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Major Assignment Spring 2020

Q1: What is a Reecurrence Relation? And explain Repeated Substitution Method with the help of an example.

Ans: A Reccurrence relation is an equation that recursivly define a sequence where next term is a function of Previous term.

For Example.

$$a_1 = a_0 + 2$$
$$a_2 = a_1 + 2$$
$$a_3 = a_2 + 2$$
$$\vdots$$
$$a_n = a_{n-1} + 2.$$

Example $\Rightarrow$

$$a_k = a_{k-1} + 2 \quad \text{with } a_0 = 1$$

$k = 1 \ldots\ldots n$.

$$\Rightarrow a_1 = a_0 + 2 = 1 + 2$$
$$a_2 = a_1 + 2 = 1 + 2 + 2 = 1 + 2 \cdot 2$$
$$a_3 = a_2 + 2 = 1 + 2 + 2 + 2 = 1 + 3 \cdot 2.$$
$$\vdots$$
$$a_n = 1 + n \cdot 2 \Rightarrow a_n = 1 + 2n$$

Substitution Method :- Substitution method has two types.

(1) Forward Substitution.

(2) Backward Subsitution.

(1) Forward Substitution

uses the initial cond^n in the initail term and value for the next term is generated.

E.g.:- $T(n) = T(n-1) + n$, $T(0) = 0$ } Initial cond^n

Solution:-

$n = 1$, then

$T(1) = 1$

$T(1) = T(0) + 1$ — (i)

$n = 2$, then

$T(2) = T(1) + 2$, $T(2) = 3$ — (ii)

$n = 3$, then

$T(3) = T(2) + 3$, $T(3) = 6$ — (iii)

$T(n) = \frac{n(n+1)}{2}$ } Guessed from (i) (ii) and (iii)

$$T(n) = \frac{n^2}{2} + \frac{n}{2}$$

Ans: $\boxed{T(n) = O(n^2)}$

(2) Backward Substitution:

In this substitution backward values are subtituted recursively.

Example: $T(n) = T(n-1) + n$ with $T(0) = 0$ — ①

$T(n-1) = T(n-1-1) + (n-1)$ } Replace n by n-1 in ①.

$T(n-1) = T(n-2) + (n-1)$ — (ii)

$T(n-2) = T(n-2-1) + (n-2)$ — (ii)

Replace n by n-2.

$$T(n-2) = T(n-3) + (n-2)$$

$$= T(n-k) + (n-k+1) + (n-k+2) + \ldots\ldots n.$$

if $k = n$ then.

$$T(n) = \underbrace{T(0) + 1 + 2 + \ldots + n}_{\text{sum of } n \text{ natural number}}.$$

$$T(n) = \frac{n(n+1)}{2}$$

$$T(n) = \frac{n^2}{2} + \frac{n}{2}$$

$$\boxed{T(n) = O(n^2).}$$ Ans.

Q No 2 Conside.

Premises: If there was a ball game, then traveling was difficult. if they arrived on time, then traveling was nat difficult. They arrived on time.

Conclusion: There was no ball game.

p: if there was a ball game.

q: then traveling was difficult

p: if they arrived on time.

¬q: then traveling was nat difficult.

Conclusion: ¬p: There was not ball game.

Inference Rules.

Symbolic representation.

Modus Tollens.

$$\frac{\begin{array}{c} p \rightarrow q \\ \neg q \end{array}}{\neg p} \quad \text{OR} \quad \big[(p \rightarrow q) \wedge \neg q\big] \rightarrow \neg p$$

Q No 3:- Consider.

Premises: If Calghoom has wide Support, then he'll be asked to run for the Senate. If Claghorn Yells "Eureka" in Iowa, he will not be asked to run for the Senate.

Conclusion: Claghorn deos not have wide support.

Claghorn Yells "Eureka" in lawa.

p: Clashorn has wide support.

q: He,ll be asked to run for the Senate.

r: Calaghorn Yells "Eureka" in lawa

$\neg q$: Heill not be asked to run for the Senate.

Conclusion:

$\neg p$: Claghorn does nat have wide support.

Inference Rules.

$$\begin{array}{l} p \rightarrow q \\ r \rightarrow \neg q \\ \hline \therefore \neg p \end{array}$$

OR $[(p \rightarrow q) \wedge (r \rightarrow \neg q)]$
$\rightarrow \neg p$.

OR. $[(p \rightarrow q) \wedge (r \rightarrow \neg q)] \rightarrow \neg p$.

It is Hypothetical Syllogism.

End of th Answer.

Q No 4

Pigeonhole Principle.

Theorem: If n pigeons are assigned to m Pigeonholes and $m < n$ then at least one pigeonhole contains two or more pigeons.

