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Question 1 (a):

Express the equation of plane passing through the points

A(2, -2, 1)

B (-1, 0, 3)

C (5, -3, 4)

Solution :

The non-Parallel vectors

$\vec{AB} = (-3, 2, 2)$

$\vec{AC} = (3, -1, 3)$

The perpendicular vector is;

$n = \vec{AB} \times \vec{AC}$

$$n = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$n = i(6+2) - j(-9-6) + k(3-6)$

$n = 8i + 15j - 3k$

$n = (8, 15, -3)$

Now

$A(x_0, y_0, z_0) = (2, -2, 1)$

$n(a, b, c) = (8, 15, -3)$

Solution of Eq. Plane is;

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

putting the values

$$\cancel{8(x-2)+15(y+3)-3(z-1)=0}$$

$$\cancel{8x+15y-3z-16+45+3=0}$$

$$\cancel{8x+15y-3z+32=0}$$ ~~Answer.~~

$$8(x-2)+15(y+2)-3(z-1)=0$$

$$8x+15y-3z-16+30+3=0$$

$$8x+15y-3z+17=0$$ Answer.

Question 1 (b):

Express a pair of planes whose intersection is the given line;

$$x=2-3t$$

$$y=3+t$$

$$z=2-4t$$

P.T.O

Solution:

As

$$x = 2-3t$$

So

$$x-2 = 3t$$

$$t = \frac{x-2}{-3}$$

Similarly

$$y = 3+t$$

$$y-3 = t$$

$$t = \frac{y-3}{1}$$

And

$$z = 2-4t$$

$$z-2 = -4t$$

$$t = \frac{z-2}{-4}$$

So

$$\frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-2}{-4}$$

for 1st plane take 1st & 2nd

$$\frac{x-2}{-3} = \frac{y-3}{1}$$

By cross Multiplication

$$x-2 = -3y+9$$

$$\boxed{x+3y-11=0}$$

for 2nd plane take 1st & 3rd

$$\frac{x-2}{-3} = \frac{z-2}{-4}$$

$$-4x+8 = -3z+6$$

$$-4x+3z+2 = 0$$

OR

$$\boxed{4x-3z-2=0}$$

Question 4:

Find an equation of the plane passing through the point $(-1,3,2)$ & perpendicular to the vector $n=(0,1,-3)$

Solution:

$$(-1,3,2) \quad n=(0,1,-3)$$

Equation of plane

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

Given that

$$P(x_0, y_0, z_0) = (-1,3,2)$$
$$n(a,b,c) = (0,1,-3)$$

putting values

$a(x_0 - ($

$\Rightarrow$ 0

$\Rightarrow$

$\Rightarrow$

Question

$L(x,y)$

illustra

transfor

Solutio

$0(x-(-1))+1(y-3)+(-3)(z-2)=0$

$\Rightarrow \quad 0+y-3-3z+6$

$\Rightarrow \quad y-3z-3+6$

$\Rightarrow \quad \boxed{y-3z+3}$ Answer.

Question 2:

$L(x,y) = (x+1, y, x+y)$

illustrate that "L" is linear transformation

Solution:

$$L(x,y) = (x+1, y, x+y)$$

Let

$u = (x_1, y_1)$

$v = (x_2, y_2)$

$u+v = (x_1, y_1) + (x_2, y_2)$

$u+v = (x_1+x_2, y_1+y_2)$

$L(u+v) = L(x_1+x_2, y_1+y_2)$

$L(u+v) = (x_1+x_2+1, y_1+y_2, x_1+x_2+y_1+y_2) \rightarrow (i)$

Given that

$u = (x_1, y_1)$

$L(u) = L(x_1, y_1) = (x_1+1, y_1, x_1+y_1)$

$L(v) = L(x_2, y_2) = (x_2+1, y_2, x_2+y_2)$

$L(u)+L(v) = (x_1+x_2+2, y_1+y_2, x_1+x_2+y_1+y_2) \rightarrow (ii)$

Since eq (i) $\neq$ (ii)

So not L.T <u>Answer</u>

Question 5:

Find an

Eigen ve

$A = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Solution:

As we

Ax

So

$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ -2x \end{bmatrix}$

Question 5:

Find an Eigen values & Eigen vectors of matrix

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

Solution:

