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Module = 18

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Question No 1 a

Sol:-

$$y_h(n) = C_1(-1)^n + C_2(4)^n$$

an exponential sequence of the same from as x(n), Normally, we could assume a solution of the form

$$y_p(n) = K(4)^n u(n)$$

we absorve that yp(n) is already contained in the homogenous solution.
Thus we assume that

$$y_p(n) = kn(4)^n u(n).$$

we obtain.

$$kn(4)^n u(n) - 3k(n-1)(4)^{n-1} u(n-1) - 4k(n-2)(4)^{n-2} u(n-2) = (4)^n u(n) + 2(4)^{n-1} u(n-1).$$

To determine k we evaluate this equation for any $n \geq 2$. To simplify the arthimetic we selet $n=2$ from which we obtain $\boxed{k = 6/5}$

There fore,

$$y_p(n) = \frac{6}{5} n (4)^n u(n)$$

Total solution to the difference equation is obtained

$$y(n) = C_1(-1)^n + C_2(4)^n + \frac{6}{5} n(4)^n \quad n \geq 0$$

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where the constants c_1 and c_2 are determined that the initial conditions

$$y(0) = 3y(-1) + 4y(-2) + 1$$
$$y(1) = 3y(0) + 4y(-1) + 6$$
$$= 13y(-1) + 12y(-2) + 9$$

on other hand evaluated at $n=0$ and $n=1$

$$y(0) = c_1 + c_2$$
$$y(1) = -c_1 + 4c_2 + \frac{24}{5}$$

Hence $c_1 = -\frac{1}{25}$ and $c_2 = \frac{26}{25}$

finally we have the zero state response to the forcing function.

$x(n) = (4)^n u(n)$ in the form.

$$y_{zs}(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5}n(4)^n \quad n \geq 0$$

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Question (1 b)

Soln:

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

$$y(z) = \frac{X(z)}{1-0.6z^{-1}+0.08z^{-2}}.$$

Impulse response.

$$x(n) = \delta(n)$$

$$x(z) = 1$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0.6z^{-1}+0.08z^{-2}}$$

$$= \frac{1}{(1-\frac{1}{5}z^{-1})(1-\frac{2}{5}z^{-1})}$$

$$H(z) = \frac{-1}{1-\frac{1}{5}z^{-1}} + \frac{2}{1-\frac{2}{5}z^{-1}}$$

$$h(n) = \left[-1\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right]u(n).$$

Unit Step response:

$$\delta(n) = \sum_{k=0}^{n} h(n-k), \; n \geq 0$$

$$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= \left\{\frac{1}{0.12}\left[\left(\frac{2}{5}\right)^{n+1} - 1\right] - \frac{1}{0.16}\left[\left(\frac{1}{5}\right)^{n+1} - 1\right]\right\}u(n)$$

Question 2a.

Soln,

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$

Hence, $x(n) = \left[4(2)^n - 3 - n\right]u(n)$.

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Question 2b :-

first we eliminate the negative powers of 'z' by multiplying both numerator and denominator by z^2

Thus we obtain $X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$

The poles of $X(z)$ are $p_1 = 1$ and $p_2 = 0.5$

consequently, the expansion will be

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{2}{(z-1)} - \frac{1}{(z-0.5)}$$

(obtained by applying partial fraction)

$$\Rightarrow X(z) = \frac{2z}{(z-1)} - \frac{z}{(z-0.5)}.$$

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Question 3a.

Sol.:

At $\omega = 0$ we have.

$$H(0) = \frac{b_0}{(1-p)^2} = 1.$$

Hence

$$b_0 = (1-p)^2$$

At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4) + jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2} + jp/\sqrt{2})^2}$$

Hence

$$\frac{(1-p)^4}{[(1-p/\sqrt{2})^2 + p^2/2]^2} = 1/2.$$

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or equivalently

$\sqrt{2}(1-p)^2 = 1+p^2 - \sqrt{2}p.$

$p = 0.32$.

$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$

———— x ———— x ————