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Department: BS(CS)

Subject : Discrete structure

Assignment No: 02

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Semester : 2nd

## Question = 1

### Recurrence Relation $\Rightarrow$

A recurrence relation for the sequence  $\{a_n\}$  is an equation that express  $a_n$  in terms of one or more of the Previous of the sequence, namely,  $a_0, a_1, \dots, a_{n-1}$ , for all integer  $n$  with  $n \geq n_0$  is a nonnegative integer. A sequence is called a solution of a recurrence relation if its terms satisfy the recurrence relation.

**Example  $\Rightarrow$**  Let  $\{a_n\}$  be a sequence the satisfies the recurrence relation relation  $a_n = a_{n-1} + 3$  for  $n = 1, 2, 3, \dots$  and suppose that  $a_0 = 2$ . what are  $a_1, a_2$  and  $a_3$ .

we see recurrence relation

$$a_1 = a_0 + 3 = 2 + 3 = 5$$

$$\text{then } a_2 = 5 + 3 \quad a_3 = 8 + 3 = 11$$

Using Substitution Method:

$$T(n) = \begin{cases} T(n/2) + c & \text{if } n > 1 \\ 1 & \text{if } n = 1 \end{cases}$$

$$T(n) = T(n/2) + c$$

$$T(n/2) = T(n/4) + c$$

$$T(n/4) = T(n/8) + c$$

we now  $T(n)$  is equal to  $T(n/2) + c$   
and again  $T(n/2)$  is equal to  $T(n/4) + c$   
we write

$$T(n) = T(n/2) + c$$

we write

$$T(n) = T(n/4) + c + c$$

$$= T(n/2^2) + 2c$$

then

$$= T(n/2^3) + 3c$$

then

$$= T(n/2^4) + 4c$$

⋮  
k

$$T(n/2^k) + kc$$

$$\therefore n = 2^k$$

we log both side

$$T(n/n) + kc$$

$$\log n = \log 2^k$$

$$= T(1) + kc$$

$$n = k \log 2$$

$$= 1 + \log_2 n c$$

and c constant

$$= O(\log_2 n)$$

order

## Question : 2

Premises: If there was a ball game, then traveling was difficult. If they arrived on time, then traveling was not difficult. They arrived on time.

Conclusion: There was no ball game.

Determine symbolically using the rule of inference.

$P$ : There was a ball game

$Q$ : Traveling was difficult.

$Y$ : They arrived on time.

Sol:

1)  $P \rightarrow Q$  Premises

2)  $Y \rightarrow \sim Q$  Premises

3)  $Y$  Premises

4)  $\sim Q$  2, 3 Modus Ponens

5)  $\sim Q \rightarrow \sim P$  1, Contrapositive

$\sim P$

4, 5 Modus Ponens

Conclusion:  $\sim P$  = There was no ball game

### Question : 3

Premises: If Claghorn has wide support, then he'll be asked to run for the Senate. If Claghorn yells "Eureka" in Iowa, he will not be asked to run for the Senate. Claghorn yells "Eureka" in Iowa.

Conclusion: Claghorn does not have wide support.

Determine logically symbolically using rule of inference.

Sol:

P: Claghorn has wide support.

Q: Claghorn is asked to run for the Senate.

Y: Claghorn yells "Eureka".

$P \rightarrow Q$  Premises

$Y \rightarrow \sim Q$  Premises

Y Premises

$\sim Q$  2, 3 Modus Ponens

$\sim Q \rightarrow \sim P$  1 Contrapositive

$\sim P$  5, 6 Modus Ponens

Conclusion  $\sim P$  = Claghorn does not have wide support.

Question : 4

## Pigeon hole Principal:

Consider a flock of Pigeons nestled in a set of  $n$  Pigeonhole. If there are  $n$  Pigeon, then it is Possible for all of the Pigeon to rest happily in separate Pigeonholes. However, if at least one more Pigeon arrives, making a total of more than  $n$  Pigeon, then at least one of the Pigeonhole, inevitably, will end up with more than one Pigeon.

The idea that having more Pigeons than Pigeonhole requires a Pigeonhole with more than one Pigeon is seemingly trivial, but it turns out to be important enough that it has a name.

### Theorems of Pigeonhole Principal:

If more than  $n$  object are Placed into  $n$  boxes, then at least one box must contain more than one object.

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At first glance, the Pigeonhole Principle also known as Dirichlet's Principle in honor of the eponymous German mathematician might appear to be too obvious to be useful; indeed, the Power of the Principle comes from cleverly choosing the "boxes" and "object". Even though the Principle itself is quite simple, it is not always clear if it is useful and, if so, how. For instance, consider the following example:

### Example of Pigeonhole Principle:

A box contains three pairs of socks colored red, blue, and white, respectively. Suppose I take out the socks without looking at them. How many socks must I take out to be sure that they will include a matching pair?

If I take only 2 or 3 socks, it is possible that they are different. For example: they may be one red and one blue; or one red, one blue and one white. But if I

take out 4 Socks, these must include a matching Pair. Here the 4 chosen socks are the "object" and the 3 colors are the "boxes"; by PPI, it follows that at least two of the four chosen must have the same colour and hence must be a matching Pair. Thus the minimum number of socks to be taken out is 4.

**Ans:** There are  $26 \times 26 = 676$  different possibilities set of two initials that can be obtained using the 26 letter A, B, C, ..., Z. So the number of student should be greater than 676. Thus the minimum number of student is 677.

**Example 2** How many student do you need in school to guarantee that there are least 2 student who have the same first two initials?



## Question : 5

The light in classroom are controlled by two switches; one at the back and one at the front of the room. Moving either switches to the opposite position turns the light off if they are on and on if they are off. Assume the light have been installed so that when both switches are in down position, the light are off.

Design a circuit to contrall the switches.

Sol:

Let  $P$  and  $Q$  be switches

Let  $R$  be the light-

Let 0 represent that the switch is in the down position and let 1 represent that the switch is in the up position.

$R=0$  represent that light are off and

$R=1$  represent that the light are on.

If both switches are in the down position, then the light are ~~turned on~~ are off

$P=0$  and  $Q=0$ , then  $R=0$

If one of the switches is turned in the up position, then the light are turned on.

$$P = 1 \text{ and } Q = 0, \text{ then } R = 1$$

$$P = 0 \text{ and } Q = 1, \text{ then } R = 1$$

If one of the switches was in the up position (light are on) and the other switch is also turned into the up position, then the light should turn off again.

$$P = 1 \text{ and } Q = 1 \text{ then } R = 0$$

Combining this information is an input/output table, we then obtain.

Input		<del>output</del>	output
P	Q		R
1	1		0
1	0		1
0	1		1
0	0		0

Boolean Expression.

Negation  $\sim P$  : Not P

Disjunction  $P \vee Q$  : P or Q

Conjunction  $P \wedge Q$  : P and Q

we note that the second and third row of the given input/output result in an output of 1.

The second row corresponds with  $P=1$   
 $Q=0$ . If  $Q=0$  then the negation has opposite value thus  $\sim Q=1$ .

Thus we obtain  $R=1$  in the first row if  $P=1$  and  $\sim Q=1$ : or equivalently if their conjunction  $P \wedge \sim Q$  is equal to 1.

$$P \wedge \sim Q$$

The third row corresponds with  $P=0$   
 and  $Q=1$

if  $P=0$ , then the negation has the opposite value, thus  $\sim P=1$

Thus, we obtain  $R=1$  in the third row if  $\sim P=1$  and  $Q=1$   
 or equivalently if their conjunction

$$\sim P \wedge Q$$

The Boolean expression corresponding to the given table is then the disjunction of the two

Previous statement, as we either have second row or the third row when  $S = 1$

$$(P \wedge \sim Q) \vee (\sim P \wedge Q)$$

Circuit  $\Rightarrow$

A NOT gate output the negation  $\sim a$  of the input  $a$ .

An OR gate output disjunction  $a \vee b$  of the input  $a$  or  $b$ .

