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PAPER :- Intro to structural Dynamics & Earth quake Engg

Submitted to :-
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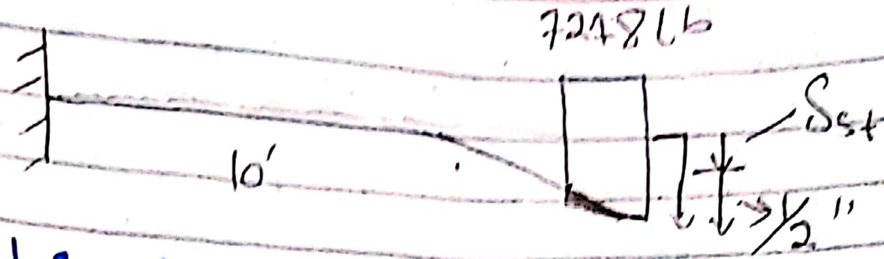
Dated :-
29/06/2020

Q1 :



SOLUTION :

The general E.O.M for SDOF system is

$$ku + c\dot{u} + m\ddot{u} = P(t)$$

In our case system is undamped ($c=0$) undergoing free vibration $P(t) = 0$

Hence general EOM become $ku + m\ddot{u} = 0 \rightarrow$ eq (1)

$$k = 3EI/L^3$$

$$= \frac{3 \times 2900\ k/in^2 \times 150\ in^4}{(10 \times 12\ in)^3}$$

$$= 7.55\ k/in$$

In order to eliminate the chances of mistake during calculation it is more appropriate to use fundamental unit like lb, ft sec or kg, m, sec.

$$k = 7.55\ k/in = 90625\ lb/ft$$

$$m = \frac{7278\ lb\ sec^2}{32.2\ ft}$$

$$m = 226.02\ slug$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{226.02}} = 20.02 \text{ rad/sec}$$

$$T_n = 2\pi/\omega_n = 2\pi/20.02 = 0.313 \text{ sec}$$

Subsituting the corresponding value eq (1)

$$90625 + 226.02\,\ddot{u} = 0$$

where "k" is in lb/ft & "m" is in lb sec²/ft

General solution to the EOM for undamped free vibration is,

$$u(t) = u(0)\cos(\omega_n t) + \dot{u}(0)/\omega_{(n)} \sin(\omega_n t)$$

$$u(0) = 1/2'' = 1/24 \text{ ft} \quad \& \quad \dot{u}(0) = 0$$

$$u(t) = \left(1/24\right) \times \cos(20.02t) + 0 = \left(1/24\right) \times \cos(20.02t)$$

Equivalent static force at any time "t" is

$$f_s(t) = k \cdot u(t) = \frac{90625 \times \cos(20.02t)}{24}$$

$$f_s(t) = 3776\cos(20.02t)$$

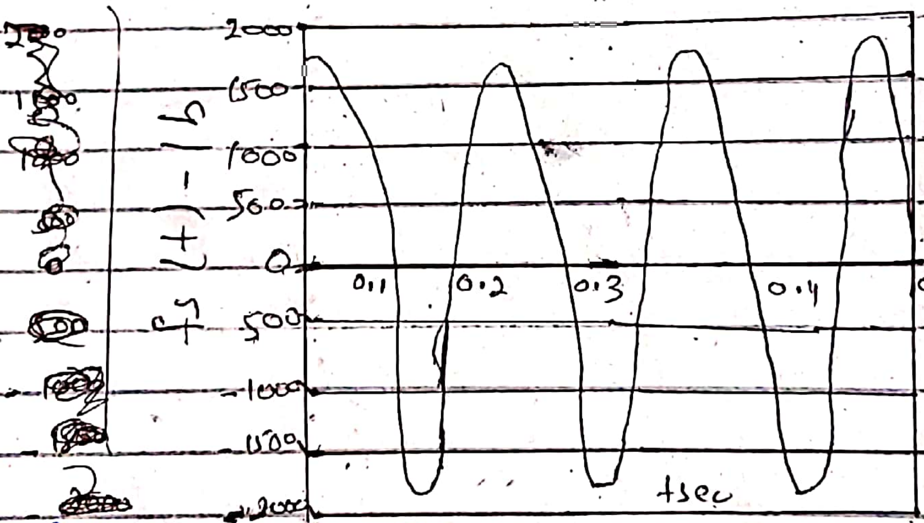
Amplitude of dynamic displacement u_0 for undamped free vibration is

$$u_0 = \sqrt{\left[(u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2\right]} = \sqrt{\left((1/24)^2 + 0\right)} = 1/24 \text{ ft}$$

