ID # 16236

Program : Bs (SE)

Section : (A)

Examination : Mid Term

Paper : Calculas And Analytical Geometry.

Instructor : Sir Abrar Khan.

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Q1: $\frac{dy}{dx}$

a) : $\frac{3x^3-5x^2+5}{x^2+1}$ . x

Sol: As:

$\frac{3x^3-5x^2+5}{x^2+1}$ , diff. with x.

- Apply Quotient Rule;

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - g' \cdot f}{g^2}$$

$$\Rightarrow \frac{3x^3-5x^2+5}{x^2+1} = \frac{\frac{d}{dx}(3x^3-5x^2+5)(x^2+1) - \frac{d}{dx}(x^2+1)\cdot(3x^3-5x^2+5)}{(x^2+1)^2}$$

$$= \frac{(9x^2-10x)(x^2+1)-(2x)(3x^3-5x^2+5)}{(x^2+1)^2}$$

$$= \frac{9x^4+9x^2-\cancel{10x^3}-10x-6x^4+\cancel{10x^3}-\cancel{10x}}{(x^2+1)^2}$$

$$= \frac{3x^4+9x^2-20x}{(x^2+1)^2}$$

Result

(b) : $\dfrac{(x^2+1)^2}{x^2-1}$ : $x$

Sol. As

$$\frac{(x^2+1)^2}{x^2-1} , x$$

Applying Quotient Rule:

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - g' f}{g^2}$$

$$\Rightarrow \left(\frac{(x^2+1)^2}{(x^2-1)}\right)' = \frac{\frac{d}{dx}(x^2+1)^2 \cdot (x^2-1) - \frac{d}{dx}(x^2-1) \cdot (x^2+1)^2}{(x^2-1)^2}$$

$$= \frac{2(x^2+1)\frac{d}{dx}(x^2+1) \cdot (x^2-1) - (2x)(x^2+1)^2}{(x^2-1)^2}$$

$$= \frac{(2x^2+2)(2x) \cdot (x^2-1) - (2x)(x^2+1)^2}{(x^2-1)^2}$$

$$= \frac{4x(x^2+1)(x^2-1) - 2x(x^2+1)^2}{(x^2-1)^2}$$

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$$= \frac{4x^5 - 4x - 2x(x^4+2x^2+1) - (2x)(x^4+2x^2+1)}{(x^2-1)^2}$$

$$= \frac{\cancel{4x^5} - 4x - \cancel{2x^5} - 4x^3 - 2x - \cancel{2x^5} - 4x^3 - 2x}{(x^2-1)^2}$$

$$= \frac{-40x^5 - 4x^3 - 8x}{(x^2-1)^2}$$

$$= \frac{-8x^3 - 8x}{(x^2-1)^2}$$ Result

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(a) $\frac{dy}{dx} = ?$ , $y = (1+2\sqrt{x})^3 . x^{2/3}$

Let

$$U = (1+2\sqrt{x})^3 x^{2/3}$$

Diff w.r.t "x"

$$\frac{dU}{dx} = (1+2\sqrt{x})^3 \frac{2}{3} x^{-1/2} + x^{2/3} \frac{(1+2\sqrt{x})^2 \cancel{(2)}}{\cancel{2}\sqrt{x}}$$

$$\boxed{\frac{dU}{dx} = \frac{2(1+2\sqrt{x})^3}{3(\sqrt{x})} + \frac{x^{2/3}(1+2\sqrt{x})^2}{\sqrt{x}}}$$

$$y = U$$

$$\frac{dy}{dU} = \frac{dU}{dU}$$

$$\boxed{\frac{dy}{dU} = 1}$$

$$\frac{dy}{dx} = \frac{dy}{dU} \cdot \frac{dU}{dx}$$

$$\frac{dy}{dx} = \frac{2(1+2\sqrt{x})^3}{5\sqrt{x}} + x^{2/3}\frac{(1+2\sqrt{x})^2}{\sqrt{x}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{x}}\left(\frac{2(1+2\sqrt{x})^3}{3} + x^{2/3}(1+2\sqrt{x})^2\right)}$$

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(b) Let $y = \sqrt{\frac{1-x}{1+x}}$

Let $u = \frac{1+x}{1-x}$

Diff w.r.t "x"

$$\frac{du}{dx} = \frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2}$$

$$\frac{du}{dx} = \frac{1-\cancel{x}+1+\cancel{x}}{(1-x)^2}$$

$$\boxed{\frac{du}{dx} = \frac{2}{(1-x)^2}}$$

And

$$y = \sqrt{u} = u^{1/2}$$

$$\frac{dy}{du} = \frac{1}{2} u^{-1/2}$$

$$\frac{dy}{du} = \frac{1}{2\sqrt{\frac{1-x}{1+x}}}$$

$$\boxed{\frac{dy}{du} = \frac{(1+x)^{-1/2}}{2(1-x)^{1/2}}}$$

Now

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{(1+x)^{-1/2}}{\cancel{2}(1-x)^{1/2}} \cdot \frac{\cancel{2}}{(1-x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{a(1+x)^{-1/2}}{(1-x)^{5/2}}}$$

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Q:3 Integration;

a) $\int \frac{1}{\sqrt{x^3}}\, dx$

Sol: As: $\int \frac{1}{\sqrt{x^3}}\, dx$

$$= \int \frac{1}{(x)^{3/2}} \cdot dx$$

$$= \int (x)^{-3/2} \cdot dx$$

Applying Power Rule

$$\int x^a\, dx = \frac{x^{a+1}}{a+1} \; ; \; a \neq 1$$

$$= \frac{x^{-3/2+1}}{-3/2+1}$$

$$= \frac{x^{-1/2}}{-1/2}$$

$$= -2x^{-1/2} = -2/\sqrt{x} + c$$ Result

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(b) : $\int \frac{1}{(6x+7)^6} \cdot dx$

Sol:- As. $\int \frac{1}{(6x+7)^6} \cdot dx$

Apply (Integral Substitution) u-substitution;

$u = 6x+7$

$\therefore \int \frac{1}{(6x+7)^6} \cdot dx \quad ; \quad u = 6x+7$

$\therefore = \int \frac{1}{6u^6} du$

$= \frac{1}{6} \cdot \int \frac{1}{u^6} \cdot du$

$= 1/6 \cdot \int 1 (u)^{-6} \cdot du$ : Apply Power Rule:

$= 1/6 \cdot \frac{(u)^{-6+1}}{-6+1}$

$= \frac{1}{6} \cdot \frac{(6x+7)^{-6+1}}{(-6+1)}$

$= \frac{1}{6} \cdot \frac{(6x+7)^{-5}}{(-5)}$

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$$= \frac{1}{6} \cdot (-1) \frac{(6x+7)^{-5}}{(5)}$$

$$= -\frac{1}{30(6x+7)^{5}} + C$$

$\Rightarrow$ Result

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