ID: 14192

Name: Salman Khan.

BS: CS

Date: 28/9/2020

Course: Differential Equations.

Q1a) Define 2nd order linear homogenous / non-homogenous differential equation along with examples?

Ans: The equation of the form

$$a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y = 0$$

where $a_2(x)$, $a_1(x)$ and $a_0(x)$ are function of the variable $x$ or constant

Ex: 1 $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 3y = 0$

Ex: 2 $3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

Non-Homogenous:-

General form: $a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$

are called non-homogenous differential equation of order 2.

Ex: 1 $\frac{d^2y}{dx^2} + \frac{3dy}{dx} + 5y = x^2$

$\frac{d^2y}{dx^2} + \frac{5dy}{dx} + 6y = e^{3x}$

are non-Homogenous Differential equation.

Q. 1(b)

(i) $4y'' - 6y' + 7y = 0$

Solution: $4y'' - 6y' + 7y = 0$ — (i)

Auxillary equation of (i) is

$$4m^2 - 6m + 7 = 0$$

As $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{6 \pm \sqrt{6^2 - 4*4*7}}{2*4}$$

$$m = \frac{6 \pm \sqrt{36 - 112}}{8}$$

$$m = \frac{6 \pm \sqrt{-76}}{8}$$

$$m = \frac{6 \pm 2i\sqrt{19}}{8}$$

$$m = \frac{3 \pm i\sqrt{19}}{4}$$

$$m = \frac{3 + i\sqrt{19}}{4}$$

$$m = \frac{3 - i\sqrt{19}}{4}$$

Root are complex conjugate

Solution will be

$$y = e^{\frac{3}{4}x}\left(C_1 \cos\frac{\sqrt{19}}{4}x + C_2 \sin\frac{\sqrt{19}}{4}x\right)$$

Ans!

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Part b (ii)

Sol: $y'' - 4y' - 12y = 3e^{5x}$

$K^2 - 4K - 12 = 0$

$K^2 - 6K + 2K - 12 = 0$

$K(K-6) + 2(K-6) = 0$

$(K-6)(K+2) = 0$

$K = 6, 2$.

General Solution is:

$y_1(x) = C_1 e^{6x} + C_2 e^{2x}$

Particular Solution is:

$y_1 = Axe^{5x}$

$y_1 = Ae^{5x} + 5Axe^{5x}$

$y_1 = (A + 5Ax)e^{5x}$

$y_1 = 5Ae^{5x} + (5Ae^{5x} + 25Axe^{5x})$

$y_1 = 10Ae^{5x} + 25Axe^{5x} = (10A + 25Ax)e^{5x}$

Substiting the fln $y_1$ & its derivative in the differential equation.

$y'' - 4y' - 12y = 3e^{5x}$

$(10Ae^{5x} + 25Axe^{5x}) - 4(A + 5Ax)e^{5x} - 12Axe^{5x} = 3e^{5x}$

$10Ae^{5x} \, 4 \, Ae^{5x} + 25Axe^{5} - 20Axe^{5x} - 12Axe^{5x} = 3e^{5x}$.

$6Ae^{5x} + \overset{-7}{8}Axe^{5x} = 3e^{5x}$

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$y_1 = Axe^{5x}$

$y_1 = \frac{1}{2} x e^{5x}.$

$A = \frac{1}{2}$

$$\boxed{y_1 = C_1 e^{6x} + C_2 e^{2x} + \frac{x}{2} e^{5x}}$$

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Q2

(i) Sol: $16y'' - 40y' + 25y = 0, \quad y(0) = 3 \quad y'(0) = 9/4$

let $y(x) = e^{mx}$

then $y(x) = me^{mx}; \quad y'' = m^2 e^{mx}$

from eq(1) $16m^2 e^{mx} - 40\, me^{mx} + 25e^{mx} = 0$

$16m^2 e^{mx} - 20m^{mx} - 20me^{mx} + 25e^{mx} = 0$

$16m^2 - 20m - 20m + 25 = 0$

$4m(4m-5) - 5(4m-5) = 0$

$(4m-5) = 0 \Rightarrow m = 5/4$

$(4m-5) = 0 \Rightarrow m = 5/4$

$y(x = C_1 e^{m_1 x} + C_2 e^{m_2 x}$

$\boxed{y(x) = C_1 e^{\frac{5}{4}x} + C_2 e^{\frac{5}{4}x}}$ Ans

Q2

(ii) $y'' + 14y' + 49y = 0$ , $y(-4) = -1$ $y'(-4) = 5$

$y(x) = e^{mx} \rightarrow y' = me^{mx}$; $y' = m^2 e^{mx}$

$m^2 e^{mx} + 14me^{mx} + 49e^{mx} = 0$

$m^2 + 14m + 49 = 0$

$m^2 + 7m + 7m + 49 = 0$ $m(m+7) + 7(m+7)$

$= m = -7$

$y(x) - C_1 e^{7x} + C_2 x e^{-7x} \Rightarrow y''(x) = 7C_1 e^{x}$

$y(-4) = -1$, $-1 = C_1 e^{-7x} + C_2 (-4) e^{-7(-4)}$ $C_2 e^{-7x} - 7C_2 x e^{-7x}$