An AND gate output the conjunction  $a \wedge b$  of the input  $a$  and  $b$ .

$$(P \wedge \sim Q) \vee (\sim P \wedge Q)$$

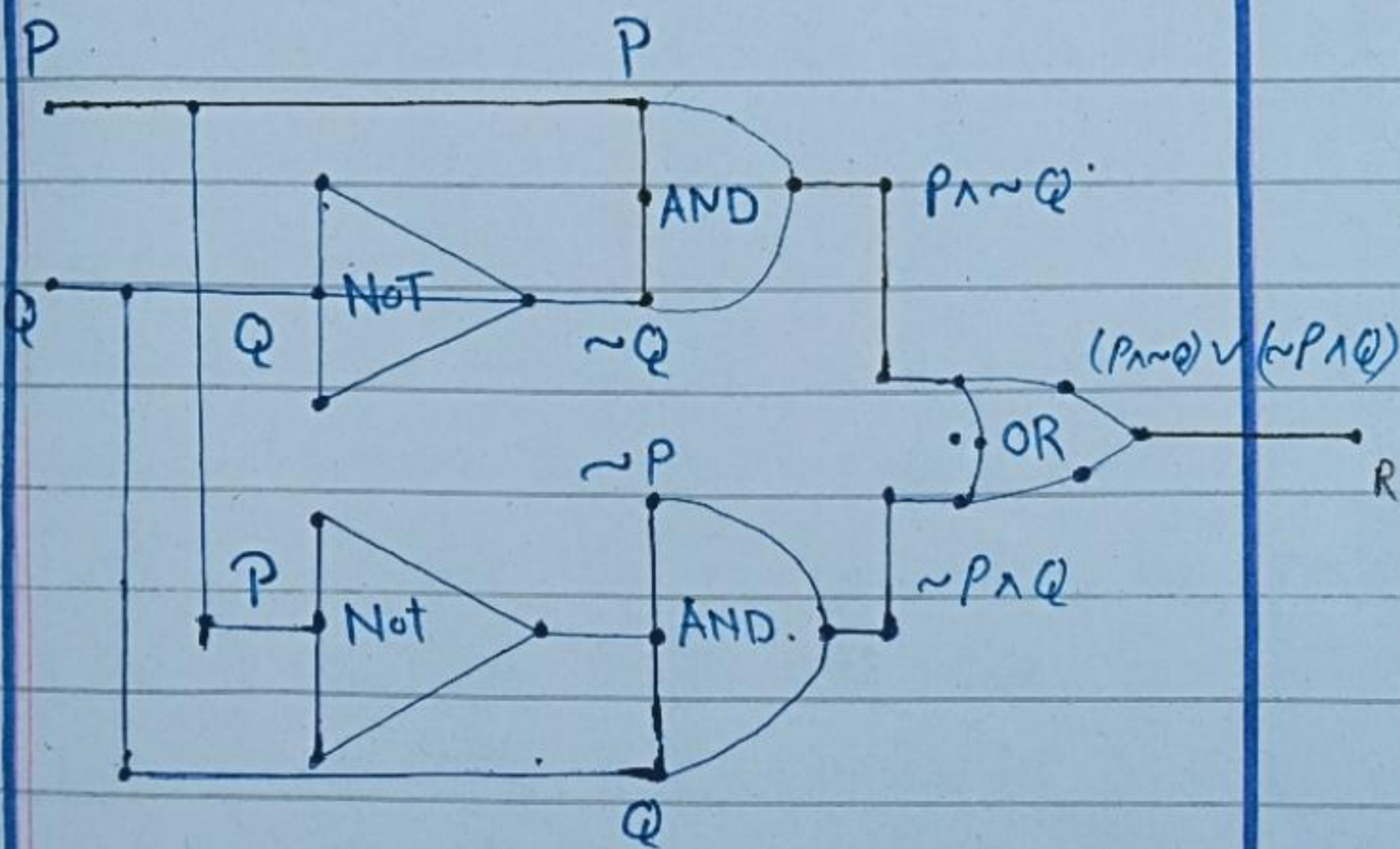
The negation will occur before the conjunction and disjunction, thus we first negate  $P$  by using a NOT gate with input  $P$  and we negate  $Q$  by using a NOT gate with input  $Q$ .

Next, the conjunction will occur before the disjunction in  $(P \wedge \sim Q) \vee (\sim P \wedge Q)$  due to the two pairs of brackets.

We then need to use an AND gate with input  $P$  and  $\sim Q$  to form  $P \wedge \sim Q$ .

While we use an AND gate with input  $\sim P$  and  $Q$  to form  $\sim P \wedge Q$ .

Finally, we need to take the disjunction of  $P \wedge \sim Q$  and  $\sim P \wedge Q$  (output AND gate) by using an OR gate.



Design circuit Draw

$$\text{Result} = (P \wedge \sim Q) \vee (\sim P \wedge Q)$$

### Question : 6

An alarm system has three different control panels in three different locations. To enable the system, switches in at least two of the panels must be in the on position. If fewer than two are in the on position, the system is disabled. Design a circuit to control switches.

Sol:

Let  $P$ ,  $Q$  and  $R$  be the three control panels. Let  $S$  be the system.

Let 0 represent that the switch in the control panel is in the off position and let 1 represent that the switch in the control panel is in the on position.

$S=0$  represent that the system is disabled and  $S=1$  represent that the system is enabled.

If two or three of the control panels have the switches in the on position, then the system is enabled.

$P = 0$  and  $Q = 1$  and  $R = 1$ , then  $S = 1$

$P = 1$  and  $Q = 0$  and  $R = 1$ , then  $S = 1$

$P = 1$  and  $Q = 1$  and  $R = 0$ , then  $S = 1$

$P = 1$  and  $Q = 1$  and  $R = 1$ , then  $S = 1$

If one or more of the control Panel have the switch in the on Position, then the system is disable.

$P = 1$  and  $Q = 0$  and  $R = 0$ , then  $S = 0$

$P = 0$  and  $Q = 1$  and  $R = 0$ , then  $S = 0$

$P = 0$  and  $Q = 0$  and  $R = 1$ , then  $S = 0$

$P = 0$  and  $Q = 0$  and  $R = 1$ , then  $S = 0$

Combining this information in an input/output table, we obtain

input			output
P	Q	R	S
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

## Boolean Expression:

Negation  $\sim P$ : not P

Disjunction  $P \vee Q$ : P or Q

Conjunction  $P \wedge Q$ : P and Q

IF two of the three control Panels have the switches in the position.

(S=1) then either  $P \wedge Q$ ,  $P \wedge R$  or  $Q \wedge R$  is equal to 1 (or all three are true at the same time when all three control Panels have the switches in the on position). The Boolean expression is then the Disjunction of these three statements.

$$(P \wedge Q) \vee (Q \wedge R) \vee (P \wedge R)$$

note: You should could also use each row of the table to represent one the term in the disjunction  $(P \wedge Q) \vee (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (P \wedge Q \wedge R) \vee (\sim P \wedge Q \wedge R)$ , but this will result in a much larger and more complex circuit.



## circuit

A NOT gate output the negation  $\sim a$  of input  $a$ .

An OR gate output the disjunction  $a \vee b$  of the input  $a$  and  $b$ .

An AND gate output the conjunction  $a \wedge b$  of the input  $a$  and  $b$ .

