Solve the following objective type questions.

i. The order of matrix A is $m \times p$ and the order of B is $p \times n$. Then the order of matrix AB is?

Answer:-

$m \times n$

ii:- The number of non-zero rows in an Echelon form?

Answer:-

One.

iii:- If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Answer:- $\begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = a \times 1 - 2 \times 4$

$= a - 8 = 0$.

$\Rightarrow a = 8$.

iv:- If $A \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Answer:- $|A| = 2i(-i) - i(i)$

$= -2i^2 - i^2$

$= -2(-1) - (-1) \Rightarrow 2 + 1 \Rightarrow 3 \Rightarrow \boxed{|A| = |3|}$

V:- The matrix $= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Answer:-

Scalar matrix.

$1 \cdot h^2 - a'b)$.

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vi :- Solution of $\frac{dy}{dx} + 2xy = y$?

Answer :-

$$\frac{dy}{dx} + 2xy = y.$$

Separating variables.

$$\Rightarrow \frac{dy}{dx} + 2xy = y.$$

$$\Rightarrow \frac{dy}{dx} = y - 2xy.$$

$$\Rightarrow \frac{dy}{dx} = y(1-2x).$$

$$\Rightarrow \frac{dy}{y} = (1-2x)\,dx.$$

$$\Rightarrow \int \frac{1}{y}\,dy = \int 1\,dx - \int 2x\,dx.$$

$$\Rightarrow \ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + C$$

$$\Rightarrow \ln y = x - x^2 + C.$$

Vii :- The order and degree of differential equation $\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is ?

Answer :-

Order = 1

Degree = 3.

Viii :- The order and degree of differential equation $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is ?

Answer :-

Order = 2

Degree = 1.

Xi:- The differential equation $2\frac{dy}{dx} + x^2 y = 2x+3$, $y(0) = 5$ is ?

Answer

$$2y' + x^2 y = x^2+3 \;,\; y(0) = 5$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{x^2+3}{2}.$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{1}{2}(x^2+3)$$

$$\mu = \frac{x^2}{2}.$$

$$e^{\int \frac{x^2}{2} dx} = e^{x^3/6}$$

$$e^{x^3/6}y' + e^{x^3/6}\left(\frac{x^2}{2}\right)y = \frac{1}{2}e^{x^3/6}(x^2+3)$$

$$y(x) = \frac{e^{x^3/6}x^2 + 3e^{x^3/6} + C}{2e^{x^3/6}}$$

$$y(0) = \frac{0+3}{2} = 3/2.$$

$$\boxed{y(x) = \frac{e^{x^3/6}x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + \frac{3}{2}.}$$

X:- $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Answer:-

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}.$$

$$1(bc^2 - cb^2) - 1(ac^2 - a^2c) + 1(ab^2 - a^2b).$$

$$= bc^2 - cb^2 - ac^2 + a^2c + ab^2 - a^2b.$$

$$= a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2$$

$$= a^2(c-b) + b^2(a-c) + c^2(b-a).$$

Question No 2

Part A

Express the Determinant.

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c.

Solution:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}.$$

Expand by R_1.

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$

$= ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^2$

$+ a^2cb^3 - a^3b^2c.$

Common abc.

$\Rightarrow abc(bc^2 - b^2c - ac^2 - a^2c + ab^2 - a^2b).$

$\Rightarrow \boxed{abc[bc(c-b) - ac + ab(b-a)]}.$

Answer.

Question 2

Part B.

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}.$$

Characteristic equ $\rightarrow [A - \lambda I] = 0 \longrightarrow$ (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Now take determiment.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix}$$

Expand by R_1.

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} -(-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow (B)$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R_1.

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\left[((3-\lambda)(2-\lambda)-(-1)(-1)+1((-1)(2-\lambda)-(-1)(-1)) \right.$$
$$\left. -1((-1)(-1)-(-1)(3-\lambda))\right]$$

$$= (3-\lambda(6-3\lambda-2\lambda+\lambda 2-1)+(2+\lambda-1)-(+1+3-\lambda)$$

$= (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$

$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$

$= \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \longrightarrow (a)$

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1.

