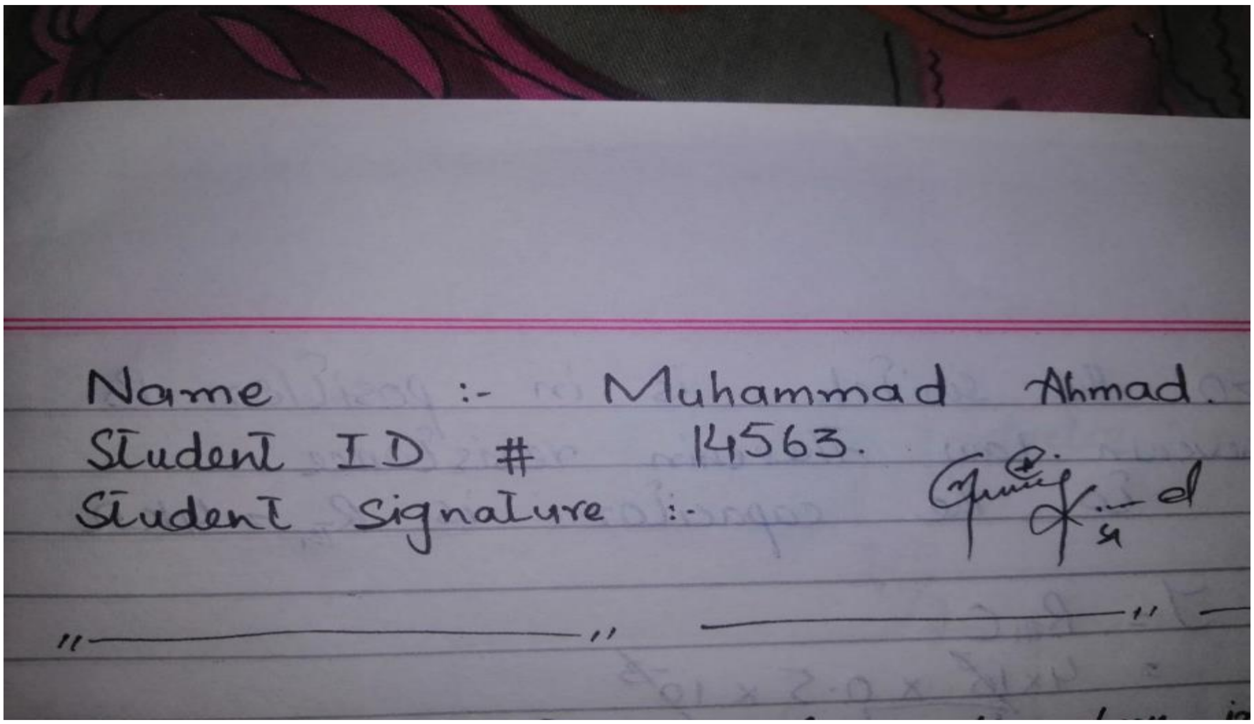


QNo1(a) Show with a help of an equation that the differentiable of a function in time domain.

We know that differentiation in the domain corresponds to multiplication JW in frequency domain. We might suspect that multiplication by $J\tau$. the time domain corresponds roughly to differentiation in frequency domain.

As we know.

$$X(JW) = \int_{-\infty}^{\infty} x(\tau)\, e^{-JW\tau}\, d\tau$$

Differentiating both sides w.r.t ("w").

$$\frac{dx}{dw}(JW) = \int_{-\infty}^{\infty} -J\tau\, x(\tau)\, e^{-JW\tau}\, d\tau$$

$$\frac{dx}{dw}(JW) = -J\tau \int_{-\infty}^{\infty} x(t)\, e^{-JW\tau}\, d\tau$$

$$\frac{dx}{dw}(JW) = -J\tau\, f\{x(\tau)\}.$$

$$\Rightarrow -J\tau\, x(\tau) \xleftrightarrow{f} \frac{d}{d\tau} x(JW).$$

Q.No 1 (b)

If $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$.
$h[n] = 3\delta[n] - \delta[n-1] + 2\delta[n-2]$.

find $Y(z)$ & $Y(n)$.

Solution:-

$x(n) = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$
$h(n) = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$.

find $Y(z)$ and $Y(n)$.