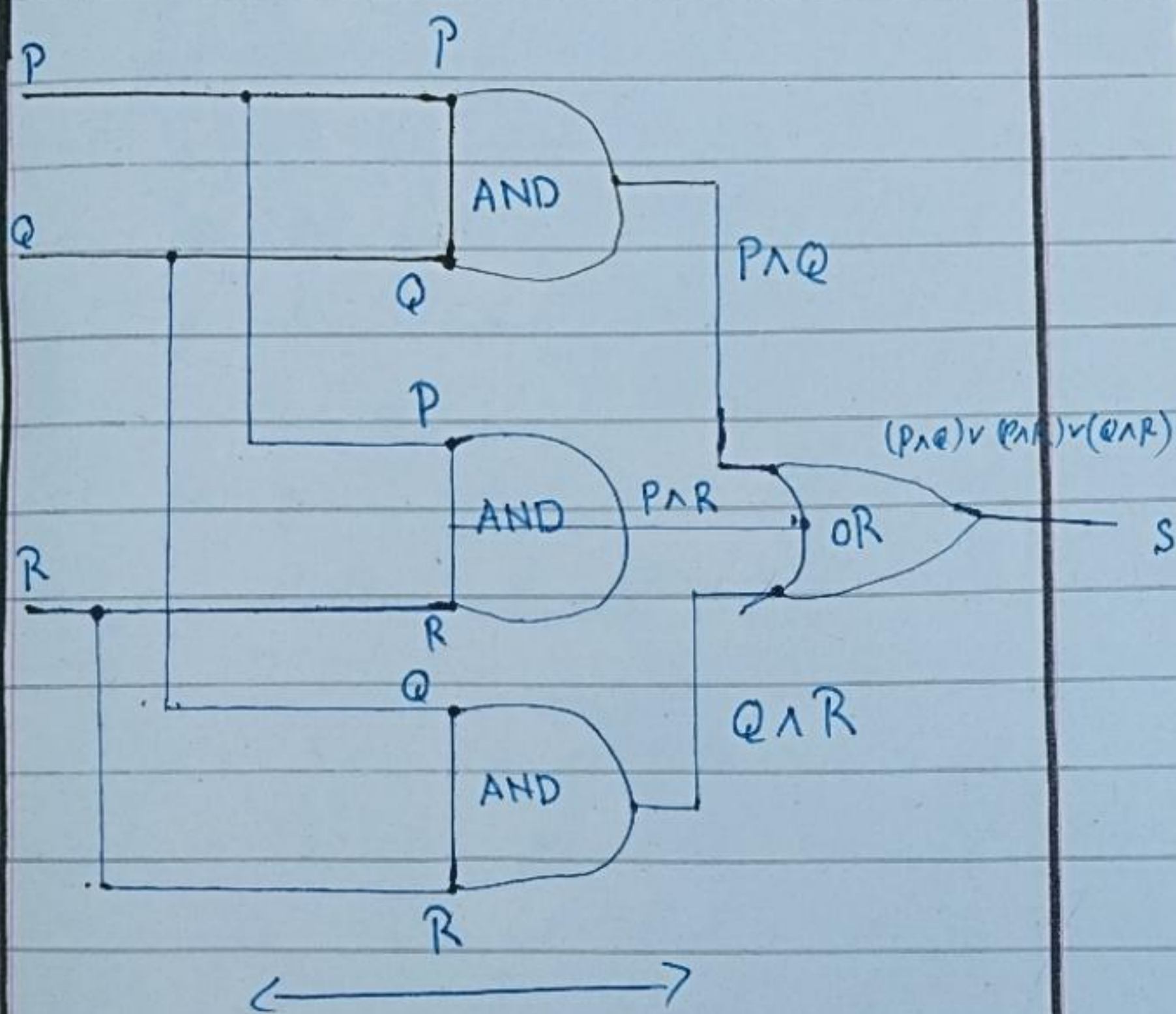
$$(P \wedge \sim Q) \vee (\sim P \wedge Q)$$

The negation will occur before the conjunction and disjunction, thus we first negate  $P$  by using a NOT gate with input  $P$  and we negate  $Q$  by using a NOT gate with input  $Q$ .

Next, the conjunction will occur before the disjunction in  $(P \wedge \sim Q) \vee (\sim P \wedge Q)$  due to the two pair of brackets.

We then need use an AND gate with input  $P$  and  $\sim Q$  to form  $P \wedge \sim Q$ . while we use an AND gate with input  $\sim P$  and  $Q$  to form  $\sim P \wedge Q$ .

Finally, we need to take the disjunction of  $P \wedge \sim Q$  and  $\sim P \wedge Q$  (output AND gate) by using an OR gate.



Design circuit

$$(P \wedge Q) \vee (P \wedge R) \vee (Q \wedge R)$$

## Question 7

Given the output signal for the circuit.

## (A) Answer

A Not gate change the input from a 0 to 1 or from a 1 to a 0.

An OR gate output a 1 if at least one of the input is a 1 else it output 0.

An AND gate output a 1 if both input are a 1 else it output 0.

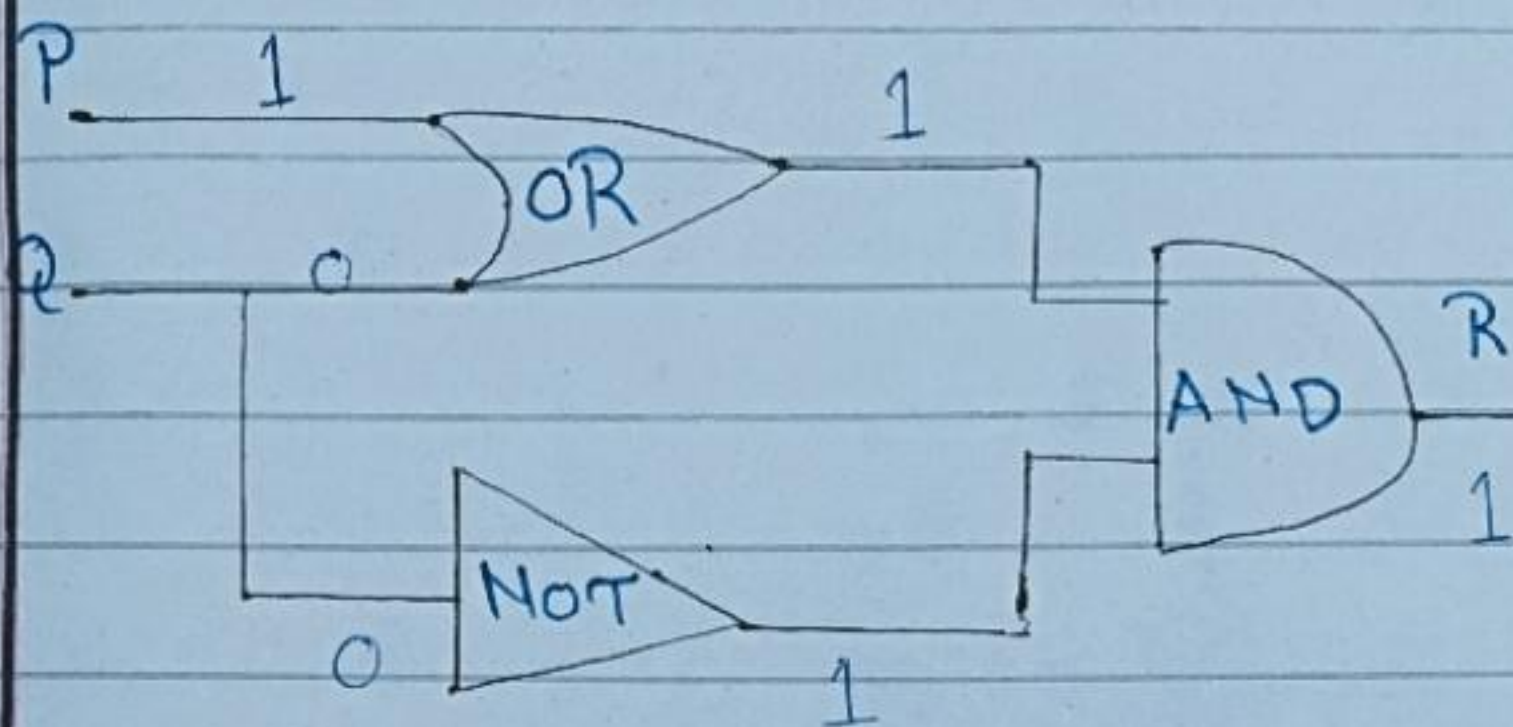
$$P = 1 \quad Q = 0$$

we note that the OR gate receive on input of 1 from P and an input of 0 from Q. Since at least one of the input is a 1. The OR gate will be output a 1.

we note that the 'not gate' receive on input of 0 (from Q) a 1 thus the not gate output a 1.