Proof: Suppose each pigeonhole contains almost one pigeon. then m pigeon will be accomadated in m Pigeonholes one pigeon in each Pigeonhole

But $m < n$, not all Pigeons have been assigned Pigeonholes

This is a Contradiction.

[Hence, there is at least one pigeonhole which contains two or more Pigeons.]

eg:- If 7 colours are used to Paint 50 bicycles Show that at least 8 of them will of same colour.

Solution:- Let colours denoted Pigeonholes and bicycles denote Pigeons and $m < n$.

$\frac{50}{7} = 7$ ∴ 7 Bicycles each of colours

remainder 1 will be a

Colour from the 7.

∴ 8 bicycle may have a

Same colour.

The extended Pigeonhole Principle

e.g:- Show that in a group of 50 student at least 5 are born in Same month.

Solution:- There are $n = 12$ (month) Pigeonholes

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and $kn+1 = 50$ Pigeons.

$\therefore k \cdot 12 + 1 = 50$

$k \cdot 12 = 50 - 1$

$k = \frac{49}{12} = 4 \cdot$ | remainder $= 1$.

$\therefore$ At least $4+1=5$ student are born in same month.

Ans.

Q No: 7: Give the out Put signals for the circuit.

Circuit No 1st -

input signals. $P = 1$ and $Q = 0$.

$R = ?$

p	q	$p \vee q$	$\neg q$	$p \wedge q = R$
1	0	1	1	1 = 1

So the output signal is 1

$\boxed{R = 1}$ Ans

2nd circuit: input signals

$P = 1, \ Q = 0, \ R = 0$

output: $S = ?$

P	Q	$\neg Q$	$(P \wedge \neg Q) \vee R \rightarrow$	S
1	0	1	1	1

So output signal is

$\boxed{S = 1}$ Ans -

3rd circuit: input signals -

input = $P = 0, \ Q = 0, \ R = 0$.

P	Q	R	P ∨ Q	Q ∧ R	((P ∨ Q) ∨ ¬P ∧ Q))
0	0	0	0	0	1

So output signal

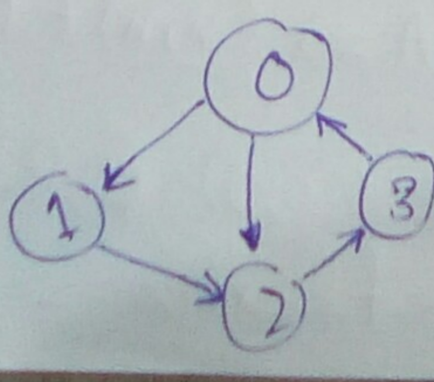
$\boxed{S = 1}$ Ans

Q No 8 :. A number of relations are defined on the set $A = \{0, 1, 2, 3\}$ for each relation;

ⓐ Draw the directed graph.

A directed graph contains direction to the adjacent vertices by $\longrightarrow$ (Arrow Symbol)

A graph, which has ordered pair of vertices.



$E(G) = \{(0,1), (1,2), (2,3),$
$(3,0), (0,2)\}$

(b) Determine whether the relation is reflexive.

To check the reflexivety

$A = \{0, 1, 2, 3\}$ as $R = \{(a, b) : b = a+1\}$ is reflexive.

Sol: let $A = \{0, 1, 2, 3\}$

$R \{(a, b) : b = a+1\}$

$R = \{(0, 1), (2, 3)\}$

Reflexive: $\forall n \in A \Rightarrow (n, n) \in R$

for $0 \in A \Rightarrow (0,0) \notin R \Rightarrow R$ is not reflexive

(c) Determine, whether the relation is symmetric.

if Symmetric : $\forall (a,b) \in R \Rightarrow (b,a) \in R$.

$R = \{(0,1), (2,3)\}$.

Symmetric; let $(0,1) \in R \Rightarrow (1,0) \notin R$

$\Rightarrow R$.

is not symmetric Relation.

(d) Transitive:- $\{(0,1), (2,3)\}$

(a,b) & (b,c). (a,c)

For $(0,1) \in R$ and $(1,2) \notin R \Rightarrow (1,3) \notin R$

So R is not transitive

Give a counterexample in each case in which the relasion does not satisfy one of the Properties

(1) $R1 = \{(0,0), (0,1), (0,3), (1,1), (1,0), (2,3), (3,3)\}$.

R1 is not a reflexive relation because it has no (2,2) Pair.

R1 is not a Symmetric relation because it does not have (3,0), (3,2) Pairs

R1 is a Transitive because it has ~~(1,0), (0,0)~~ (0,1), (1,0) etc.

② $R_2 = \{(0,0), (0,1), (1,1), (1,2), (2,2), (2,3)\}$.

(i) R_2 is not reflexive because it does not contain (3,3) Pair.

