

Q1

Sol: The pressure drop  $\Delta p$  is expected to depend upon the gate opening  $h$ , the overall depth  $d$ , the velocity  $V$ , density  $\rho$  and viscosity  $\mu$ .

the relevant variables

$$\Delta p, h, d, V, \rho, \mu$$

Write down dimensions.

$$\Delta p \quad ML^{-1}T^{-2}$$

$$h \quad L$$

$$d \quad L$$

$$V \quad LT^{-1}$$

$$\rho \quad ML^{-3}$$

$$\mu \quad ML^{-1}T^{-1}$$

Number of variables  $n=6$

Number of independent dimensions:  $m=3$  (M, L and T)

Number of non-dimensional groups:  $n-m=3$

Choose  $m (=3)$  scaling variables

geometric ( $d$ ); kinematic/time depend ( $V$ ); dynamic/mass-dependent ( $\rho$ )

Form dimensionless groups by non dimensionlising the remaining variables:  
 $\Delta p, h$  and  $\mu$

$$\Pi_1 = \Delta p d^a V^b \rho^c$$

$$M^0 L^0 T^0 = (ML^{-1}T^{-2})^a (L)^b (LT^{-1})^c (ML^{-3})^c$$

$$= M^{1+c} L^{-1+a+b-3c} T^{-2-b-c}$$

$$M: 0 = 1+c \quad \Rightarrow c = -1$$

$$T: 0 = -2-b \quad \Rightarrow b = -2$$

$$L: 0 = -1+a+b-3c \quad \Rightarrow a = 1+3c-b$$

$$\Pi_1 = \Delta p V^{-2} \rho^{-1} = \frac{\Delta p}{\rho V^2}$$

$$\Pi_2 = \frac{h}{d} \quad (\text{by inspection since } h \text{ is a length})$$

$$\Pi_3 = \mu d^a V^b \rho^c \quad (\text{probably obvious by now, but here goes anyway})$$

$$\begin{aligned} M^0 L^0 T^0 &= (M L^{-1} T^{-1})^c (L)^a (L T^{-1})^b (M L^{-3})^c \\ &= M^{1+c} L^{-1+a+b-3c} T^{-1-b} \end{aligned}$$

$$M: 0 = 1 + c \quad \Rightarrow c = -1$$

$$T: 0 = -1 - b + 0 \quad \Rightarrow b = -1$$

$$L: 0 = -1 + a + b - 3c \quad \Rightarrow a = 1 + 3c - b = -1$$

$$\Rightarrow \Pi_3 = \mu d^{-1} V^{-1} \rho^{-1} = \frac{\mu}{\rho V d}$$

Recognition of the Reynolds number suggests that we replace  $\Pi_3$  by

$$\Pi_3 = (\Pi_3)^{-1} = \frac{\rho V d}{\mu}$$

Hence, dimensional analysis yields

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

$$\frac{\Delta p}{\rho V^2} = f\left(\frac{h}{d}, \frac{\rho V d}{\mu}\right)$$

(a) Dynamic similarity requires that all non-dimensional groups be the same in model and prototype,

$$\Pi_1 = \left(\frac{\Delta p}{\rho V^2}\right)_p = \left(\frac{\Delta p}{\rho V^2}\right)_m$$

$$\Pi_2 = \left(\frac{h}{d}\right)_p = \left(\frac{h}{d}\right)_m \quad \text{automatic if similar shape i.e. "geometric similarity" }$$

$$\Pi_3 = \left( \frac{\rho V d}{\mu} \right)_p = \left( \frac{\rho V d}{\mu} \right)_m$$

From the last, we have a velocity ratio

$$\frac{V_p}{V_m} = \frac{(U/\rho)_p}{(U/\rho)_m} = \frac{d_m}{d_p} = \frac{0.002/800}{1.0 \times 10^{-3}} \times \frac{1}{5} = 0.5$$

Hence

$$V_m = \frac{V_p}{0.5} = \frac{3.0}{0.5} = 6.0 \text{ m s}^{-1}$$

(b) The ratio of the quantities of flow is

$$\frac{Q_p}{Q_m} = \frac{(\text{velocity} \times \text{area})_p}{(\text{velocity} \times \text{area})_m} = \frac{V_p}{V_m} \left( \frac{d_p}{d_m} \right)^2 = 0.5 \times 5^2 = 12.5$$

