Question No 1.

Find the solution of the following.

(a) The sum of two numbers is K. Find the minimum value of the sum of thier cubes.

Solution.

Let x and y = the numbers

Z = Sum of thier cubes.

$K = x+y$

$y = K-x$

$Z = x^3+y^3$

$Z = x^3+(K-x)^3$

$\frac{dz}{dx} = 3x^2+3(K-x)^2(-1)=0$

$x^2-(K^2-2Kx+x^2)=0$

$x = \frac{1}{2}K.$

$y = K-\frac{1}{2}K.$

$y = 1/2\, K$

$Z = (1/2K)^3 + (1/2K)^3$

$Z = \frac{1}{4}K^3$

(b) The sum of two positive numbers is 2. Find the smallest value possible for the sum of cubes of one number and the square of other.

Solution:

Let x and y = the numbers.

$x+y=2 \longrightarrow$ Equation (1)

$1+y'=0, \quad y'=-1$

$Z=x^3+y^2 \longrightarrow$ equation (2)

$$\frac{dz}{dx}=3x^2+2yy'=0.$$

$3x^2+2y(-1)=0.$

$y=3/2\,x^2$

From eq (1)

$x+3/2\,x^2=2$

$2x+3x^2=4$

$3x^2+2x-4=0.$

$x=0.8685$ and -1.5352

Use $x=0.8685$

$y=3/2\,(0.8685)$

$y=1.1315$

$Z = (0.8685)^3 + 1.1315^2$

$Z = 1.9354$ Answer

Question No 2: Find the solution of the following.

ⓐ Let $f(x)$ be a differentiable function such that $f(3) = 12$, $f'(3) = -2$. Estimate the value of $f(3.5)$. Using local approximation at $a = 3$.

Solution: The Linear approximation is given by the equation.

$f(x) \cong L(x)$

$= f(a) + f'(a)(x-a)$.

we just need to plug in the known values and calculate the value of $f(3.5)$:

$L(x) = f(3) + f'(3)(x-3)$.

$= 12 - 2(x-3) \Rightarrow 18 - 2x$

Then

$f(3.5) \cong 18 - 2.3.5 = 11$ Ans

6. Estimate $\sqrt[3]{9}$ using a linear approximation at $a = 8$.

Solution,
let $f(x) = \sqrt[3]{x}$. The linear approximation ~~is~~ given at the point $a = 8$ is given by

$f(x) \approx L(x)$

$= f(8) + f'(8)(x-8)$

Find the derivative.

$$f'(x) = (\sqrt[3]{x})' = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}.$$

Compute the value of the derivative at $a = 8$.

$$f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{12}$$

Substituting this, we get function $L(x)$ in the form.

$$f(x) \approx L(x) = 2 + \frac{1}{12}(x-8)$$

$$= \frac{x}{12} + \frac{4}{3}$$

Hence.

$$\sqrt[3]{9} \approx L(9) = \frac{9}{12} + \frac{4}{3} = \frac{9+16}{12} = \frac{25}{12}$$

QNo 3: Solve the following differential equation

$$2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0.$$

Solution: $(2y + x^2 + 1)\, dy/dx + (2xy - 9x^2) = 0$

we Can Rewrite this as.

$$(2y + x^2 + 1)\,dy + (2xy - 9x^2)\,dx = 0.$$

Check for exactness:

Let $M = 2xy - 9x^2$

Let $N = 2y + x^2 + 1$.

$$\frac{dM}{dy} = \frac{dN}{dx}$$

$$d(2xy - 9x^2)/dy = d(2y + x^2 + 1)/dx.$$

$$2x = 2x$$

Thus it is Indeed exact. To Further Solve.

To get $g(y)$, we differentiate it partially with respect to y.

$$\frac{dF}{dx} = M = 2xy - 9x^2.$$

$\int dF = \int (2xy - 9x^2)\, dx$

$F = (x^2)y - 3x^3 + g(y).$

To get $g(y)$, we differlate it partially with respect to y:

$$\frac{dF}{dy} = \frac{d}{dy}\left((x^2)y - 3x^3 + g(y)\right) = N$$

$$\frac{dF}{dy} = x^2 + g'(y) = N$$

$$= x^2 + g'(y) = 2y + x^2 + 1.$$

$$= g'(y) = 2y + 1.$$

Integrating,

$$g(y) = y^2 + y + c.$$

Therefore

$$F = (x^2)y - 3x^3 + y^2 + y + c$$

Ans