Department of Electrical Engineering

Final term exam

Date: 23/09/2020

Course Details

Course Title:	Signals & Systems	**Module:**	04
Instructor:	Engr. Mujatab Ihsan	**Total Marks:**	50

Student Details

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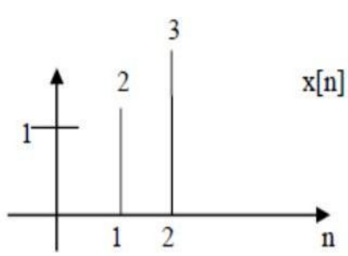
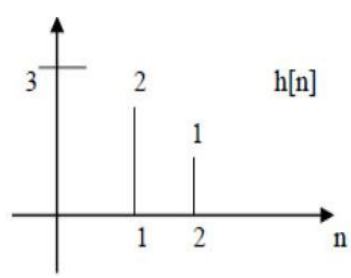
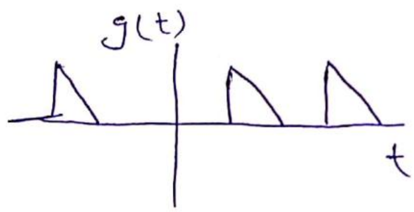
Q1.		**Identify** the basic difference between a periodic and an aperiodic signal using examples.	Marks 06 CLO 1
Q2.		$f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ \pi & 0 \le x \le \pi \end{cases}$ **Retrieve** the Fourier series for the given function.	Marks 12 CLO 3
Q3.		If $X(z) = \dfrac{2z^2 + 2z}{(z^2 + 2z - 3)}$ **Retrieve** x[n] using inverse Z-transform method.	Marks 10 CLO 3
Q4.		If $x[n] = 4\delta[n] - 3\delta[n-1] + 4\delta[n-2]$ $h[n] = 2\delta[n-1] - 3\delta[n-2] + 2\delta[n-3]$ **Produce** Y(z) and y[n]	Marks 10 CLO 3
Q5.		**Evaluate** y[n] using convolution summation.  	Marks 12 CLO 2

Que # 01

Answer

① Periodic Signal:-

A Signal is Periodic if $g(t) = g(t + t_0)$ for all t and and Some positive Constant T_0



② APeriodic Signal:-

A Signal which does not repeat itself after a Specific Value of time is Said to be an APeriodic Signal

Que # 02

Sol:-

$$f(x) = \begin{cases} 0 & -\pi \leq x \leq 0 \\ \pi & 0 \leq x \leq \pi \end{cases}$$

Ans:- $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\,dx$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx \right]$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} \pi\,dx \right]$$

$$= \frac{1}{2\pi} \left[0 + \pi x \Big|_{0}^{\pi} \right]$$

$$= \frac{1}{2\pi} \left[\pi(\pi - 0) \right]$$

$$= \frac{1}{2\cancel{\pi}} \times \cancel{\pi}$$

$$\boxed{a_0 = \frac{\pi}{2}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} (0)\cos nx\,dx + \int_{0}^{\pi} (\pi)\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[0 + \int_{0}^{\pi} \pi\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\pi\int_{0}^{\pi} \cos nx\,dx\right]$$

$$= \frac{1}{\cancel{\pi}}\left[\cancel{\pi}\,\frac{\sin nx}{n}\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{n}\left[\sin n(\pi) - \sin n(0)\right]$$

$$= \frac{1}{n}\left[0 - 0\right]$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} f(x) \sin nx \, dx + \int_{0}^{\pi} f(x) \sin nx \, dx \right]$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} (0) \sin nx \, dx + \int_{0}^{\pi} \pi \sin nx \, dx \right]$$

$$= \frac{1}{\pi} \left[0 + \pi \int_{0}^{\pi} \sin nx \, dx \right]$$

$$= \frac{1}{\pi} \left[\pi \left(-\frac{\cos nx}{n} \right) \Big|_{0}^{\pi} \right]$$

$$= \frac{-\cancel{\pi}}{n\cancel{\pi}} \left[\cos nx \Big|_{0}^{\pi} \right]$$

$$\cancel{= \frac{1}{n} \left[\cos nx \Big|_{0}^{\pi} \right]} = \frac{1}{n} \left[\cos n(\pi) - \cos n(0) \right]$$

$$\cancel{= \frac{1}{\pi}} \quad b_n = \begin{cases} 0 & \text{if } n \text{ Even} \\ \frac{2}{n} & \text{if } n \text{ odd} \end{cases}$$