(c) Finally, for the pressure drop.

$$\Pi_1 = \left( \frac{\Delta P}{\rho V^2} \right)_p = \left( \frac{\Delta P}{\rho V^2} \right)_m \Rightarrow \frac{(\Delta P)_p}{(\Delta P)_m} = \frac{\rho_p}{\rho_m} \left( \frac{V_p}{V_m} \right)^2 = \frac{800}{1000} \times 0.5^2 = 0.2$$

Hence

$$\Delta P_a = 0.2 \times \Delta P_m = 0.2 \times 60 = 12.0 \text{ kPa}$$

D2

7648,

Given data

Max depth of reservoir is = 76 m

Specific gravity = 2.4

$\rho_{av} = 764 \text{ T/m}^3$

Height of wave = 1.2 m

So

$$\begin{aligned} 1) \quad H_{\text{limiting}} &= \frac{\rho_{av}}{\gamma_w (G - W + 1)} \\ &= \frac{764 \times 1000}{1000(2.4 - 0 + 1)} \end{aligned}$$

$$H_{\text{limiting}} = 224.70$$

2) Top width "a"

$$\begin{aligned} \text{Free board} &= 1.5 \times h_w \\ &= 1.5 \times 1.2 \\ &= 1.8 \end{aligned}$$

$$\begin{aligned} \text{Height of dam} &= H_w + \text{F.B.} \\ &= 76 + 1.8 \\ &= 77.8 \end{aligned}$$

$$\begin{aligned} a &= 14\% \text{ of HD} \\ &= 0.14 \times 77.8 \\ &= 10.892 \end{aligned}$$

3) Base width: →

$$b = \frac{Hw}{UG} = \frac{76}{0.7 \times 2.4}$$

$$= 45.23 \text{ m}$$

$$= 46$$

4) For no tension criteria:

$$b' = \frac{Hw}{\sqrt{G}} = \frac{76}{\sqrt{2.4}}$$

$$\Rightarrow 49.05$$

Depth of vertical portion on up side:

$$h = 2a\sqrt{G \cdot w}$$

$$= 2 \times 10.892 \sqrt{2.4 - 0}$$

$$= 33.74$$

$$= 34 \text{ m}$$

Up stream off set:

$$= \frac{a}{16}$$

$$= \frac{10.892}{16}$$

$$= 0.6$$

Depth of below the water level to the end of inclined portion

$$U/s = 3.14 a \sqrt{G}$$

$$= 3.14 \times 10.892 \sqrt{2.4}$$

$$= 52.92$$

$$\tan \theta = \frac{b'}{H} = \frac{49.05}{76}$$

$$\theta = \tan^{-1}(0.64) \\ = \del{11.3} 32.61^\circ$$

Depth of vertical portion on D/s (from WL on u/s side)