$-1 = C_1 e^{28} + (-4C_2 e^{28}$

$y'(-4) = 5$, $y'' = -7C_1 e^{-7x} + C_2 e^{-7x}$ $C_2 e^{-7x} x$

$5 = -7C_1 e^{-7(-4)} + C_2 e^{-7(-4)} - 7C_2 e^{-7(-4)} (-4)$

$5 = 7C_1 e^{28} + C_2 e^{28} + 29$ $C_2 e^{28}$

$5 = 7C_1 e^{28} + C_2 e^{28}$ $29Ce^{28}$.

Q2 (iii) $y'' - 4y' + 9y = 0$ $y(0) = 0$ $y'(0) = -8$

Solution:- $y'' - 4y' + 9y = 0$

$y = e^{mx}$

$y' = me^{mx}, y = m^2 e^{mx}$

$m^2 e^{mx} - 4me^{mx} + 9e^{mx} = 0$

$m^2 - 4m + 9 = 0$

$$m = \frac{-(-4) \pm \sqrt{4^2 - 4*1*9}}{2*1}$$

$$m = \frac{4 \pm \sqrt{16 - 36}}{2}$$

$$m = \frac{4 \pm 2i\sqrt{5}}{2}$$

$m = 2 \pm i\sqrt{5}$

$y = e^{2x}(c_1 \cos\sqrt{5}\,x + c_2 \sin\sqrt{5}\,x)$

Ans !

Q2(iv) $y'' - 8y' + 17y = 0$

Solution:- $y'' - 8y' + 17y = 0$

$m^2 e^{mx} - 8me^{mx} + 17e^{mx} = 0$

$m^2 - 8m + 17 = 0$

$m = \dfrac{8 \pm \sqrt{8^2 - 68}}{2 * 1}$

$m = \dfrac{8 \pm \sqrt{64 - 68}}{2}$

$m = \dfrac{8 \pm \sqrt{4}}{2}$

$m = \dfrac{8 \pm 2i}{2}$

$m = 4 \pm i$

So $y = e^{4x}(c_1 \cos x + C_2 \sin x)$

Ans.

10/a

Q3 Define laplace tranform along with two example.

Ans The laplace tranform of a function f(t) defined for all real number t ≥ 0, is the function F(s), which is a unilateral tranform defined by

e.g $f(s) = \int_0^{\infty} f(t) e^{-st} dt$

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Q3 A.

(1) f(t)=6(e^-5t)+e^3t+5(t^3)-9

Sol: $f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$

$f(5) = L\{f(t)\} = 6(L\{e^{3t}\} + 5L\{t^3\} - 9L\{1\}$

$= 6\frac{1}{5-(-5)} + \frac{1}{5-3} + 5\frac{3!}{5^{3+1}} - 9\frac{1}{5}$

$= \frac{6}{5+5} + \frac{1}{5-3} + \frac{30}{5^4} - \frac{9}{5}$ Ans.

(2) g(t) = 4cos(4t) - 9sin(4t) + 2cos(10t)

Sol. $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

$G(s) = L\{g(t)\} = 4L\{\cos(4t)\} - 9L\{\sin(4t)\} + 2L\{\cos(10t)\}$

$= 4\frac{5}{5^2+4^2} - 9\frac{4}{5+4^2} + 2\frac{5}{5^2+10^2}$

$= \frac{45-36}{5^2+16} + \frac{25}{5^2+100}$ Ans

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(3) h(t) = e^3t + cos(6t) - e^(3t)cos(6t)

Sol: $h(t) = L\{h(t)\} = L\{e^{3t}\} + L\{\cos(6t)\} - L\{e^{3t}\cos(6t)\}$

$= \frac{1}{5-3} + \frac{5}{5+6^2} - \frac{5-3}{(5-3)^2+3^2}$

$= \frac{1}{5-3} + \frac{2}{5^2+9} - \frac{5-3}{(5-3)^2+9}$ Ans.

Q4: Solve the following IVP using laplace Tranform.

(i) $y'' - 10y' + 9y = 5t,\ y(0) = 1,\ y'(0) = 2$

Sol:-

The First step in using laplace transforms to solve an IVP is to take the transform of every term in differential equation.

$$L\{y''\} - 10L\{y'\} + 9L\{y\} = L\{5t\}$$

using the appropriate formulas from our table of laplace transforms gives us the following.

$$s^2Y(s) - sy(0) - y'(0) - 10(sY(s) - y(0)) + 9Y(s) = \frac{5}{s^2}$$

Plug in the initial conditions and collect all the terms that have a $Y(s)$ in them

$$(s^2 - 10s + 9)Y(s) + s - 12 = \frac{5}{s^2}$$

$$Y(s) = \frac{5}{s^2(s-9)(s-1)} + \frac{12-s}{(s-9)(s-1)}$$

At the present is concerened to recall just what we're trying to do. we are trying to find the solution $y(t)$. to an IVP. what we have manage to find all the terms is not that solution, that the laplace transform.

