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Section :- A

PAPER : differential equation

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Q No 1 :-

THE WAVE Equation :-

The wave equation is an important second-order linear partial differential equation for the description of wave. - as they occur in classical physic - such as machanical wave - (e.g water waves, Sound wave and sesmic wave.

→ we generally visit beach and if we stand on an ocean shore and take a snapshots of the wave by 1- Dimensional equation.

Show as - - - - - - -
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equation.

$$\frac{\partial^2 w}{\partial t^2} = \frac{c^2 \partial^2 w}{\partial x^2}$$

(i) $w = \sin(x+ct) + \cos(2x+2ct)$

(ii) $w = \tan(2x+ct)$

i: $w = \sin(x+ct) + \cos(2x+2ct)$

Solution:- $w = \sin(x+ct) + \cos(2x+2ct)$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\Rightarrow \frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) 4c^2$$

$$\Rightarrow \frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct)$$

$$\Rightarrow \frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= \left[-\sin(x+ct) - 4\cos(x+2ct)\right]$$

$$\Rightarrow \frac{\partial^2 w}{\partial t^2} = +c^2\left[-\sin(x+ct) - 4(2x+2ct)\right]$$

$$c^2 \cdot \frac{\partial^2 w}{\partial x^2}$$

Hence

$$\frac{\partial^2 w}{\partial t^2} = \frac{c^2 \partial^2 w}{\partial x^2}$$

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Q No 1 (PART B)

$w = \tan(2x + ct)$

Solution:-

As we know that

$w = \tan(2x + ct)$

taking derivative w.r.t "t"

$$\Rightarrow \frac{\delta w}{\delta t} = \frac{\delta}{\delta t} \tan(2x + ct) \cdot c$$

$$\Rightarrow \frac{\delta w}{\delta t} = \sec^2(2x + ct) \cdot c$$

$$\Rightarrow \frac{\delta w}{\delta t} = c \sec^2(2x + ct)$$

Again taking derivative

$$= \frac{\delta^2 w}{\delta t^2} = c \cdot 2 \sec(2x + ct) - \sec(2x + ct) \cdot \tan(2x + ct) \, c$$

$$\Rightarrow \frac{\delta^2 w}{\delta t^2} = 2c^2 \sec^2(2x+ct) \cdot \tan(2x+ct)$$

Now $w = \tan(2x+ct)$

Taking derivative w.r.t x

$$\Rightarrow \frac{\delta w}{\delta x} = \frac{\delta}{\delta x} \tan 2x+ct$$

$$\Rightarrow \frac{\delta w}{\delta x} = \sec^2(2x+ct) \cdot 2$$

$$\Rightarrow \frac{\delta w}{\delta x} = 2\sec^2(2x+ct)$$

Again taking derivative

$$\Rightarrow \frac{\delta^2 w}{\delta x^2} = 2 \cdot 2 \sec(2x+ct) \sec(2x+ct) \tan(2x+ct) \cdot$$

$$\Rightarrow \frac{\delta^2 w}{\delta x^2} = 6\sec^2(2x+ct) \tan(2x+ct)$$

As we know wave equation

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$1 \neq 3$

So we have conluded that

$L.H.S \neq R.H.S$

Hence it is not the proof of wave equation

(6)

Q No 2

Solution Answer to the Question #02

$$F(x) = \begin{cases} x; & -\pi < x \leq 0 \\ 2x; & 0 \leq x \leq \pi \end{cases}$$

⇒ we have to find fourier's co-efficient, a_0, a_n and b_n

Now

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left(\frac{x^2}{2}\right)_{-\pi}^{0} + \frac{2}{\pi}\left(\frac{x^2}{2}\right)_{0}^{\pi}$$

$$\frac{1}{\cancel{\pi}}\left[0 - \frac{\pi^2}{2}\right] + \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\pi^2}{\cancel{2}} - 0\right]$$

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$$\boxed{a_0 = -\frac{\pi}{2} + \pi = \frac{\pi}{2}} \to (1)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \cos nx \, dx$$

$$-\frac{1}{\pi} \int_{-\pi}^{0} (x \cos nx \, dx) + \frac{1}{\pi} \int_{0}^{\pi} (2x \cos nx) \, dx$$

$$= \frac{1}{\pi} \left[x \left(\frac{\sin nx}{x} \right) - \left(-\frac{\cos nx}{n^2} \right) \right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi} \left[x \left(\frac{\sin 3x}{x} \right) - \left(-\frac{\cos nx}{n^2} \right) \right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi} \left[\frac{\cos(0)}{n^2} - \frac{\cos nx}{n^2} + \frac{2}{\pi} \left[\frac{\cos nx}{n^2} \right. \right.$$

$$\left. \frac{\cos(0)}{n^2} \right]$$

$$= \frac{1}{\pi} \left[\frac{1 - (-1)^n + 2(-1)^n - 2}{n^2} \right]$$

$$= \frac{(-1)^n - 1}{\pi n^2}$$

so

$$a_n = \left\{ \begin{array}{ll} \frac{-2}{\pi n^2} & ; \text{ if } n \text{ is odd} \\ 0 & ; \text{ if } n \text{ is even} \end{array} \right\} \rightarrow ②$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin dx = \frac{1}{\pi} \int_{-\pi}^{0} x \sin nx \, dx$$

$$+ \frac{2}{\pi} \int_{0}^{\pi} x \sin nx$$

$$= \frac{1}{\pi} \left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right) \right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi} \left[x\left(\frac{-\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right) \right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi} \left[\frac{-\pi \cos n\pi}{n} \right] + \frac{2}{\pi} \left[\frac{-\pi \cos n\pi}{n} \right]$$

$$= \frac{-3 \cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

So the required Fourier is

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2x-1)}{(2x-1)^2} + 3 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin x^2}{n}$$