$X_{(z)} = 2 - 4z^{-2} + 2z^{-3}$
$h(z) = 3 + z^{-1} + 2z^{-2}$

Now

$Y(z) = (H(z)) \times X(z)$.
$= (3 + (z^{-1}) + 2z^{-2})(2 - 4z^{-2} + 2z^{-3})$.
$= 6 - 12z^{-2} + 6z^{-3} + 2z^{-1} - 4z^{-3} + 2z^{-4}$
$+ 4z^{-2} - 8z^{-4} + 4z^{-5}$.

$Y(z) = 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$.

Now for $y[n]$. we use delay property.

$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2]$
$+ 2\delta[n-3] - 6\delta[n-4] + 4\delta[n-5]$.

Ans.

Q No 2 :

Fourier series.

$$f(x) = \begin{cases} -\pi/2 & -\pi \leq x \leq 0 \\ \pi/2 & 0 \leq x \leq \pi \end{cases}.$$

$$a_0 = \frac{1}{\pi} \int_{\pi}^{\pi} fx \, dx.$$

$$a = \frac{1}{\pi} \int_{-\pi}^{0} f(x) \, dx + \frac{1}{\pi} \int_{0}^{\pi} f(x) \, dx.$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} -\frac{\pi}{2} \, dx + \frac{1}{\pi} \int_{0}^{\pi} \frac{\pi}{2} \, dx.$$

$$= -\frac{\cancel{\pi}}{2\cancel{\pi}} \int_{-\pi}^{0} dx + \frac{\cancel{\pi}}{2\cancel{\pi}} \int_{0}^{\pi} dx.$$

$$= -\frac{1}{2} \int_{-\pi}^{0} dx + \frac{1}{2} \int_{0}^{\pi} dx.$$

$$= -\frac{1}{2} \left(x \Big|_{-\pi}^{0} \right) + \frac{1}{2} \left(x \Big|_{0}^{\pi} \right).$$

$$= -\frac{1}{2} (0 - (-\pi) + \frac{1}{2} (\pi - 0)$$

$$= -\frac{1}{2} (+\pi) + \frac{1}{2} \pi.$$

$$= -\frac{1}{\cancel{2}}\pi + \frac{\pi}{\cancel{2}}.$$

$$= 0.$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0}\left(\frac{-\pi}{2}\right)\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}\frac{\pi}{2}\cos nx\,dx.$$

$$= \left(\frac{1}{\pi}\right)\left(-\frac{\pi}{2}\right)\int_{-\pi}^{0}\cos^{n}x\,dx + \left(\frac{1}{\pi}\right)\left(\frac{\pi}{2}\right)\int_{0}^{\pi}\cos nx\,dx.$$

by part

$$= \frac{x\sin nx}{n} - \int \frac{x\sin nx}{n}\,dx.$$

Then

$$a_n = -\frac{1}{2}\left[\left(\frac{\sin xn}{n}\Big|_{-\pi}^{0} + \left(\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right)\Big|_{-\pi}^{0} + \frac{\pi}{2}\left(\frac{\sin nx}{n}\right)\Big|_{0}^{\pi} - \frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right)\Big|_{0}^{\pi}\right]$$

The value of $\sin nx$ at $x = 0$ or $x = \pm\pi$ are zero therefore.

$$a_n = -\frac{1}{2n^2}\left[1 - \cos\pi n\right]_{-\pi}^{0} - (\cos n\pi)\Big|_{0}^{\pi}\Big].$$

$$= -\frac{1}{2n^2}\left(\cos 0 - \cos(-\pi n) - \cos\pi n + \cos 0\right).$$

$$= -\frac{2}{\pi n^2}\left[1 - \cos\pi n\right]$$

$$= -\frac{2}{\pi n^2}\left[1 - (-1)^n\right].$$

When $n = 2k$, then $a_{2k} = 0$

When $n = 2k+1$ then $a_{2k+1} = \frac{4}{\pi n^2}$

$k = 0, 1, 2, 3$

$$f(x) = \frac{4}{\pi}\sum_{k=0}^{\infty}\frac{\cos(2k+1)x}{(2k+1)^2}$$

QNO3

If $$X(z) = \frac{2z^2+2z}{z^2+2z-3}$$

$$X(z) = \frac{2z(z+1)}{z^2+2z-3}$$

$$X(z) = \frac{z+1}{z^2+2z-3}$$

$$\frac{X(z)}{2z} = \frac{z+1}{(z-1)(z+3)}$$

As

$$\frac{z+1}{(z-1)(z+3)} = \frac{A}{(z-1)} + \frac{B}{(z+3)}$$

$$z+1 = A(z+3) + B(z-1)$$

Now put $z = 1$ in.