Finally, we than note that the AND gate receive on input of 1

from the OR gate and an input 1 from the NOT gate. As both inputs are a 1, the AND gate will output a 1 as well thus output signal is  $R=1$ .



Input		Output
P	Q	R
1	0	0
1	1	1
0	0	0
0	1	0

(B) Answer  $Q = 7$   
 Input signal  $P = 1$   $Q = 0$   $R = 0$

A 'NOT' gate changes input from 0 to 1 or 1 to 0. An 'OR gate' outputs 1 if at least one of input 1.

A 'AND gate' outputs 1 if both inputs are the same.  $P = 1$   $Q = 0$ ,  $R = 0$

We note that a NOT gate received an input 0 (from Q). Thus the NOT gate

is output 1. We note that

AND gate received an input 1 from (P) and input of 1 from Q.

Since both inputs are 1, the output is 1.

Lastly, we note that an OR gate receives an input of 1 from the AND gate

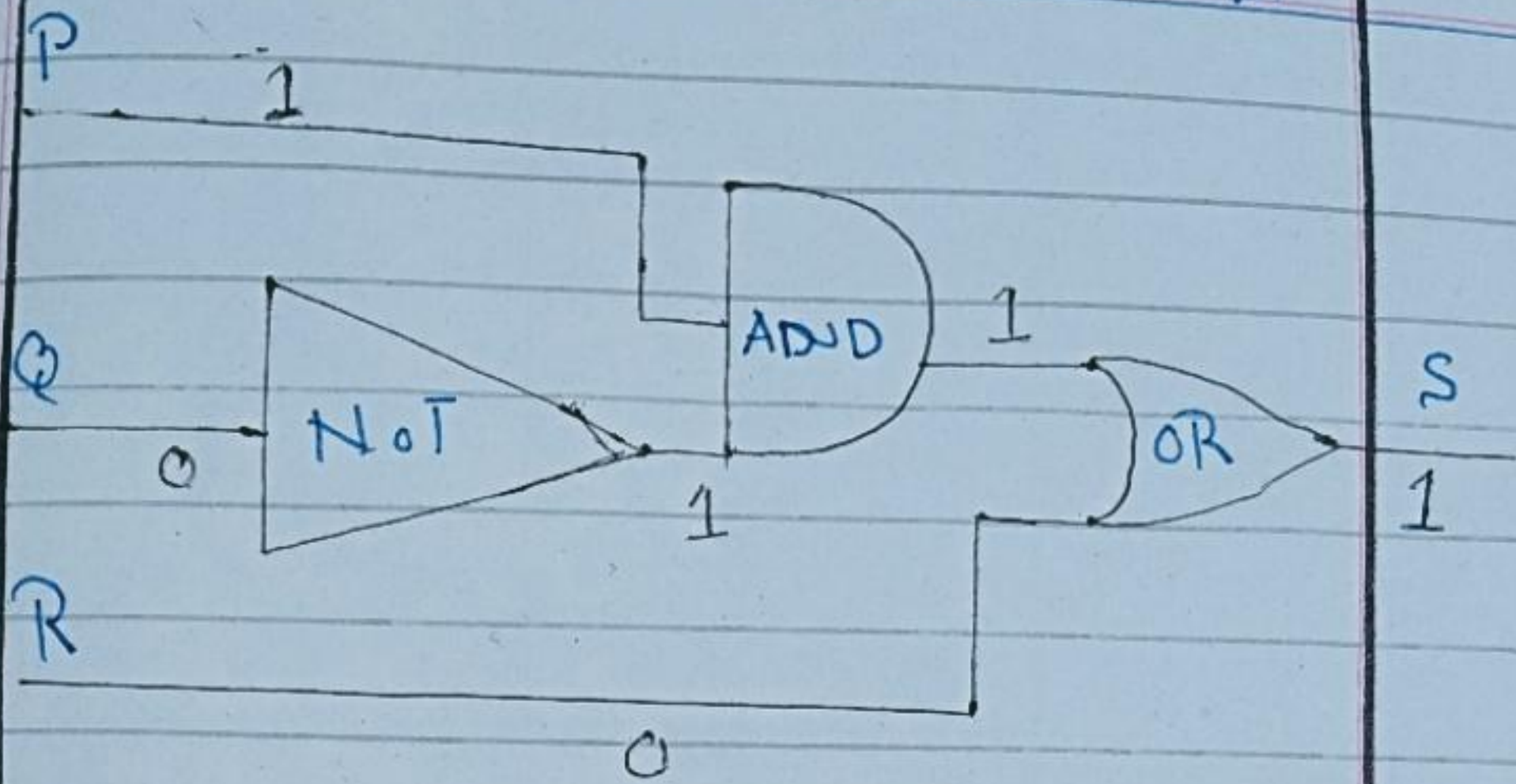
and an input of 0 from R. As at least one of the inputs is 1, the

OR gate will output 1 as well.

Thus the output is  $S = 1$ .

Similarly, we can derive the output of each gate for every

possible input P, Q, and R.



P	Q	R	output Not gate	output AND	output OR
1	1	1	0	0	1
1	1	0	0	0	0
1	0	1	1	1	1
1	0	0	1	1	1
0	1	1	0	0	1
0	1	0	0	0	0
0	0	1	1	0	1
0	0	0	1	0	0

Input			Output
P	Q	R	S
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

(C) Q = 7 Answer

input signal  $P=0$ ,  $Q=0$ ,  $R=0$

A 'NOT gate' change the input from 1 to 0

or 0 to 1. An 'OR' gate output 1

if at least one of them 1.

An 'AND' gate output 1 if both

input is 1.

we note that the left most OR gate

receive on input 0 (from P) and

on input 0 (from Q). Since

at least one of the input is a

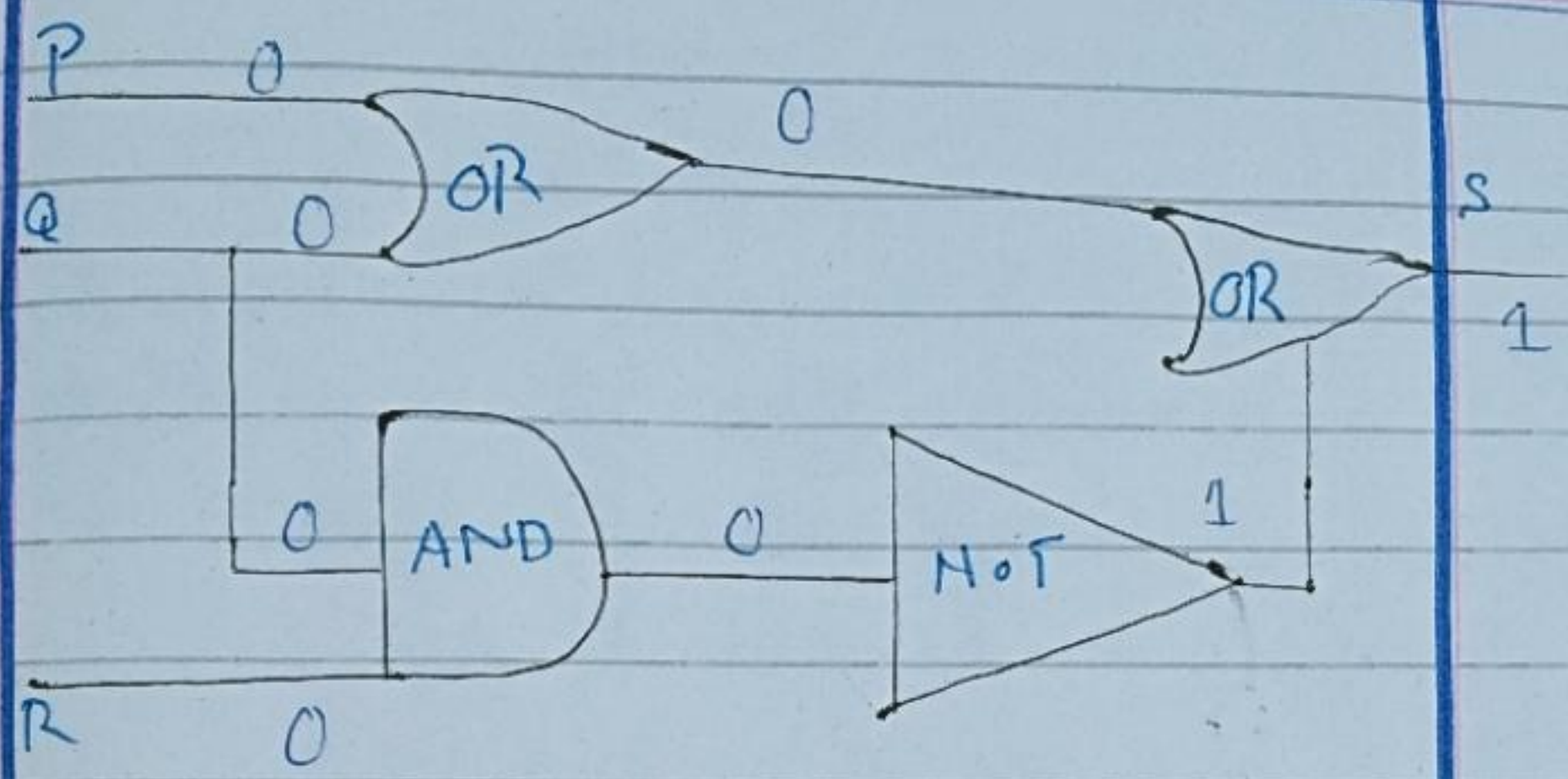
0. the OR gate will output 0.