Q No 3:-

Solve the intial value problem.

$$y'' - 4y' + 3y = 8\sin 3x$$

$$y(0) = 1, \quad y'(0) = 2$$

Solution:-

$$y'' - 4y' + 13y = 8\sin 3x \quad \cdots\cdots \text{(1)}$$

Associated Homogenious Eq (1)

$$y'' - 4y' + 13y = 0 \quad \cdots\cdots \text{(2)}$$

Into Auxiliary equation

put $y = m$ in (2)

$$m^2 - 4m + 13 = 0$$

$\Rightarrow$ use quadratic formula.

$$a = 1, \quad b = -4, \quad c = 13$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2}$$

$$m = \frac{4 \pm \sqrt{16-52}}{2}$$

$$m = \frac{4 \pm \sqrt{-36}\, i}{2} = \frac{4+6i}{2}$$

$m = 2 \pm 3i$, $m = 2-3i$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \quad \text{------- (A)}$$

let $y_p = A \cos 3x + B \sin 3x \rightarrow$ (A)

Diff. with respect to x

$$y_p' = -3A \sin 3x + 3B \cos 3x$$

Again diff with respect to "x"

$$y_p'' = -9A \cos 3x - 9B \sin 3x$$

put in (1)

$\Rightarrow (-9A \cos 3x - 9B \sin 3x) - 4(-3A \sin 3x + 3B \cos 3x) + 13(A \cos 3x + B \sin 3x) = 8 \sin 3x$

$\Rightarrow -9A \cos 3x - 12B \cos 3x + 13A \cos 3x - 9B \sin 3x + 12A \sin 3x + 13B \sin 3x = 8 \sin 3x$

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$\Rightarrow (-9A-12B+13A)\cos 3x + (-9B+12A+13B)\sin 3x = 8\sin 3x$

$\Rightarrow (4A-12B)\cos 3x + (4B+12A)\sin 3x = 8\sin 3x$

Comparing coefficients.

$\sin 3x \Rightarrow \quad 4B+12A = 8 \Rightarrow (a)$

$\cos 3x \Rightarrow \quad 4A - 12B = 0 = 4A = 12B$

$\rightarrow \boxed{A = 3B} \rightarrow (b)$

put (b) in (a)

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$\boxed{B = 1/5} = (c)$

put (c) in (b)

$\boxed{\Rightarrow A = 3/5} \rightarrow (d)$

put c and d in (*)

$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (B)$

B

The G·sol is

$y = y_c + y_p$

$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5}$
$\cos 3x + 1/5 \sin 3x \rightarrow$ (C)

Now we need to find the values of c_1 and c_2 for this put $x=0$ and $y=1$ in (C)

$1 = e^{x(0)}(c_1 \cos 3x(0) + c_2 \sin 3(0)) + \frac{3}{5}$
$\cos 3(0) + 1/5 \sin 3(0)$

$1 = ((c_1(1) + c_2(0) + \frac{3}{5}(1) + \frac{1}{5}(0)$

$1 = c_1 + 3/5$

$c_1 = 1 - \frac{3}{5}$

$c_1 = 2/5 \rightarrow$ (**)

Diff (C) w.r.t "x"

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x)$
$+ c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$
$-\frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x \rightarrow$ (D)

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put $y=2$, $x=0$ in ①

$$y = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x$$

put $y=2$, $x=0$

$$\Rightarrow 2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)) + c_2(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0)) - \frac{6}{5}\sin 3x(0) + \frac{3}{5}\cos 3(0)$$

$$2 = c_1(2) + c_2(3) - 0 + \frac{3}{5}$$

$$2c_1 = 2/5$$

$$2 = \frac{4}{5} + 3c_2 + \frac{3}{5}$$

$$2 = \frac{7}{5} + 3c_2$$

$$3c_2 = 2 - \frac{7}{5}$$

$$\boxed{c_2 = 3/15} \rightarrow ★★★$$

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Put ★★ and ★★★ in (c)

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right)$$

$$+\frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$$

$$\boxed{y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x}$$

Required general solution

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Q: No (4)

Solution:-

$$(D^2 - DD')Z = \cos x \cos 2y$$

the given PDE can be rewrite as:

$$D(D-D')z = \cos x \cos 2y$$

in CF in given by

$$CF = \phi_1(y) + \phi_2(y+x)$$

while its PI is given by

$$PI = \frac{1}{(D^2 - DD')} \cdot \frac{1}{2}\left[\cos(x+2y) + \cos(x-2y)\right]$$

$$= \frac{1}{2}\left(\frac{1}{(-1+2)}\cos(x+2y) + \frac{1}{(-1-2)}\cos(x-2y)\right)$$

$$= \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

Hence the Complete solution of the given PDE is given by $=$

$$Z = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$