$$1+1 = A(1+3) + B\cancel{(1-1)}.$$

$$\frac{\cancel{2}}{\cancel{4}} = \frac{\cancel{4}A}{\cancel{4}} + 0$$

$$\boxed{A = \frac{1}{2}}$$

Now Put $z = -3$

$$-3+1 = A(-3+3) + B(-3-1)$$

$$\frac{\cancel{2}}{\cancel{4}} = \frac{\cancel{4}B}{\cancel{4}}$$

$$B = \frac{1}{2}$$

So put A & B

$$\frac{z+1}{(z-1)(z+3)} = \frac{1}{2}\frac{z}{z-1} + \frac{1}{2}\frac{z}{z+3}.$$

So the Inverse. of z Transform.

$$x[n] = \frac{1}{2}u(n] + \frac{1}{2}(3)^{n}$$

OR.

$$x[n] = \frac{1}{2}e^{-1t} - \frac{1}{2}e^{3t}$$

Ans.

Q No 4:-

$A=\begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$ $B=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C\begin{bmatrix} 1 & 2 \end{bmatrix}$ $D=[0]$.

Sol:- $G(s) = C(SI - A)^{-1} B + D.$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s-(-2) & +1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s(s+2)-((1)-1)} \times \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s^2+2s+1} \begin{bmatrix} s \\ 1 \end{bmatrix}.$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}.$$

$$= \frac{1}{s^2+2s+1} \times \begin{bmatrix} s & 2 \end{bmatrix}.$$

$$= \boxed{\frac{s+2}{s^2+2s+1}}$$

QNO5:-

$x(t) = e^{-a|t|}$ $\quad a>0$

$X(J\omega) = ?$ or $u(t)$.

Solution:-

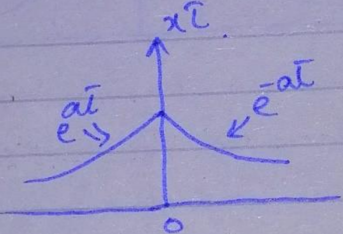
The Fourier Transform of the given function $x(t)$ is given by:

$$u(t) = \int_{-\infty}^{\infty} x(t)\, e^{-J\omega\tau}\, d\tau.$$

$$= \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-J\omega\tau}\, d\tau.$$

Remember

$$e^{-a|t|} = \begin{cases} e^{-a\tau} & \text{for } \tau \geq 0 \\ e^{-a(-\tau)} = e^{a\tau} & \text{for } \tau < 0 \end{cases}$$



$$x(J\omega) = \int_{-\infty}^{0} e^{a\tau}\, e^{-J\omega\tau}\, d\tau + \int_{0}^{\infty} e^{-a\tau}\, e^{-J\omega\tau}\, d\tau.$$

$$= \int_{-\infty}^{0} e^{(a-J\omega)\tau}\, d\tau + \int_{0}^{\infty} e^{-(a+J\omega)\tau}\, d\tau.$$

$$= \frac{e^{(a-J\omega)\tau}}{a-J\omega}\Bigg|_{-\infty}^{0} + \frac{e^{-(a+J\omega)\tau}}{-(a+J\omega)}\Bigg|_{0}^{\infty}$$

$$= \frac{1}{(a-J\omega)}\left[e^{0} - e^{-\infty} - \frac{1}{a+J\omega}\left[e^{-\infty} - e^{0}\right]\right.$$

$$= \frac{1}{a - j\omega}[1-0] - \frac{1}{a + j\omega}[0-1].$$

$$= \frac{1}{a - j\omega} + \frac{1}{a + j\omega}.$$

$$\frac{a + \cancel{j\omega} + a - \cancel{j\omega}}{a^2 - (j\omega)^2}$$

$$x(j\omega) = \frac{2a}{a^2 + \omega^2}$$