As we know that

$$Ax = \lambda x$$

So

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

then

$x_1 + x_2 = \lambda x_1 \rightarrow (i)$

$-2x_1 + 4x_2 = \lambda x_2 \rightarrow (ii)$

So

$x_1 - \lambda x_1 + x_2 = 0$

$= (1-\lambda) x_1 + x_2 = 0$

&

$-2x_1 + 4x_2 - \lambda x_2 = 0$

$= -2x_1 + (4-\lambda) x_2 = 0$

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Characteristic equation

$1-\lambda$

-2

$(1-\lambda$

$4-\lambda -$

$\lambda^2 - 5$

$\lambda^2 - 3$

$\lambda(\lambda -$

These

Now

of

$$\begin{vmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(4-\lambda)+2 = 0$$

$$4-\lambda-4\lambda+\lambda^2+2=0$$

$$\lambda^2-5\lambda+6=0$$

$$\lambda^2-3\lambda-2\lambda+6=0$$

$$\lambda(\lambda-3)-2(\lambda-3)=0$$

$$(\lambda-2)(\lambda-3)=0$$

$$\lambda-2=0, \quad \lambda-3=0$$

$$\lambda=2, \quad \lambda_1=3$$

These are Eigen values

Now finding Eigen vectors.
of $\lambda_1 = 3$ Put in (i) & (ii)

then $x_1 + x_2 = 3x_1 \rightarrow (i)$

$= -2x_1 + x_2 = 0$

$\Rightarrow 2x_1 - x_2 = 0$

$\Rightarrow -2x_1 + 4x_2 = 3x_2 \rightarrow (ii)$

$\Rightarrow -2x_1 + x_2 = 0$

$\Rightarrow 2x_1 - x_2 = 0$

$x_1 = 1/2\, x_2$

let $x_2 = r$

where $r \neq 0$

So

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/2\, r \\ r \end{bmatrix}$$

Eigen vector for $\lambda_2 = 2$

put in (i) & (ii)

$x_1 + x_2 = 2x_1 \rightarrow (i)$

$-2x_1 + 4x_2 = 2x_2 \rightarrow (ii)$

$= -x_1 + x_2 = 0 \rightarrow (i)$

$\Rightarrow x_1 - x_2 = 0$

$\Rightarrow x_1 = x_2$

$= -2x_1 + 4x_2 = 2x_2 \rightarrow (ii)$

$= -2x_1 + 2x_2 = 0$

$\Rightarrow x_1 - x_2 = 0$

$x_1 = x_2$

$x_1 = \gamma$ then $x_2 = \gamma$

So

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \gamma \\ \gamma \end{bmatrix}$$

Question 3:

Using the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

then interupret to decode the message
77, 54, 38, 71, 49, 29, 68, 51, 33, 76, 48, 40, 86, 53, 52.

Answer:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

To decode the above message we have to break it into five vectors in R^3:

$$\begin{bmatrix}77\\54\\38\end{bmatrix} \quad \begin{bmatrix}71\\49\\29\end{bmatrix} \quad \begin{bmatrix}68\\51\\33\end{bmatrix} \begin{bmatrix}76\\48\\40\end{bmatrix} \quad \begin{bmatrix}86\\53\\52\end{bmatrix}$$

So solve the equation.

$$L(x_1) = \begin{bmatrix}77\\54\\38\end{bmatrix} = Ax_1$$

for x_1. Since A is nonsingular

$$x_1 = A^{-1}\begin{bmatrix}77\\54\\38\end{bmatrix} = \begin{bmatrix}0 & 1 & -1\\2 & -2 & -1\\-1 & 1 & 1\end{bmatrix}\begin{bmatrix}77\\54\\38\end{bmatrix} = \begin{bmatrix}16\\8\\15\end{bmatrix}$$

Similarly

$$x_2 = A^{-1}\begin{bmatrix}71\\49\\29\end{bmatrix} = \begin{bmatrix}20\\15\\7\end{bmatrix}$$

$$x_3 = \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \\ 16 \end{bmatrix}$$

$$x_4 = \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 12 \end{bmatrix}$$

$$x_5 = \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

using our correspondence between letters & numbers, we received the following message

PHOTOGRAPH PLANS

Answer