we note that gate the AND gate receive an input of 0 from A and an input 0 (from B.) since both input are 0. AND gate will output 0.

we then note that the NOT gate receive an input of 0 from the AND gate thus the NOT gate will output 1.

Lastly, we then note that the right most OR gate receive an input of 0 from the left most OR gate on input of 1 from the NOT gate. As at least one of the OR gate will output 1 as well and thus the output signal  $S=1$





Input                      Output

P	Q	R	S
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

## Question 8 Answer

$$A = \{0, 1, 2, 3\}$$

A relation  $R$  on a set  $A$  is reflexive if  $(a, a) \in R$  for every element  $a \in R$

A relation  $R$  on a set  $A$  is symmetric if  $(b, a) \in R$  whenever  $(a, b) \in R$

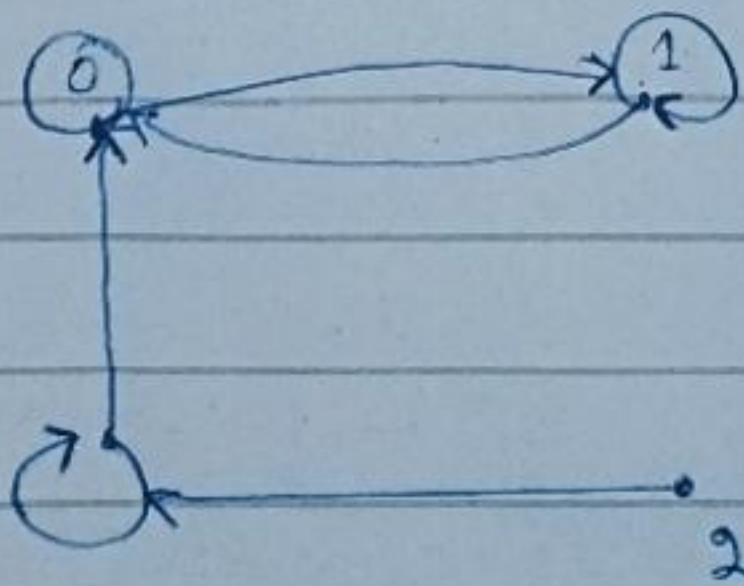
A relation  $R$  on a set  $A$  is transitive if  $(a, b) \in R$  and  $(b, c) \in R$  implies  $(a, c) \in R$ .

$$A = \{0, 1, 2, 3\}$$

$$R_1 = \{(0,0), (0,1), (0,3), (1,1), (1,2), (2,3), (3,3)\}$$

(a) Directed Graph  $\Rightarrow$

we note that  $0 \in A$  contain 4 element and thus we will draw 4 point. we label these point  $(0, 1, 2, 3)$  which are the elements of  $A$ . For every element  $(x, y) \in R$  with  $x \neq y$ , we draw an arrow from  $x$  to  $y$ .



(B) The relation  $R_1$  is reflexive if  $(a,a) \in R_1$  for every point  $a \in A$ .

Since  $A = \{0, 1, 2, 3\}$ ,  $R_1$  is reflexive if it contains  $(0,0), (1,1), (2,2), (3,3)$  we note that  $R_1$  does not contain  $(2,2)$  and thus  $R_1$  is not reflexive.  $R_1$  is not reflexive.

(C) The relation  $R_1$  on a set  $A$  is symmetric if  $(b,a) \in R_1$  whenever  $(a,b) \in R_1$ .

We note that  $(0,3) \in R_1$  while  $(3,0) \notin R_1$  thus  $R_1$  is not symmetric.

$R_1$  is not Reflexive.

(D) The relation  $R_1$  on a set is transitive if  $(a,b) \in R_1$  and  $(b,c) \in R_1$

$\in R_1$  implies  $(a,c) \in R_1$ .

We note that  $(1,0) \in R_1$  and  $(0,3) \in R_1$ , while  $(1,3) \notin R_1$  and thus

$R_1$  is not transitive.

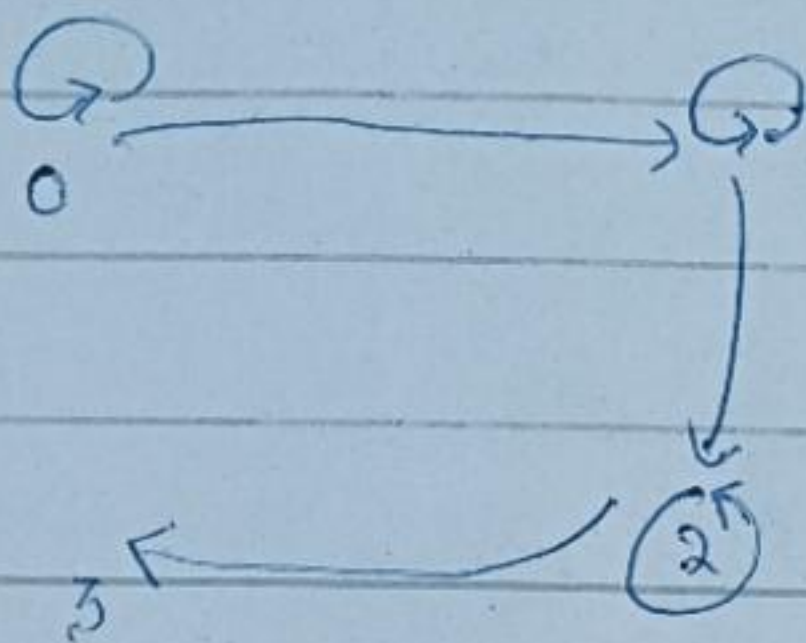
$R_1$  is not transitive.

$$R_2 = \{ (0,0), (0,1), (1,1), (1,2), (2,2), (2,3) \}$$

$$A = \{ 0, 1, 2, 3 \}$$

$$R_2 = \{ (0,0), (0,1), (1,1), (1,2), (2,2), (2,3) \}$$

(a) Directed Graph



(b) No,  $R_2$  is not reflexive because there is no loop at 3.

(c) No,  $R_2$  is not symmetric because between two number where there exists an allow, the allow goes only one direction.