(ii) R_2 is not a Symmetric Relation because it has not (1,0), (2,1), (3,2) Pairs.

(iii) R_2 is not a Trasitive relasion because it has no (x,z) Pair.

(3) $R_3 = \{(2,3),(3,2)\}$

R_3 is a Symmetric relation because it contains (2,3) and (3,2) Pairs.

R_3 is not a reflexive Relation because it does not contains (2,2),(3,3) Pairs

R_3 is not a transitive relation because it does not contains (y,z) Parir.

④ $R_4 = \{ (1,2), (2,1), (1,3), (3,1) \}$.

R_4 is a Symmetric relation Because it contains (1,2), (2,1) and (1,3), (3,1) Pairs.

⑤ $R_5 = \{ (0,0), (0,1), (0,2), (1,2) \}$.

R_5 is a transitive relation Because it contains

$(x,y), (y,z) = (0,1), (1,2)$

Pairs.

⑥ $R_6 = \{(0,1),(0,2)\}$

R_6 is not a Reflexive nor Symmetric and nor transitive. Because it does not the Properties of these three relations.

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⑦

$R_7 = \{(0,3),(2,3)\}$.

R_7 is not a Reflexive nor Symmetric and nor Transitive. Because it does not Contains these the Properties.

(8) $R_8 = \{(0,0),(1,1)\}$.

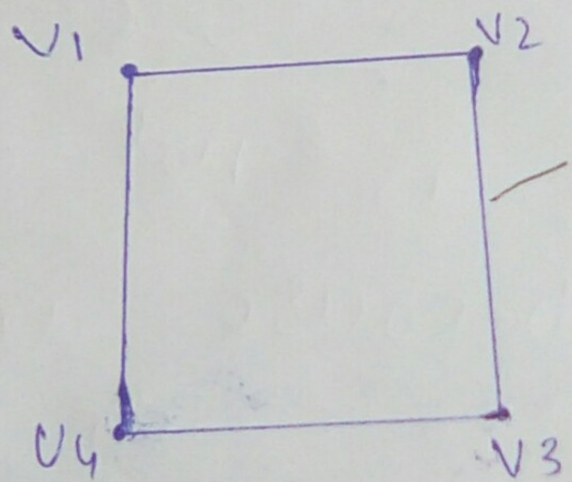
R_8 is a Reflexive Because it contains (0,0) and (1,1) Pairs.

R_8 is a Symmetric because it contains the Properties of Symmetric.

End.

Q9 :- Find which of the following graphs are bipartite. Redraw the bipartite graphs so that there bipartite nature is evident

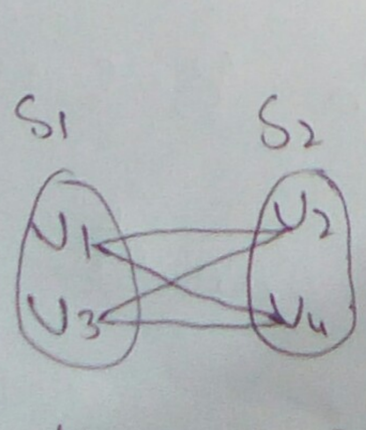
(a)



$S_1 = \{V_1, V_3\}$ and

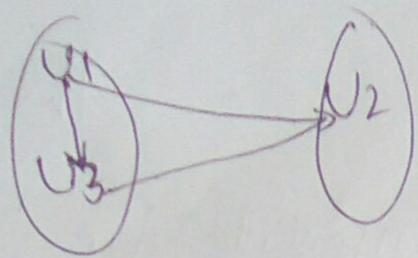
$S_2 = \{V_2, V_3\}$

It is a bipartite graph.



From this graph we can say graph (a) is a bipartite graph.

(b)

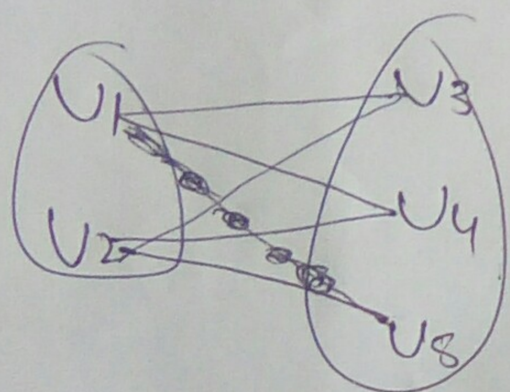


This is not a bipartite graph Because V_1 and V_3 are the belonges to Same partition.

(c) Graph C is not a bipartite graph because V_1, V_2, V_3, directly Connected to each other.

(d) Graph d is not a bipartate graph. Because V_3 is not connecte to all vertices.

(e) Graph (e) is ~~[illegible]~~ not a bipartite grap. v_5 is connected to v_1.



(f) Graph (f) is not a bipartite graph.