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$\Rightarrow -1\,(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$

$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$

$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \longrightarrow (b)$

$$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1.

$$-\left[-1\begin{vmatrix}-1 & -1\\ -1 & 2-\lambda\end{vmatrix} - (-1)\begin{vmatrix}3-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda.$$

$$= \boxed{-\lambda^2+6\lambda-8} \longrightarrow c$$

Put a, b and c in B.

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right)-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8.$$

$$= -2\lambda^3+(6\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda$$
$$-8-\lambda^2+16\lambda-8.$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda$$
$$+6\lambda+6\lambda+16-16.$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0.$$

By synthatic division.
we get:-

$$\lambda(\lambda-2)(\lambda^2-8\lambda+18)=0$$

$$(\lambda=0)$$

$$\lambda-2=0$$

$$\boxed{\lambda = 2}$$

$$\lambda^2-8\lambda+16=0$$

By facterization method.

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$(\lambda-4)(\lambda-4)$$

$$\lambda=4 \quad , \quad \lambda=4.$$

$$\boxed{\begin{array}{l} \lambda_1=0 \; , \; \lambda_2=2 \\ \lambda_3=4 \; , \; \lambda_4=4. \end{array}}$$

Answer.

Question No 3

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$x=2,\ y=6.$

Solution:-

$$(x^2+3y^2)dx - 2xy\,dy = 0.$$

$$\Rightarrow (x^2+3y^2)\,dx = 2xy\,dy.$$

Dividing both sides by $2xy\,dx$.

we get.

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}.$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{x}{2y} + \frac{3y}{2x}.$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow (*)$$

let $y = Vx.$

Diff :

$$dy = Vdx + xdv.$$

Diving by dx.

$$\boxed{\frac{dy}{dx} = V + x\frac{dv}{dx}} \longrightarrow (a).$$

Put (a) in (*).

$$V + \frac{xdv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}V} + 3\frac{V\cancel{x}}{\cancel{x}}\right].$$

$$V + \frac{xdv}{dx} = \frac{1}{2}\left[\frac{1}{V} + 3V\right].$$

Multiplying both sides by "2".

$$2v + 2x\frac{dv}{dx} = \frac{1}{V} + 3V.$$

$$2x\frac{dv}{dx} = \frac{1}{V} + 3V - 2V.$$

$$2x\frac{dv}{dx} = \frac{1}{V} + V$$

$$2x\frac{dv}{dx} = \frac{1+V^2}{V}.$$

Multiplying both sides by $\frac{dx}{dv}$

we get:

$$2x\, dv = \frac{1+V^2}{V}\, dx.$$

Multiplying both sides by $\frac{V}{x(1+V^2)}$.

we get:

$$\frac{V}{1+V^2}\, dv = \frac{1}{x}\, dx.$$

Take "$\int$" on both sides.

$$\int \frac{2v}{1+V^2}\, dv = \int \frac{1}{\bar{x}}\, dx + c$$

$$\ln|1+V^2| = \ln x + \ln c$$

Take "e" on both sides.

$$e^{\ln|1+V^2|} = e^{\ln|xc|}$$

$$\boxed{1+V^2 = Xc}$$

$$1+V^2 = Xc.$$

Put $V = Y/x$.

$$1+\left(\frac{y}{x}\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc.$$

$$x^2+y^2 = x^3c \longrightarrow (**)$$

Put $x=2$, $y=6$ in eq^n $(**)$.

$$(4)+(36) = 8c.$$

$$c = \frac{40}{8}.$$

$$\boxed{c = 5} \longrightarrow \text{Put in } (**).$$

So

$$x^2+y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1).$$

Taking "$\sqrt{}$" on both sides.

$$y = +x\sqrt{5x-1}, \quad y = -x\sqrt{5x-1}$$

OR.

$$\boxed{y = \pm x\sqrt{5x-1}}$$

Answer.