Name	Attaullah khan
ID	7933
Section	B
Sub	Diff equation
Dept	Civil
Semister	4th
Exam	Final term

(1)

Question No # (01)

Part (i)

$$w = \sin(x+ct) + \cos(2x+2ct)$$

$$\boxed{\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}}$$

$$\frac{\partial w}{\partial t} = \cos(x+ct)+c - \sin(2x+2ct)+2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct)+c^2 - \cos(2x+2ct)+4c^2 \quad ----(1)$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct)+2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct$$

$$= [-\sin(x+ct) - 4.\cos(x+2ct)]$$

(2)

$$\frac{\partial^2 w}{\partial t^2} = +c^2\left[-\sin(x+ct) - 4(\cos(2x+2ct)\right]$$

$$\boxed{c^2 \cdot \frac{\partial^2 w}{\partial x^2}}$$

Part (II)
" "

$$w = \tan(2x+ct)$$

$$\frac{\partial w}{\partial t} = \underline{\partial \partial^2}(2x+ct)\frac{\partial}{\partial t}(2x+ct)$$

$$= c\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = l2.\sec(2x+ct)\frac{2\sec}{\partial t}(2x+ct)$$

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$\Rightarrow 2c^2 \sec(2x+ct) \sec(2x+ct) \tan(2x+ct)$

$\Rightarrow \frac{\partial^2 w}{\partial t^2} = 2c^2 \sec^2(2x+ct) \tan(2x+ct)$

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = 2 \cdot 2\sec(2x+ct) \cdot \sec(2x+ct)$$
$$\tan(2x+ct)$$

$$= 8\sec^2(2x+ct) \tan(2x+ct)$$

$\Rightarrow 2c^2 \sec^2(2x+ct) \tan(2x+ct)$

$\neq c^2\, 8\sec^2(2x+ct) \tan(2x+ct)$

So its not satisfied

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Question No # 2

Given

$$f(x) = \begin{cases} x ; & -\pi < x \leq 0 \\ 2x ; & 0 \leq x \leq \pi \end{cases}$$

Sol:

First find the constant

$$a_0 = ?$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \, dx$$

$$a_0 = \frac{1}{\pi} \left(\frac{x^2}{2} \Big|_{-\pi}^{0} + \frac{2x^2}{2} \Big|_{0}^{\pi} \right)$$

(2)

$$a_0 = \frac{1}{\pi}\left[\left(\frac{-\pi^2}{2} - \frac{-0^2}{2}\right) + \left(\pi^2 - 0^2\right)\right]$$

$$a_0 = \frac{1}{\pi}\left(\frac{\pi^2}{2} + \pi^2\right)$$

$$a_0 = \frac{1}{\pi}\left(\frac{-\pi^2 + 2\pi^2}{2}\right)$$

$$a_0 = \frac{1}{\pi}\left(\frac{8\pi^2}{2}\right)$$

$$a_0 = \frac{\cancel{8}\cancel{\pi}^2}{2\cancel{\pi}}$$

$$\boxed{a_0 = \frac{\cancel{8}1\pi}{2}}$$

$$\boxed{a_0 = \frac{\pi}{2}}$$

(3)

Determine the coefficient of an and b_n

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos nx \, dx$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} x \cos nx \, dx + \int_{0}^{\pi} 2x \cos nx \, dx\right]$$

The fun is odd, so an $= 0$

Now

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} \sin nx \, dx\right]$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} x \sin nx \, dx + \int_{0}^{\pi} 2x \sin nx \, dx\right.$$

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$$b_n = \frac{1}{\pi}\left(2\pi + \frac{1}{\pi}(2\pi)\right)$$

$$b_n = 4$$

Now

$$f(x) = \frac{a_0}{2} + \left[\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx\right]$$

$$f(x) = \frac{\frac{1}{2}\pi}{2} + \left[\sum_{n=1}^{\infty} 0 \cos nx + 4 \sin nx\right]$$

$$f(x) = \frac{1}{\pi} + 4\sum_{n=1}^{\infty} \sin nx$$

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(1)

Question No # 03

Sol:

$$y'' - 4y' + 13y = 8\sin 3x \longrightarrow (1)$$

$$y(0) = 1 \text{ and } y'(0) = 2$$

Associated Homogenous eq of (1) is

$$y'' - 4y' + 13y = 0 \longrightarrow (2)$$

The eq (2) change in auxilaury eq.

Put $y = m$ in eq (2)

$$m^2 - 4m + 13 = 0$$

use quadratic formula

(2)

$a = 1$, $b = -4$, $c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$m = \frac{+4 \pm \sqrt{-36}}{2}$$

$$m = \frac{4 \pm \sqrt{36i}}{2}$$

$$m = \frac{4 \pm 6i}{2}$$

$$m = 2 \pm 3i$$

$$m_1 = 2 + 3i$$

$$m_2 = 2 \mp 3i$$

(3)

$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \longrightarrow (A)$

Let

$y_p = A \cos 3x + B \sin 3x \longrightarrow (*)$

Diff w.r.t "x"

$y'_p = -3A \sin 3x + 3B \cos 3x$

Again Diff w.r.t x

$y''_p = -9A \cos 3x - 9B \sin 3x$

Put in (1)