Amplitude of equivalent static force

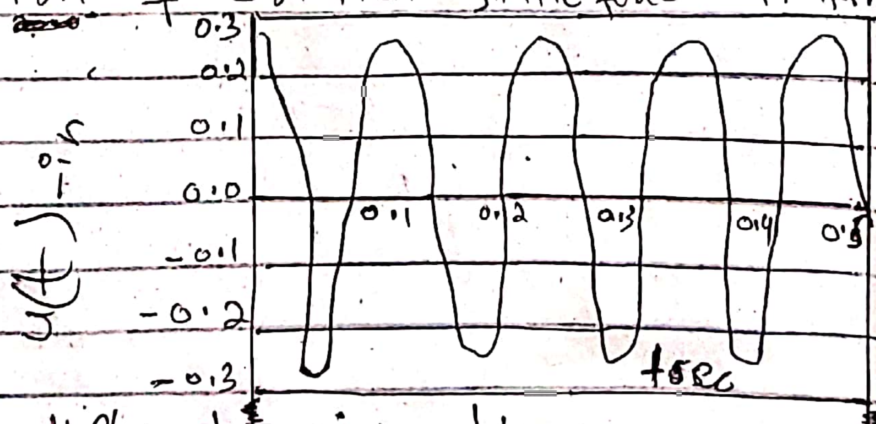
$$k u_0 = 90625 \times 1/24 = 3776 \text{ lb}$$

UNDAMPED FREE VIBRATION



UNDAMPED FREE VIBRATION:

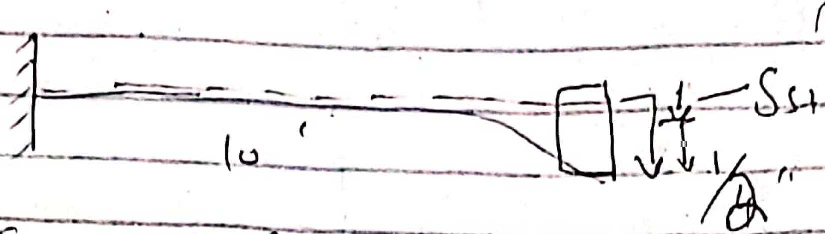
Variation of Equivalent static force with time



Variation of displacement with time

X

Q2:-



Sol:-

E.M.O for damped free vibration

$$ku + c\dot{u} + m\ddot{u} = 0 \longrightarrow eq\ (1)$$

It is known &

$k = 90625$ lb/ft & $m = 226.02$ lb.sec²/ft

$c = \zeta \times 2 m \omega_n = 2 \times 226.02 \times 20.02 \times 0.025$

$c = 226.24$ sec/ft

By subsitituting values of k, c & m in eq (1) we get.

$$90625\, u + 226.24\, \dot{u} + 226.02\, \ddot{u} = 0$$

solution to the E.O.M for damped free vibration is:

$$u(t) = e^{-\zeta \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\zeta\omega_n\right]\cdot\sin(\omega_D t)\right]$$

~~ω_D = 54.9 rad/sec~~ $\boxed{\omega_D = 20.02 \text{ rad/sec}}$

~~$u(t) = e^{-0.025 \times 54.9 t}\left[\frac{1}{24} \times \cos(54.9t)\right]$~~

$$u(t) = e^{-0.025 \times 20.02}\left[\frac{1}{24} \times \cos(20.02\,t) + \frac{1}{20.02} \times 0.025 \times 20.02 \times \sin(20.02)\right]$$

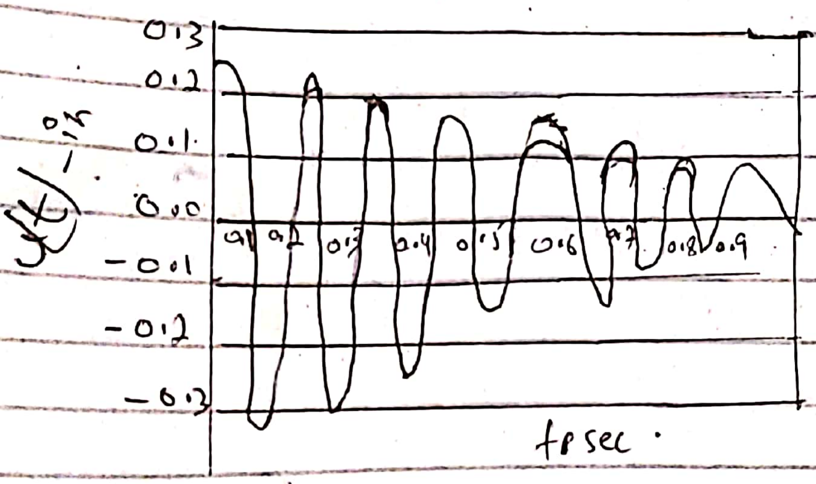
$$u(t) = e^{-0.500\,t}\left[0.041 \times \cos(20.02t) + 0.0208 \times \sin(20.02t)\right]$$

