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Subject: Linear Algebra

Program: BS, SE 2

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Q(1) $x_1 - 3rd\text{-}ID\, x_2 + x_3 = 0$

$2x_2 - 8x_3 = 8$

$5x_1 - 5x_3 = 10$

Sol:- def

$ID = 3145$

Then 3rd ID = 4

Then $\Rightarrow$ $x_1 - 4x_2 + x_3 = 0$

$2x_2 - 8x_3 = 8$

$5x_1 \quad - 5x_3 = 10$

The augmented matrix of the system

$$\left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$$R \left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 5 & 0 & -5 & 10 \end{array}\right] \to \frac{1}{2}R_2$$

$$R \left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 20 & -10 & 10 \end{array}\right] by\, R_3 - 5R_1$$

[PTO]

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$$R \left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 20 & -10 & 10 \end{array}\right] \text{ by } R_3 - 5R_1$$

$$R \left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 70 & -70 \end{array}\right] \text{ by } R_3 - 20R_2$$

The system (1) is reduces to equivalent system

$$x - 4x_2 + x_3 = 0 \longrightarrow (i)$$

$$x_2 - 4x_3 = 4 \longrightarrow (ii)$$

$$70x_3 = -70 \longrightarrow (iii)$$

The system is triangular form.

from eq (iii)

$$x_3 = \frac{-70}{70} = -1$$

we get $\boxed{x_3 = -1}$

$x_3 = -1$ put in eq (ii)

$\Rightarrow \quad x_2 - 4(-1) = 4$

$x_2 + 4 = 4$

$\boxed{x_2 = 0}$

put $x_3 = -1$ and $x_2 = 0$
in eq (i)

we get

$x_1 - 4(0) + (-1) = 0$

$x_1 - 0 - 1 = 0$

$\boxed{x_1 = 1}$

Thus the Solution set
is $x_1 = 1$, $x_2 = 0$ and $x_3 = -1$

Since the system has a solution. so it is consistent.

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Q no 2: find the inverse of
$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4th\text{-}ID \\ 5 & -2 & 7 \end{bmatrix}$$ 6)

Sol:
since $A^{-1} = \frac{1}{|A|} \operatorname{adj} A$

we need to find adj A
and $|A|$

~~Co-factor of every~~

our ID = 8145

So 4th - ID = 5.
The given matrix become

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 5 \\ 5 & -2 & 7 \end{bmatrix}$$

Co-factor of every element of A.

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 5 \\ -2 & 7 \end{vmatrix} = 1.(-7-(-10))$$

$$= 1.(-7+10) = 3$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 5 \\ 5 & 7 \end{vmatrix} = -1.(14-25)$$

$$= -1.(-11) = 11$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = 1.(-4-(-5))$$

$$= 1.(-4+5) = 1$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = -1.(28-(-10))$$

$$= -1.(28+10) = -38$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = 1.(21-25)$$

$$= 1(-4) = -4$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix}$$

$$= -1 \cdot (-6 - 20) = -1 \cdot (-26)$$

$$= 26$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 4 & 5 \\ -1 & 5 \end{vmatrix} = 1 \cdot (20 - (-5))$$

$$= 1 \cdot (20 + 5) = 25$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 5 \\ 2 & 5 \end{vmatrix} = -1 \cdot (15 - 10)$$

$$= -1 \cdot (5) = -5$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = 1 \cdot (-3 - 8)$$

$$= -11$$

So

$$\operatorname{adj} A = \begin{bmatrix} 3 & -38 & 25 \\ 11 & -4 & -5 \\ 1 & 26 & -11 \end{bmatrix}$$

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Next we find

$|A|$

$$|A| = a_{11}A_{11} \cancel{+} a_{12}A_{12} + a_{13}A_{13}$$

$$= 3(3) \cancel{+} 4(11) + 5(1)$$

$$= 9 \cancel{+} 44 + 5 = -30$$

$$= \cancel{58}$$

Thus $A^{-1} = \frac{1}{|A|} \text{adj}\, A$

$$= \frac{1}{\cancel{58}\ 30} \cdot \begin{bmatrix} 3 & -38 & 25 \\ 11 & -4 & -5 \\ 1 & 26 & -11 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{3}{30} & \frac{-38}{30} & \frac{25}{30} \\ \frac{11}{30} & \frac{-4}{30} & \frac{-5}{30} \\ 1/30 & 26/30 & -11/30 \end{bmatrix}$$

Q 8:- Solve the following system of linear eq by Gauss-Jordan method.

