Name : Babar Zaman

ID : 13259

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Instructor : Anwar shamim

Q1: 2(a) The sum of two numbers is k. Find the minimum value of the sum of their cubes.

Ans: <u>Solution</u>:

Let

x and y = the numbers

Z = sum of their cubes

$k = x + y$

$y = k - x$

$Z = x^3 + y^3$

$Z = x^3 + (k-x)^3$

$dZ/dx = 3x^2 + 3(k-x)^2(-1) = 0$

$x2 - (k^2 - 2kx + x^2) = 0$

$x = 1/2k$

$y = k - 1/2k$

$y = 1/2k$

$Z = (1/2k)^3 + (1/2k)^3$

$Z = 1/4k^3$

Ans 1: (b) The sum of two possitive number is 2. Find the smallest value possible for the sum of the cube of one number and the square of the other.

Solution :> b>

Let x and y = the numbers

$x + y = 2$ ⟶ Equation (1)

$1 + y' = 0 \quad y' = -1$

$Z = x^3 + y^2$ ⟶ Equation (2)

$dz/dx = 3x^2 + 2yy' = 0$

$3x^2 + 2y(-1) = 0$

$y = 3/2\, x^2$

From Equation ①

$x + 3/2 x^2 = 2$

$2x + 3x^2 = 4$

$3x^2 + 2x - 4 = 0$

$x = 0.8685$ & -1.5352

Use

$x = 0.8685$

$y = 32(0.8685^2)$

$y = 1.1315$

$Z = 0.8685^3 + 1.1315^2$

$\boxed{Z = 1.9354}$ Ans

Q3 a5:) let $f(x)$ be a differentiable function such that $f(3) = 12$, $f'(3) = -2$. Estimate the value of $f(3.5)$ using the local approximation at $a=3$.

Solution: The linear approximation is given by the equation.

$$f(x) \simeq L(x)$$

$$= f(a) + f'(a)(x-a).$$

we just need to plug in the known values and calculate the value of $f(3.5)$:

$$L(x) = f(3) + f'(3)(x-3)$$

$$= 12 - 2(x-3) = 18 - 2x.$$

Then

$$f(3.5) \simeq 18 - 2 \cdot 3.5 = 11.$$

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Q3 (b) Estimate $\sqrt[3]{9}$ using a linear approximation at $a=8$.

Solution (b): Let $f(x) = \sqrt[3]{x}$.

The linear approximation at the point $a=8$ is given by

$$f(x) \approx L(x)$$
$$= f(8) + f'(8)(x-8).$$

Find the derivative:

$$f'(x) = (\sqrt[3]{x})' = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}.$$

compute the value of the derivative at $a=8$;

$$f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{12}$$

substituting this, we get the function $L(x)$ in the form.

$$f(x) \approx L(x) = 2 + \frac{1}{12}(x-8)$$
$$= \frac{x}{12} + \frac{4}{3}.$$

Hence $\sqrt[3]{9} \approx L(9) = \frac{9}{12} + \frac{4}{3} = \frac{9+16}{12}$

$$\boxed{= \frac{25}{12}}$$

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Q3 :- $2xy - 9x^2 + (2y + x^2 + 1)\, dy/dx = 0$

Solution :- $(2y + x^2 + 1)\, dy/dx + (2xy - 9x^2) = 0$

we can rewrite this as

$(2y + x^2 + 1)\, dy + (2xy - 9x^2)\, dx = 0$

check for exactness:

Let $M = 2xy - 9x^2$

Let $N = 2y + x^2 + 1$

$\partial M/\partial y = \partial N/\partial x$

$\partial(2xy - 9x^2)/\partial y = \partial(2y + x^2 + 1)/\partial x$

$2x = 2x$

Thus it is indeed exact. To

further solve,

$$\partial F/\partial x = M = 2xy - 9x^2$$

$$\int \partial F = \int (2xy - 9x^2)\,dx$$

$$F = (x^2)y - 3x^3 + g(y)$$

To get $g(y)$, we differentiate it partially with respect to y:

$$\partial F/\partial y = \partial/\partial y\left((x^2)y - 3x^3 + g(y)\right) = N$$

$$\partial F/\partial y = x^2 + g'(y) = N$$

$$x^2 + g'(y) = 2y + x^2 + 1$$

$$g'(y) = 2y + 1$$

Integrating

$$g(y) = y^2 + y + c$$

Therefore,

$$\boxed{F = (x^2)y - 3x^3 + y^2 + y + c}$$

The End.