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x \text{ ----}$$
$$b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x \text{ ------}$$

$$f(x) = \frac{\pi}{2} + 2 \sin x + \frac{2}{3} \sin 3x \quad \text{Ans}$$

Que # 03

Sol :-

Given that

$$X(z) = \frac{2z^2 + 2z}{z^2 + 2z - 3}$$

$$X(z) = \frac{2z(z+1)}{z^2 + 3z - z - 3}$$

$$X(z) = \frac{2z(z+1)}{z(z+3) - 1(z+3)}$$

$$\frac{X(z)}{z} = \frac{2(z+1)}{(z+3)(z-1)}$$

OR we can write

$$\frac{2(z+1)}{z^2 + 2z - 3} = \frac{A}{(z+3)} + \frac{B}{(z-1)}$$

Putting values

$$2(z+1) = A(z-1) + B(z+3) \Rightarrow Eq\ 1$$

Now Putting $z = 1$ in Eq $\rightarrow$ 1

$$2(1+1) = B(1+3)$$

$$4 = 4B$$

$$\frac{\cancel{4}}{\cancel{4}} = \frac{\cancel{4}B}{\cancel{4}}$$

$$1 = B \Rightarrow \boxed{B = 1}$$

Now Putting $Z=-3$ in Eq ①

$$2(-3+1) = A(-3-1)$$

$$-4 = -4A \Rightarrow \boxed{A = 1}$$

Now Putting the Value of Ⓐ & Ⓑ in Eq ①

$$\frac{2(Z+1)}{(Z+3)(Z-1)} = \frac{1}{(Z+3)} + \frac{1}{(Z-1)}$$

$$X(Z) = \frac{Z}{Z+3} + \frac{Z}{Z-1}$$

Now Inverse Z - Transform

$$\boxed{X(Z) = U[3] + 1(-1)^k}$$

Que # 04

Sol :-

$$x[n] = 4\delta[n] - 3\delta[n-1] + 4\delta[n-2]$$

$$h[n] = 2\delta[n-1] - 3\delta[n-2] + 2\delta[n-3]$$

$\Rightarrow$

$$X(z) = 4 - 3z^{-1} + 4z^{-2}$$

$$h(z) = 2z^{-1} - 3z^{-2} + 2z^{-3}$$

Now $Y(z) = Hz * X(z)$

$$y(z) = (2z^{-1} - 3z^{-2} + 2z^{-3}) \times (4 - 3z^{-1} + 4z^{-2})$$

$$= 8z^{-1} - 6z^{-2} + 8z^{-3} - 12z^{-2} + 9z^{-3} - 12z^{-4} + 8z^{-3} - 6z^{-4} + 8z^{-5}$$

$$y(z) = 8z^{-1} - 6z^{-2} - 12z^{-2} + 8z^{-3} + 9z^{-3} + 8z^{-3} - 12z^{-4} - 6z^{-4} + 8z^{-5}$$

$$Y(z) = 8z^{-1} - 18z^{-2} + 25z^{-3} - 18z^{-4} + 8z^{-5}$$

To Find $Y[n]$ use delay property

~~$Y[n] = 8\delta[n] + 4\delta[n-1]$~~

$$Y[n] = 8\delta[n-1] - 18\delta[n-2] + 25\delta[n-3] - 18\delta(n-4) + 8\delta[n-5].$$