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Sol:- augmented matrix is

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$R \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right] \text{by } \tfrac{1}{2}R_1$$

$$R \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -9 & -13 \end{array}\right] \begin{array}{l} \text{by } R_2 - R_1 \\ \text{\& by } R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & -18 & -22 \end{array}\right] \text{ by } 2R_3 + R_2$$

$$z = \frac{-22}{-18} = \frac{11}{9}$$

$$\boxed{z = \frac{11}{9}}$$

from eq (ii)

$$2y + 0z = 4$$

$$\boxed{y = 2}$$

and from (i)

$$x + 2 + 2 \cdot \frac{11}{9} = 9$$

$$x + 2 + \frac{22}{9} = 9$$

$$x + \frac{18 + 22}{9} = 9$$

$$x_1 + \frac{40}{9} = 9$$

$$x_1 = 9 - \frac{40}{9}$$

$$x_1 = \frac{81 - 40}{9} = \frac{41}{9}$$

$$\boxed{x_1 = \frac{41}{9}}$$

Q.4 Show that this matrix is Diagonalisable

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Solution:

$A = CDC^{-1}$

let $(A - \lambda I) = 4$

$$A - \lambda I_3 = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 11 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix}$$

$$= 4-\lambda \begin{vmatrix} 3-\lambda & 2 \\ 4 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} -1 & 2 \\ -2 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} -5 & 3-\lambda \\ -2 & -4 \end{vmatrix}$$

$= 4-\lambda((3-\lambda)(1-\lambda)-8) - 2(-5(1-\lambda)+4) - 2$
$((-20+2)(3-\lambda)) = 0$

$= 4-\lambda[3-3\lambda-\lambda+\lambda^2+8] - 2[-5+5\lambda+4] - 2[-20+6$
$-2\lambda)] = 0$

$= 4-\lambda[\lambda^2-4\lambda-5] - 2[1\lambda-1] - 2[-14-2\lambda] = 0$

$4\lambda^2 + 16\lambda - 26 - \lambda^3 + 4\lambda^2 + 5\lambda - 1\lambda + 2 + 28 + 4\lambda = 0$

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$-\lambda^3 + 8\lambda^2 + 15\lambda + 10 = 0$

$\lambda = 9.65$
$\lambda = -0.82$
$\lambda = -0.829$

for $\lambda = -965$

P. λ :-

$$A \cdot \lambda I_3 \begin{bmatrix} -3.65 & 2 & -2 \\ -5 & -6.65 & 2 \\ -2 & 4 & -8.85 \end{bmatrix}$$

for $\lambda = 0.82$

$$A - \lambda I_3 = \begin{bmatrix} 9.82 & 2 & -2 \\ -5 & 3.82 & 2 \\ -2 & 4 & 1.82 \end{bmatrix}$$

In the End or only One eigen
or 2 basis

so This matrix I mean A
is not diagonalizable.

Q.No:5 Determine if the following homogenous System has a non-trivial solution. Then describe the Solution Set.

$$3x_1 + 5x_2 - 4x_3 = 0$$

$$-3x_1 - 25x_2 + 4x_3 = 0$$

$$6x_1 + x_2 - 8x_3 = 0$$

Sol: we will Solve the above System by matrix method for this The System of equation can be written in matrix form as

let

$$A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & -8 \end{bmatrix}$$

first of all we have to find $|A|$.

for this

$$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

$$|A| = 3\begin{vmatrix} -25 & 4 \\ 1 & -8 \end{vmatrix} - 5\begin{vmatrix} -3 & 4 \\ 6 & -8 \end{vmatrix} - 4\begin{vmatrix} -3 & -25 \\ 6 & 1 \end{vmatrix}$$

$$|A| = 3[(-8\times-25)-(1\times4)]-5[(-8\times-3)-(6\times4)]$$
$$-4[(1\times-3)-(6\times-25)]$$

$\cancel{|A| = 3(200)}$

$$|A| = 3(200-4)-5(24-24)-4(-3+150)$$

$$|A| = 3(196)-5(0)-4(147)$$

$$|A| = 588-588$$

$$|A| = 0$$

As $|A|=0$ So the System has non-trivial solution we will find the required Solution set.

Again

$$3x_1+5x_2-4x_3=0 \rightarrow (1)$$

$$-3x_1-25x_2+4x_3=0 \rightarrow (2)$$

$$6x_1+x_2-8x_3=0 \rightarrow (3)$$

multiplying eq (3) by 5 then subtract from (1)

$$3x_1+\cancel{5x_2}-4x_3=0$$

$$\underline{\pm\, 30x_1 \cancel{\pm 5x_2} \mp 40x_3=0}$$

$$-27x_1+36x_3=0 \rightarrow (4)$$

$$\Rightarrow 9(-3x_1+4x_3)=0$$

$$-3x_1+4x_3=0 \rightarrow (4)$$

let $x_3=t$ put in (4)

$$-3x_1=-4t$$

$$\boxed{x_1=\frac{4}{3}t}$$

put $x_3 = t$ $x_1 = \frac{4}{3}t$ in eq ①

we get.

$$3\left(\frac{4}{3}t\right) + 5x_2 - 4t = 0$$

$$4t + 5x_2 - 4t = 0$$

$$5x_2 = 0$$

$$x_2 = 0$$

Thus $S.S = (x_1, x_2, x_3) = \left(\frac{4}{3}t, 0t, t\right)$

Q No 6:- Reduce the Matrix to normal form and find its Rank.

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 6 \end{bmatrix}$$

$$\sim_R \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3 \end{bmatrix} \begin{matrix} \\ \text{by } R_2 - 3R_1 \\ \text{by } R_3 - R_1 \end{matrix}$$

$$\sim_R \begin{pmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ by } -2R_3 + R_2$$

the last matrix is the echelon form of A and the number of its non-zero rows is 2. Hence Rank = 2