(d) No,  $R_2$  is not transitive though there is an allow between 0 and 1 and also allow between 1 and 2 but is no allow between 0 and 2.

$$R_3 = \{ (2,3), (3,2) \}$$

$$A = \{ 0, 1, 2, 3 \}$$

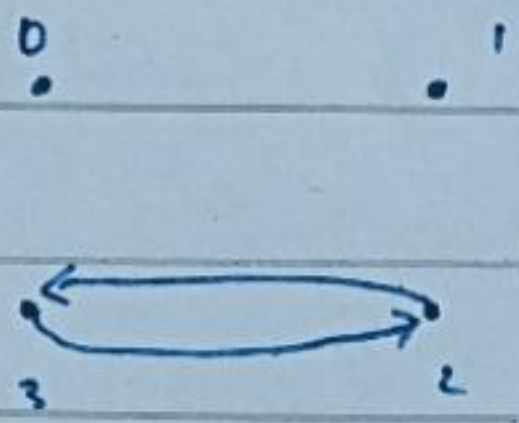
(a) Directed Graphs

we note that contain

u element and thus we will draw u point. we label these point 0, 1, 2, 3 which are the element of A.

for every element  $(x, y) \in R$  with  $x \neq y$  we draw an arrow from x to y.

For every element  $(x, x) \in R$ , we draw a loop at the point x.



(b) The relation  $R_3$  is reflexive if  $(a, a) \in R_3$  for every element  $a \in A$ .

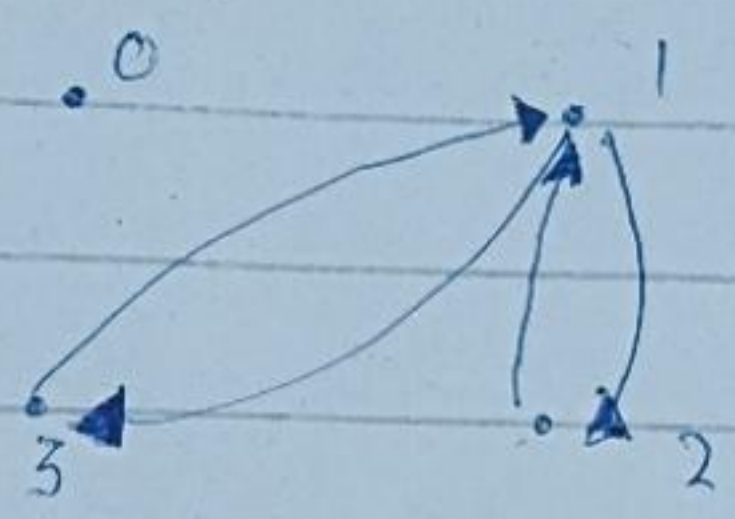
Since  $A = \{0, 1, 2, 3\}$   $R_3$  is reflexive if it contain  $(0,0), (1,1), (2,2), (3,3)$ . we note that  $R_3$  does not contain  $(0,0)$  and thus  $R_3$  is not reflexive.  $R_3$  is not reflexive.

(c) The relation  $R_3$  on a set A is symmetric if  $(b, a) \in R_3$  whenever  $(a, b) \in R_3$  we note that  $(2,3) \in R_3$  while  $(3,2) \in R_3$  and there are no other closeded pairs in  $R_3$ , thus  $R_3$  is symmetric.

$R_3$  is symmetric.

4  $R_u = \{ (1,2), (2,1), (1,3), (3,1) \}$   
 $A = \{ 0, 1, 2, 3 \}$

(a) Directed Graph:



b) No,  $R_u$  is not reflexive because there is no loop at 1.

$R_u$  is not reflexive.

c) Yes  $R_u$  is symmetric because between two number where there exist an arrow, the arrow go both way

$R_u$  is symmetric.

d) No  $R_u$  is not transitive though there are arrows between 2 and 1 and also arrows between 1 and 3 but there are no arrows between 2 and 3.

$R_u$  is not transitive.

$$5 \quad R_S = \{ (0,0), (0,1), (0,2), (1,2) \}$$

$$A = \{ 0, 1, 2, 3 \}$$

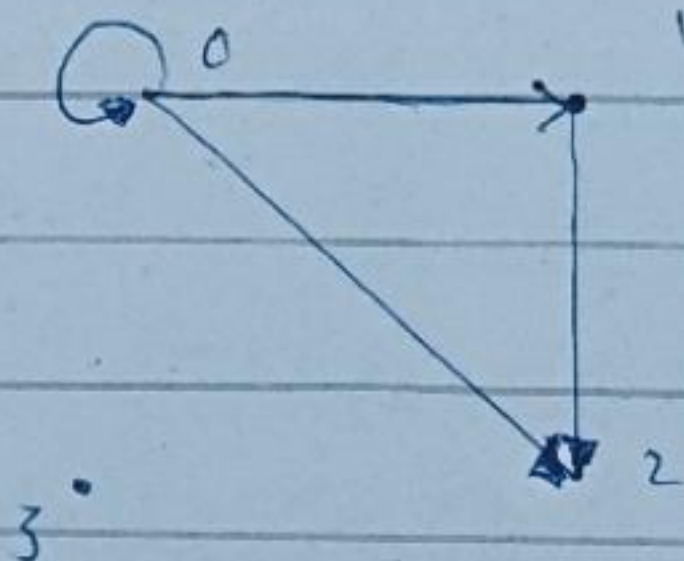
a) Directed Graph:

we note  $A$  contain 4 element  
and thus will draw 4 points.

we label these point 0, 1, 2, 3 which  
are the element of  $A$ .

For every element  $(x,y) \in R$  with  $x \neq y$   
we draw an arrow from  $x$  to  $y$ .

For every element  $(x,x) \in R$ , we draw a  
loop at the point  $x$ .



b) The relation  $R_S$  is reflexive if  
 $(a,a) \in R_S$  for every element  $a \in A$ .

Since  $A = \{ 0, 1, 2, 3 \}$   $R_S$  is reflexive  
if it contain  $(0,0), (1,1), (2,2), (3,3)$

we note  $R_S$  does not contain  $(1,1)$

and thus  $R_S$  is not  
reflexive

$R_S$  is not reflexive.

(c) The  $R_5$  on a set  $A$  is symmetric if  $(b, a) \in R_5$  whenever  $(a, b) \in R_5$

we note that  $(0, 1) \in R_5$  while  $(1, 0) \notin R_5$  and thus  $R_5$  is not symmetric

$R_5$  is not symmetric.

(d) The  $R_5$  on a set  $A$  is transitive if  $(a, b) \in R_5$  and  $(b, c) \in R_5$  implies

$(a, c) \in R_5$ .

we note that  $(0, 1) \in R_5$  and  $(1, 2) \in R_5$  while  $(0, 2) \in R_5$  and thus  $R_5$  is

transitive as there are no other pairs for which transitivity can be

checked.  $R_5$  is transitive.

$$6 \quad R_6 = \{(0, 1), (0, 2)\}$$

(a) Directed Graph:

$$A = \{0, 1, 2, 3\}$$

we note that  $A$  contains 4 elements and thus we will draw 4 points. we label these points  $0, 1, 2, 3$  which are elements

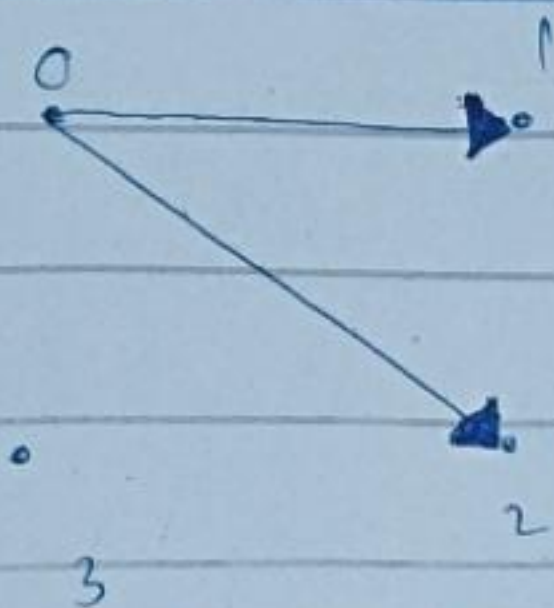
of  $A$ . For every element  $(x, y) \in R_6$  with

$x \neq y$  we draw an arrow from

$x$  to  $y$ . For every element  $(x, x)$

$\in R_6$  we draw a loop at the point  $x$ .





b) The relation  $R_6$  is reflexive if  $(a,a) \in R_6$  for every element  $a \in A$ .