$f_s(t) = k \cdot u(t) \Rightarrow 90625 \times u(t)$

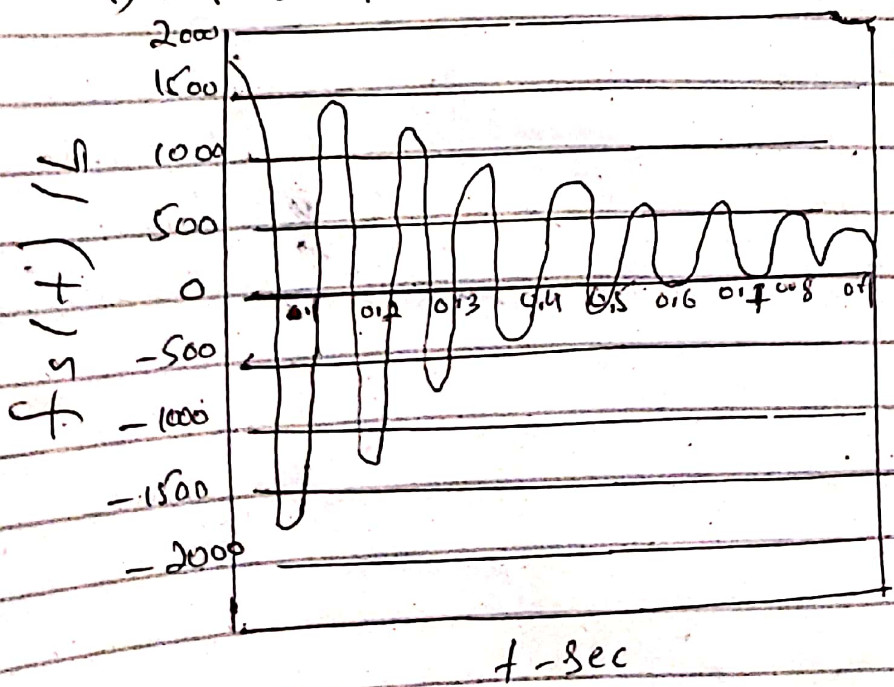
$f_s(t) = e^{-0.5000t}\left[(90625 \times 0.041)\cos(20.02t) + (90625 \times 0.0208 \sin(20.02t))\right]$

$f_s(t) = e^{-0.5000t}\left(3715.62\cos(20.02t) + 1885 \sin(20.02t)\right)$

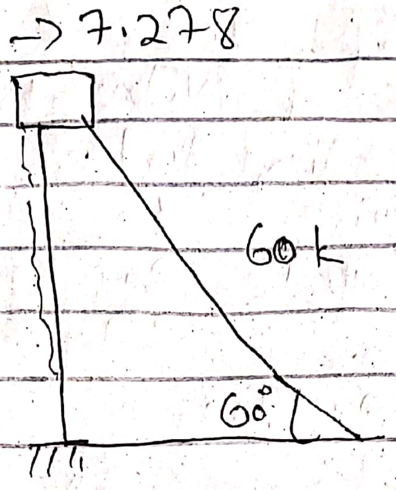
Damped free Vibration



Damped free vibration



Q3:-



cycles = 7
T = 3.57 sec
Amplitude of displacement
= 0.9"
a Damping ratios

$u_1 = 7.278$

$j = \frac{1}{2\pi \zeta} \ln \left[\frac{u_1}{u_{j+1}} \right]$

$7 = \frac{1}{2(3.14)\zeta} \ln \left(\frac{7.27}{0.9} \right)$

$7(6.28)\zeta = \ln(8.077)$

$43.96\,\zeta = 2.08$

$\zeta = \frac{2.08}{43.96}$

$\zeta = 0.04\ \%$

b) $T_n = ?$

if cycle of vibration are complete in 3.57 sec

The required to complete one cycle $\frac{3.57}{7} = T_D$

$T_D = 0.51 sec$

Now

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n \sqrt{(1-\zeta^2)}}$$

$$T_D = \frac{T_n}{\sqrt{(1-\zeta^2)}}$$

$$T_n = T_D \sqrt{(1-\zeta^2)}$$

$$= 0.51 \sqrt{1-(0.04)^2}$$

$$= 0.51 \sqrt{0.998}$$

$T_n = 0.509$ sec

(c) $k = ?$

$$k = \frac{60 \cos 60^\circ}{7.27}$$

$= 4.126$ k/in

$= 49439$ lb/ft

$$\omega_n = \sqrt{k/m} = \sqrt{k/(w/g)}$$

$$= \sqrt{\frac{kg}{w}}$$

$$w = mg/\omega_n^2$$

$$\omega_n = 2\pi/T_n$$

$$w = kg\frac{T_n^2}{4\pi^2}$$

$$w = \frac{49489\ lb}{ft} \times \frac{32.2\ ft}{sec^2}$$

$$w = 1591.93\ (0.00658)$$

$$= 10475.72\ lb$$

$$w = 10.45\ ui$$

$c = ?$

$$\zeta = \frac{c}{2m\omega_n}$$

$$c = \zeta\, 2m\omega_n$$

$$= \zeta\, 2m\,(2\pi/T_n)$$

$$c = \frac{(0.0475)\,4\,(3.14)\left(\frac{10475.7}{32.2}\right)}{0.5}$$

$c = 380.31\ lb/sec/ft$

No of cycles to reduce displacement Amplitude from 7.27 to 0.5 in $j = ?$

$$j = \frac{1}{2\pi\zeta} \ln \left(\frac{u_1}{u_{j+1}}\right)$$

$$j = \frac{1}{2\pi(0.04)} \ln \left(\frac{7.27}{0.5}\right)$$

$$j = \frac{1}{0.251} (2.67)$$

$$\boxed{j = 3.98}$$ or 9 cycles