So, in order to find the solution all that we need to do is to take the inverse tranform. Here we doing that let's ~~take~~ notice that in its Present form we will have to do partial fractions twice. However, if we combine the two terms up we will only the doing partial fractions once. Not only that, but the denominator for the contained form will be identical to the denominator of the first term. This means that we are going to partial fractions up a term that denominator no matter what so we might as well make the numenator slightly messior, and then just partial fraction once.

This is one of those things where we are apparently making the problem messior, but in the process we are going to save ourselves a fair amount of work.

$$Y(s) = \frac{s + 12s^2 - s^2}{s^2(s-9)(s-1)}$$

The partial fraction decomposition for this tranform is. $Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s-9)} + \frac{D}{(s-1)}$

setting numerators equal gives

$$s + 12s^2 - s^2 = As(s-9)(s-1)$$

Picking appropriate values of a and solving for the constants gives

$$s + 12s^2 - s^2 = As(a-9)(s-1) + B(s-9)(s-1) + Cs^2(s-1) + Ds^2(s-9)$$

Picking appropriate values of s and solving for the constant gives.

$$s = 0 \qquad s = 9B \quad \Rightarrow \quad B = \frac{5}{9}$$

$$s = 1 \qquad 16 = -8D \quad \Rightarrow \quad D = -2$$

$$s = 9 \qquad 248 = 648C \quad \Rightarrow \quad C = \frac{31}{81}$$

$$s = 2 \qquad 45 = -44 + \frac{4345}{81} \Rightarrow A = \frac{50}{81}$$

Plugging in the constants gives,

$$Y(s) = \frac{\frac{50}{81}}{s} + \frac{\frac{5}{9}}{s^2} + \frac{\frac{31}{81}}{s-9} - \frac{2}{s-1}$$

Finally taking the inverse tranform gives us the solution to the IVP

$$y(t) = \frac{50}{81} + \frac{5}{9} + \frac{31}{81}e^{9t} - 2e^t$$

Q4 (ii) Sol: $y'' - 6y' + 15y = 2\sin(3t)$. $\quad y(0) = -1 \quad y'(0) = -4$

Take the laplace tranform of everything and plug in the intical conditions.

$$s^2 Y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 15Y(s) = 2\frac{3}{s^2+9}$$

$$(s^2 - 6s + 15)Y(s) + s - 2 = \frac{6}{s^2+9}$$

$$Y(s) = \frac{As + B}{s^2+9} + \frac{Cs + D}{s^2 - 6s + 15}$$

$$-s^3 + 2s^2 - 9s + 24 = (As + B)(s^2 - 6s + 15) + (Cs + D)(s^2 + 9)$$

$$-s^3 + 2s^2 - 9s + 24 = (As + B)(s^2 - 6s + 15) + (Cs + D)(s^2 + 9)$$

$$= (A + C)s^3 + (-6A + B + D)s^2 + (15A - 6B + 9C)s + 15B + 9D$$

Solving for the Constants gives.

$$\left.\begin{array}{ll} s^3: & A + C = -1 \\ s^2: & -6A + B + D = 2 \\ s^1: & 15A - 6B + 9C = -9 \\ s^0: & 15B + 9D = 24 \end{array}\right\} \Rightarrow \begin{array}{ll} A = \frac{1}{10} & B = \frac{1}{10} \\ C = \frac{11}{10} & D = \frac{5}{2} \end{array}$$

$$Y(s) = \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11s + 25}{s^2 6s\ 15}\right)$$

$$= \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{3 + 3) + 25}{(s-3)^2 + 6}\right)$$

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$$= \frac{1}{10}\left(\frac{s}{s^2+9} + \frac{1\frac{3}{3}}{s^2+9} - \frac{11(s-3)}{(s-3)^2+6} - \frac{8\frac{\sqrt{6}}{\sqrt{6}}}{(s-3)^2+6}\right)$$

$$Y(s) = \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11s+25}{s^2-6s+15}\right)$$

$$= \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11(s-3+3)+25}{(s-3)^2+6}\right)$$

$$= \frac{1}{10}\left(\frac{s}{s^2+9} + \frac{1\frac{3}{3}}{s^2+9} - \frac{11(s-3}{(s-3)^2+6} - \frac{8\frac{\sqrt{6}}{\sqrt{6}}}{(s-3)^2+6}\right)$$

finally, take the inverse trasform

$$y(t) = \frac{1}{10}\Big(\cos(3t) + \frac{1}{3}\sin(3t) - 11\cos(\sqrt{6}t)$$

$$- \frac{8}{\sqrt{6}} e^{3t} \sin(\sqrt{6}t)\Big)$$