$\Rightarrow (-9A \cos 3x - 9B \sin 3x) - 4(-3A \sin 3x + 3B \cos 3x) + 13(A \cos 3x + B \sin 3x) = 8 \sin 3x$

(4)

$\Rightarrow -9A\cos 3x - 12B\cos 3x + 13A\cos 3x,$

$- 9B\sin 3x + 12A\sin 3x + 13B\sin 3x = 8\sin 3x.$

$\Rightarrow (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x$

$= 8\sin 3x$

$\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$

Comparing Coefficient

$\sin 3x \Rightarrow 4B + 12A = 8 \rightarrow (a)$

$\cos 3x \Rightarrow 4A - 12B = 0 \Rightarrow 4A = 12B$

$\boxed{A = 3B} \rightarrow (b)$

Put (b) in (a)

$4B + 12(3B) = 8$

$4B + 36B = 8$

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$40B = 8$

$$\boxed{B = \frac{1}{5}} \rightarrow \text{(c)}$$

Put c in (b)

$$\boxed{A = 3/5} \rightarrow \text{(d)}$$

Put (c) and (d) in eq (*)

$$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow \text{(B)}$$

The general sol is

$$y = y_c + y_p$$

$$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow \text{(C)}$$

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Now we need to find the value of c_1 and c_2 for this

Put $x=0$ and $y=1$ in (C)

$$1 = e^{2(0)}\left(c_1 \cos 3(0) + c_2 \sin 3(0)\right) + \frac{3}{5}\cos 3(0) + \frac{1}{5}\sin 3(0)$$

$$1 = \left(c_1(1) + c_2(0)\right) + \frac{3}{5}(1) + \frac{1}{5}(0)$$

$$1 = c_1 + 3/5$$

$$c_1 = 1 - 3/5$$

$$\boxed{c_1 = 2/5} \longrightarrow (\star\star)$$

(7)

Diff ⓒ w.r.t x

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2$$

$$(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$$

$$-6/5 \sin 3x + 3/5 \cos 3x$$

Put $y' = 2$, $x = 0$ in ⓓ

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x} \cdot \sin 3x)$$

$$+ c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$$

$$-6/5 \sin 3x + 3/5 \cos 3x$$

Put $y' = 2$, $x = 0$

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$$2 = C_1\left(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)\right)$$

$$+ C_2\left(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0)\right)$$

$$- 6/5 \sin 3(0) + 3/5 \cos 3(0)$$

$$2 = C_1(2) + C_2(3) - 0 + 3/5$$

$$2 = 2C_1 + 3C_2 + 3/5$$

Put $C_1 = 2/5$

$$2 = 2\frac{(2)}{5} + 3C_2 + 3/5$$

$$2 = \frac{4}{5} + 3C_2 + 3/5$$

$$2 = \frac{7}{5} + 3C_2$$

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$3c_2 = 2 - 7/5$

$3c_2 = \frac{3}{5}$

$\boxed{c_2 = \frac{3}{15}} \rightarrow$ (☆☆☆)

Put (☆☆) and (☆☆☆) in C

$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x$

$+ 1/5 \sin 3x$

$y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x$

$+ \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$

①

Question No # 04

Given:

$$(D^2 - DD')Z = \cos x \cos 2y \rightarrow ①$$

Sol:

$$(D^2 - DD')Z = \cos x \cos 2y \rightarrow ①$$

Put A.E $(D^2 - DD') = 0$

As we know that

$$\frac{D}{D'} = m \quad \text{i.e } D = m, \; D' = 1$$

$$\Rightarrow \quad m^2 - m = 0$$

$$m = 0, 1$$

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Therefore $C.F = f_1(y) + f_2(y+x)$

from eq (1)

$$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y$$

$$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} 2\cos x \cdot \cos 2y$$

$\cancel{2\cos A \cos B = \cos(A+B) + \cos(A-B)}$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$Cf = f_1(y-x) + x f_2(y-x)$

$$P.I = \frac{1}{D^2 + 2DD' + D'^2}\left[2(y-x) + \sin(x-y)\right]$$

$$= \frac{1}{(D+D')^2}\left[2(y-x) + \sin(x-y)\right]$$

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By general method

$m = -1,\ y - x = c$

$\Rightarrow \frac{1}{D+D'}\left[(2c + \sin(-c)\right] dx$

$\Rightarrow \frac{1}{D+D'}\left[2cx - (\sin c)x\right]$

Replacing by $y-x$

$\Rightarrow \frac{1}{D+D'}\left[2x(y-x) - x\sin(y-x)\right]$

again put $y - x = c$

$= \int (2xc - x\sin c)\, dx$

$\Rightarrow cx^2 - \frac{x^2}{2} \sin c$

(4)

Replacing c by $y-x$

$$= x^2(y-x) - \frac{x^2}{2}\sin(y-x)$$

$$\Rightarrow x^2y - x^3 + \frac{x^2}{2}\sin(x-y)$$

Hence the req solution is

$$Z = C.F + P.I = f_1(y-x) + xf_2(y-x)$$
$$+ x^2y - x^3 + \tfrac{1}{2}x^2 \sin(x-y)$$