Since  $A = \{0, 1, 2, 3\}$ ,  $R_6$  is reflexive if it contains  $(0,0)$ ,  $(1,1)$ ,  $(2,2)$ ,  $(3,3)$ . We note that  $R_6$  does not contain  $(0,0)$  and thus  $R_6$  is not reflexive.

$R_6$  is not reflexive.

c) The relation  $R_6$  on a set  $A$  is symmetric if  $(b,a) \in R_6$  whenever  $(a,b) \in R_6$ . We note that  $(0,1) \in R_6$  while  $(1,0) \notin R_6$  and thus  $R_6$  is not symmetric.

d) ~~The~~ The relation  $R_6$  on a set  $A$  is transitive if  $(a,b) \in R_6$  and  $(b,c) \in R_6$  implies  $(a,c) \in R_6$ . We note that the if statement  $(a,b) \in R_6$  and  $(b,c) \in R_6$  is never true as there exists now such pairs in  $R_6$ . When the if-statement is false and thus  $R_6$  is transitive.  
 $R_6$  is transitive.

$$7 \quad R_7 = \{(0, 3), (2, 3)\}$$

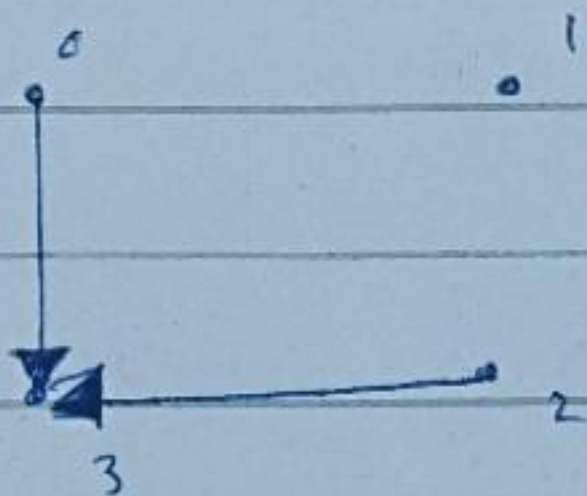
$$A = \{0, 1, 2, 3\}$$

(a) Directed Graph:

we note that  $A$  contain 4 element and thus we will draw 4 points. we label these point 0, 1, 2, 3 which are the element of  $A$ .

for every element  $(x, y) \in R$   $x \neq y$  we draw arrow  $x$  to  $y$ .

for every element  $(x, x) \in R$ , we draw a loop at the point  $x$ .



b) The relation  $R_7$  is reflexive if  $(a, a) \in R_7$  for every element  $a \in A$ .

since  $A = \{0, 1, 2, 3\}$   $R_7$  is reflexive if it contain  $(0, 0), (1, 1), (2, 2), (3, 3)$

we note that  $R_7$  does not contain

$(0, 0)$  and thus  $R_7$  is not reflexive.

$R_7$  is not reflexive.

c) The  $R_7$  on set  $A$  symmetric if  $(b,a) \in R_7$  whenever  $(a,b) \in R_7$

we note that  $(0,3) \in R_7$  while  $(3,0) \notin R_7$  and thus  $R_7$  is not symmetric

(d) The  $R_7$  on a set  $A$  is transitive if  $(a,b) \in R_7$  and  $(b,c) \in R_7$  implies  $(a,c) \in R_7$ .

We note that if statement  $(a,b) \in R_7$  and  $(b,c) \in R_7$  is never true as there exist no such pairs in  $R_7$ .

How ever an if-then statement is always true.

$R_7$  is transitive.

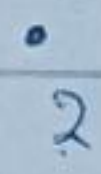
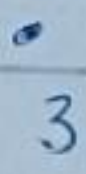
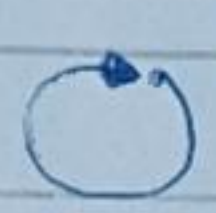
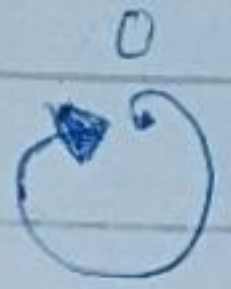
$$R_8 = \{(0,0), (1,1)\}$$

$$A = \{0, 1, 2, 3\}$$

### (a) Directed Graphs

we note that certain element and then we will draw a point. we labels these point  $0, 1, 2, 3$  which are the element of  $A$ . For every element  $(x,y) \in R$  with  $x \neq y$ . we draw an arrow from  $x$  to  $y$ .

For every loop element  $(x, x) \in R$ , we draw a loop at one point.



b) The relation  $R_S$  is reflexive  $(a, a) \in R_S$  for every element  $a \in A$ .

Since  $A = \{0, 1, 2, 3\}$   $R_S$  is reflexive if it contains  $(0, 0)$   $(1, 1)$   $(2, 2)$   $(3, 3)$  we note that  $R_S$  does not contain  $(2, 2)$  and thus  $R_S$  is not reflexive.

$R_S$  is not reflexive.

c) The relation  $R_S$  on a set  $A$  is symmetric if  $(b, c) \in R_S$  whenever  $(a, b) \in R_S$ . we note that  $(0, 0) \in R_S$  while  $(0, 0) \in R_S$  and  $(1, 1) \in R_S$  while  $(1, 1) \in R_S$ . There are no other element in  $R_S$  so the  $R_S$  is symmetric.

$R_S$  is symmetric.

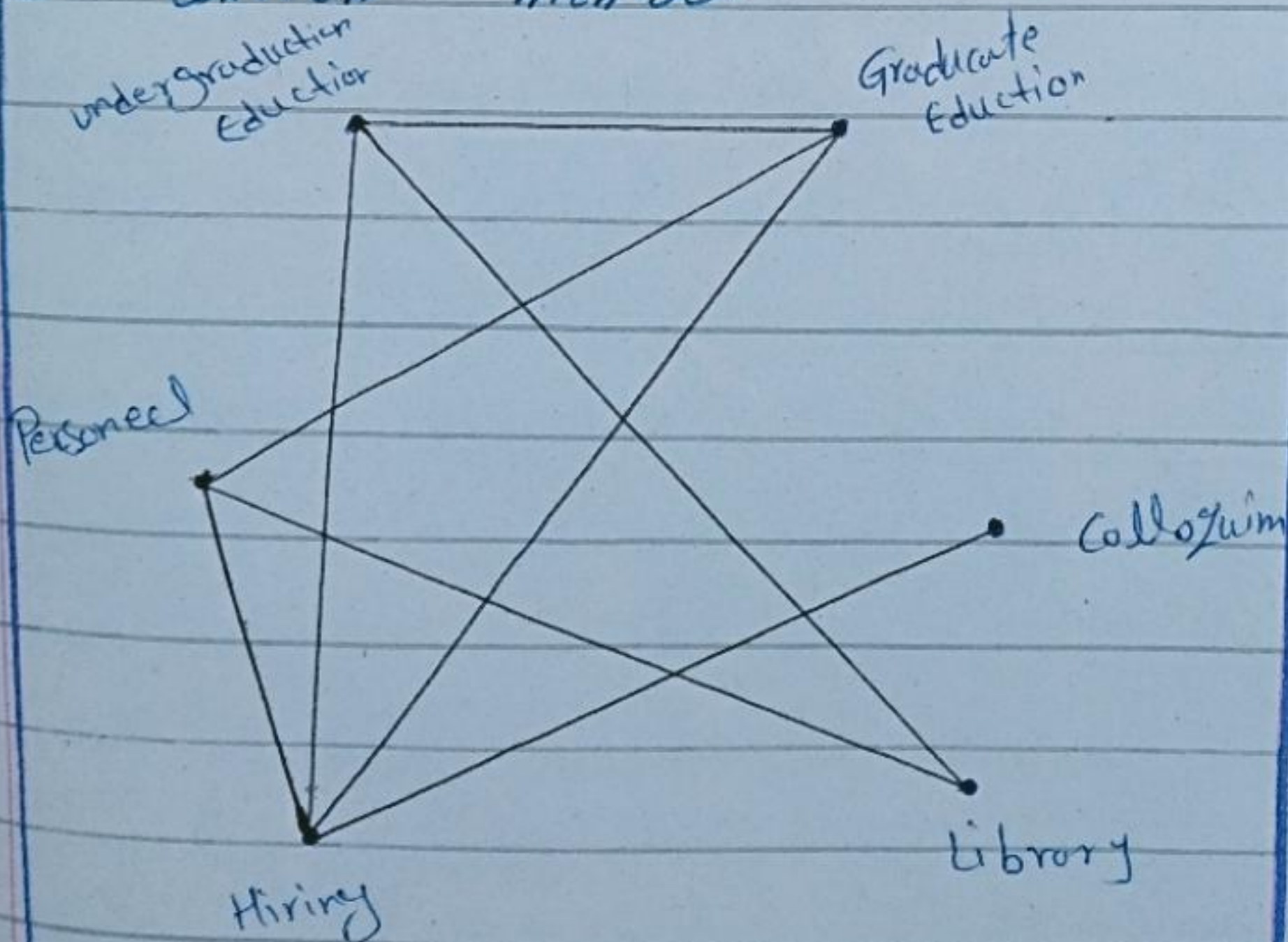
(d) The  $R_8$  on a set  $A$  is transitive if  $(a, b) \in R_8$  and  $(b, c) \in R_8$  implies  $(a, c) \in R_8$ .