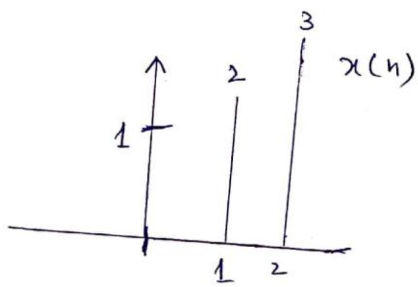
Ques # 05

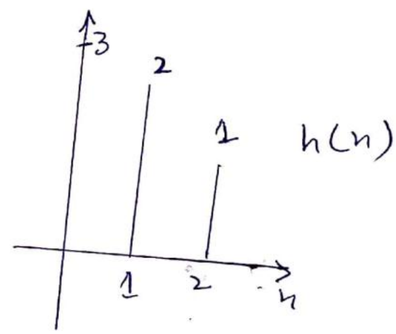
Sol :- As we know that the formula for convolution summation is given by

$$y[n] = x[n] * h[n]$$

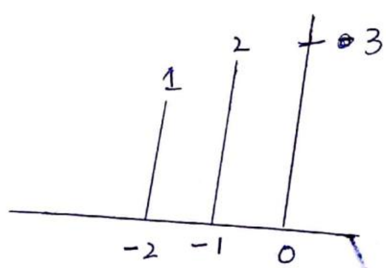
$$y[n] = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

* ~~Replace it~~ Convert $x(n)$ in $x(k)$





Ⓐ invert $h(k)$ to get $h(-k)$



$$Y[n] = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Now $y[0] = x(k)\, h(0-k)$

$= (3)(1)$

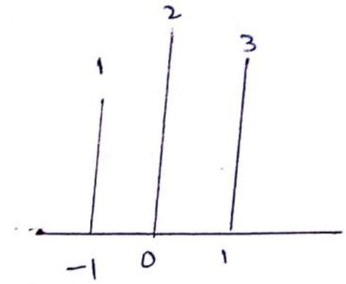
$\boxed{= 3\delta[n]}$

$$Y[1] = x(k)\,h[1-k]$$

$$= (1)(2) + (2\times 3)$$

$$2 + 6 = 8$$

$$\boxed{Y[1] = 8\delta[n-1]}$$

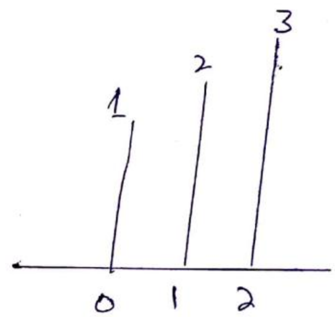


$$Y[2] = x[k]\,h[2-k]$$

$$= (2)(2) + (3\times 3)$$

$$= 4 + 9 = 13$$

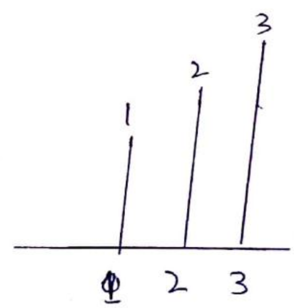
$$\boxed{Y[2] = 13\delta[n-2]}$$



$$Y[3] = x[k]\,h[3-k]$$

$$= (3\times 2) = 6$$

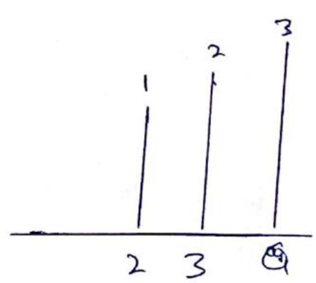
$$\boxed{Y[3] = 6\delta[n-3]}$$



$$Y[4] = x[k]\,[4-k]$$

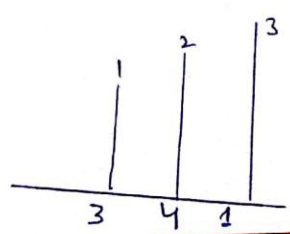
$$= (3\times 1) = 3$$

$$\boxed{Y[4] = 3\delta[n-4]}$$



$$Y[5] = x[k]\,[5-k]$$

$$\boxed{= 0}$$



So There is no overlapping between

$x[k]$ and $h[n-k]$

$$y[n] = 3\delta[n] + 8\delta[n-1] + 13\delta[n-2] + 6\delta[n-3] + 3\delta[n-4]$$

<u>Graphically</u> :-