$$\tan \theta = \frac{g}{d'} = \frac{10.892}{d'}$$

$$d' = 16.87 \text{ m}$$

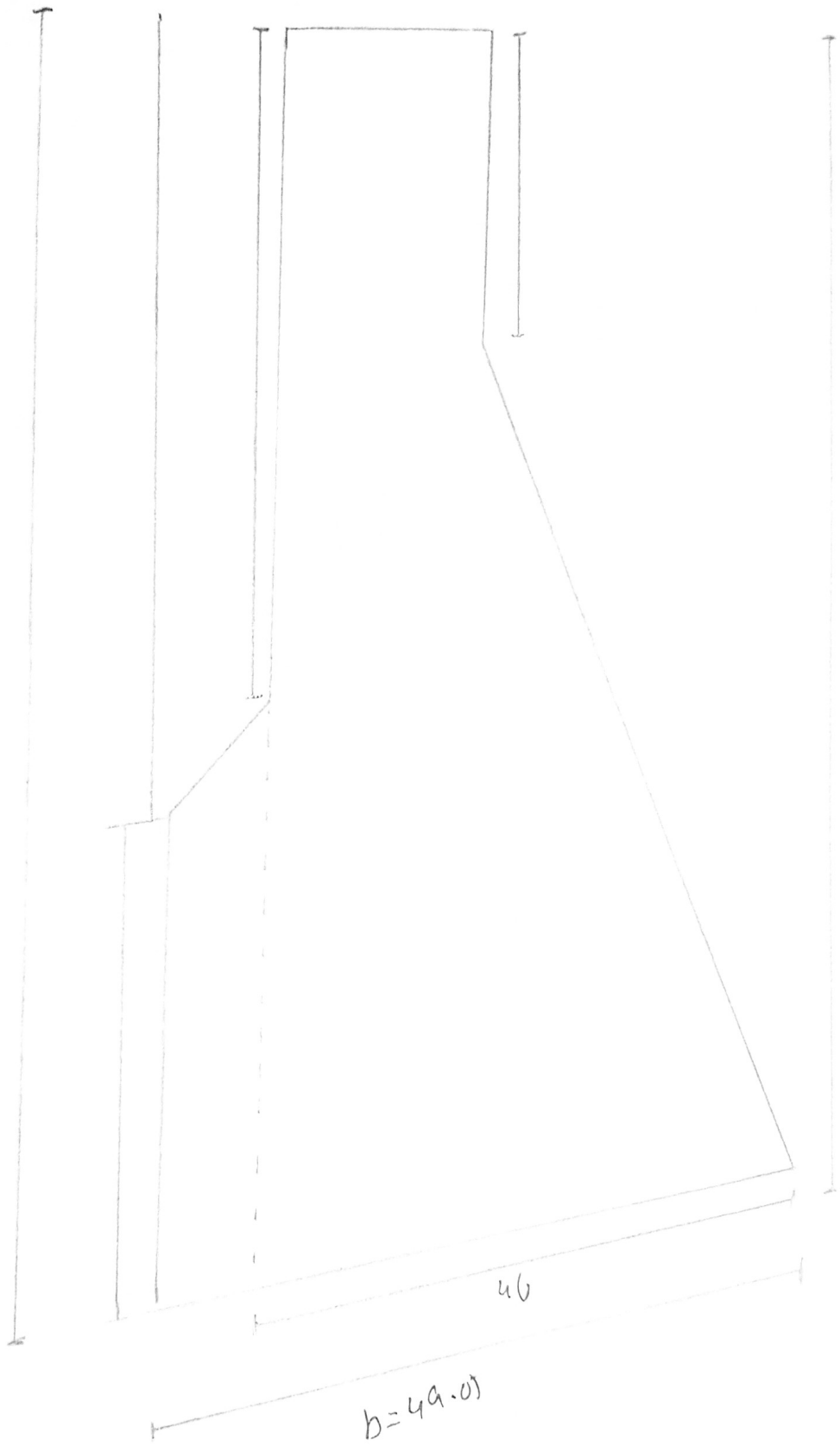
Depth of vertical portion

$$d = d' + F.B$$

$$= \del{16.87} +$$

$$= 16.87 + 1.8$$

$$= 18.67$$



Ans

Dimension analysis is a mathematical technique making use of study of dimension

→ Purpose of dimension analysis:

To obtain scaling laws so that the prototype performance can be predicted from model performance. To predict in the relationship b/w parameter. To generate non dimensional parameter that help in the relationship design of experiment and in reporting of results.

⇒ Fundamental dimension:

These are the basic quantities

For example

Time,  $T$ ; Distance,  $L$ ; Mass,  $M$ .

⇒ Secondary dimension:

Those quantity which posed more than one fundamental dimension.

Velocity,  $L/T$ , Acceleration  $L/T^2$ , Density  $M/L^3$

⇒ Similitude:

It is defined as similarity b/w the model and prototype in every respect which means model and prototype are completely similar.

It is used in testing engineering model.

Example:

Consider a submarine modeled at 1/40th scale. The operational operation in seawater at 0.5 i moving at 5 m/s. The model will be tested in fresh water at 20°C



1) Particle diameter:

The diameter of the particle is directly proportional to the fall velocity because greater the size of particle so it will tend to move faster as compared to the particle of small size thus there will be more concentration of large on particle of greater size so it will fall rapidly due to its weight.

2) Particle density.

Density of the particle is directly proportional to the rate of fall velocity since particles with high density tend to settle down early compared with particle of low density.

3) Particle concentration.

Concentration of particle size will considerably effect its fall velocity as the section having greater concentration will be settled down at the place thus causing more fall velocity comparing with section of low concentration.

4) Particle shape:

Particle having regular shape tends to be affected more than irregular shape since regular shape particles have even surface which offers very little or no friction which particles with irregular shape offers more friction as the particles with smaller surface area are more likely to be affected due to their less resistance.