we note that the statement if  $(a, b) \in R_8$  and  $(b, c) \in R_8$  is never true for two distinct pairs  $a$  there or other such pair in  $R_8$ . when if statement is false so  $R_8$  is transitive.  
 $R_8$  is transitive.

## Question = 10 Answer

We have been given 6 Committees undergraduate Education, Graduation education colloquium, Library, Hiring and Personal. Let us then represent the scheduling Problem by a graph with 6 vertices and each vertices represent one of the committees.

Next, we draw an edge between two vertices, if the two committees have a common member.



Next we will assign a color to each other vertex such that no two adjacent vertices have the

Some colour thus no two vertices that are connect by an edge will have the same colour.

we note that vertex 'Hiring' has the most edge connecting the vertices to another vertex. Thus let us colour Red.

Hiring = Red

2 Next the vertex 'Library' is the only vertex that is not connect to the vertex "Hiring" by an edge and thus we can color 'Library' the same as 'Hiring' let us color the vertex "Library" with the color Library = Red

Next we note the vertex "Personnel" is connect to vertex "Hiring" and vertex "Library" by an edge. Thus we cannot color the vertex Personnel with Red, let us then color this vertex with Purple.

Personnel = Purple

3 Next we note vertex "Graduate Education" is connect to vertex Hiring and vertex Personnel

by on edge. Thus we cannot colour the vertex Personel with Red nor with Purple, let us then color this vertex with blue.

Graduate Education = blue.

4 Next we note that vertex undergraduate education is connect to vertex by on edge. Thus we cannot colour the vertex Personel with Red nor with blue. Let us then colour this vertex with Purple.

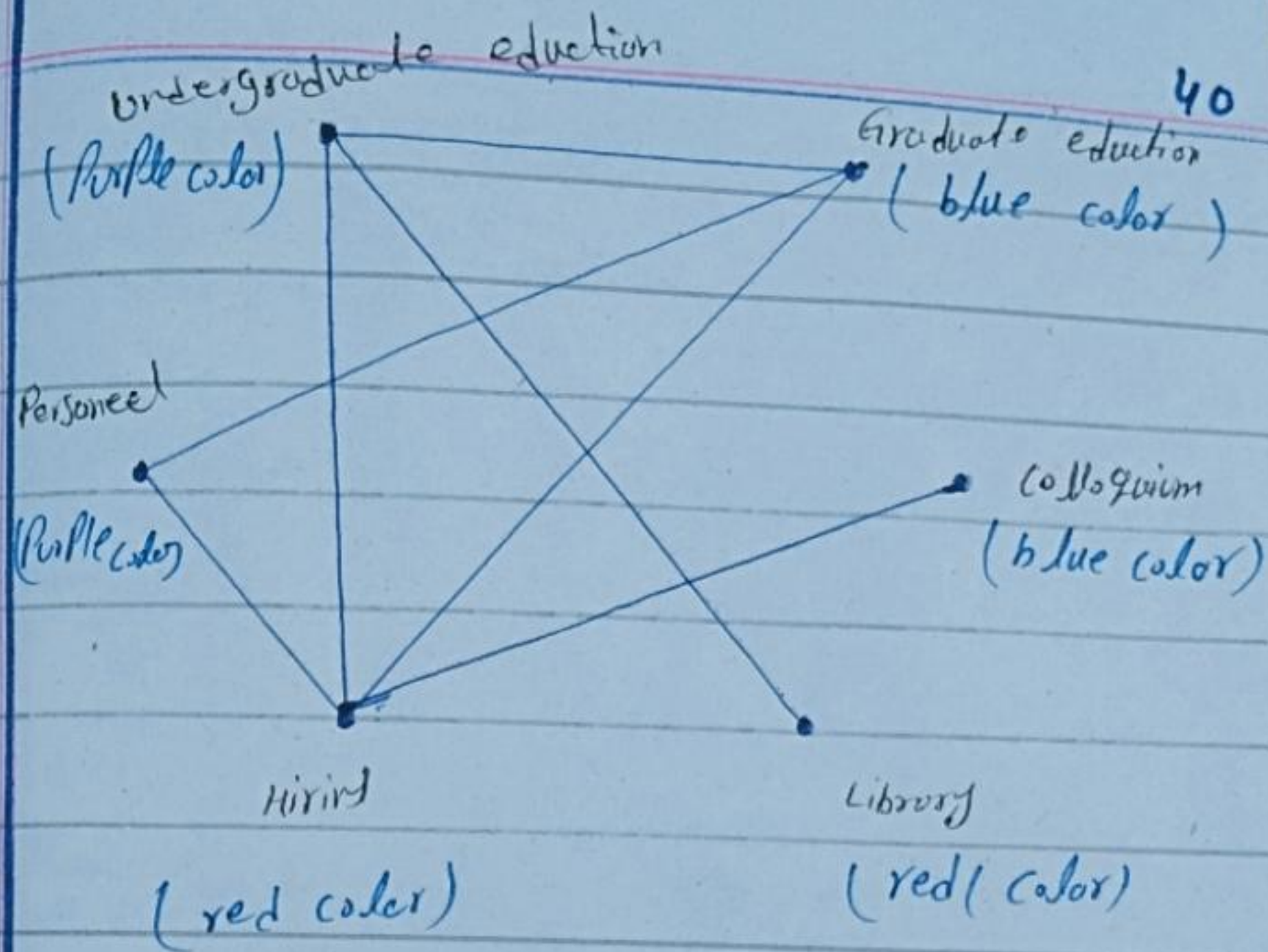
undergraduate education = Purple

And then connect the colloquium with the color blue.

Colloquium = blue

8 we then note that we colored all vertex such that no adjacent vertices have the colour and we used exactly 3 color.





we can then schedule the meeting by assigning each time slot to a colour. Thus let us assign the first time slot to the red, colour the second slot time slot to the blue colour and third time slot to the Purple colour

First time slot  $\Rightarrow$  Hiring and Library

Second time slot  $\Rightarrow$  Colloquium and Graduation

Third time slot  $\Rightarrow$  undergraduate education and Personnel.

END