

**Department of Electrical Engineering  
Assignment**

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**Course Details**

<b>Course Title:</b>	<u>Electromagnetic field</u>	<b>Module:</b>	<u>04</u>
<b>Instructor:</b>	<u></u>	<b>Total Marks:</b>	<u>30</u>

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**Student Details**

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QNO1:- The value of  $E$  at -----

$$\textcircled{c} G = 2a_x - 3a_y + 4a_z.$$

Solution:-

$\textcircled{a}$  in the direction of  $a_p =$  the incremental work is given by  $dW = -q_p E \cdot dL$ , where in this case  $dL = dp a_p = 6 \times 10^{-6} a_p$ . Thus.

$$\begin{aligned} dW &= -(20 \times 10^{-6} \text{C})(100 \text{V/m})(6 \times 10^{-6} \text{m}) \\ &= -12 \times 10^{-9} \text{J} \end{aligned}$$

$$= \boxed{-12 \text{ nJ}}$$

$\textcircled{b}$  in the direction of  $a_\phi = \hat{i}$  in this case  $dL = 2d\phi a_\phi = 6 \times 10^{-6}$  and so,

$$\begin{aligned} dW &= -(20 \times 10^{-6})(-200)(6 \times 10^{-6}) \\ &= 2.4 \times 10^{-8} \text{J} \end{aligned}$$

$$= \boxed{24 \text{ nJ}}$$

$\textcircled{c}$  in the direction of  $a_z =$  Here,  $dL = dz a_z = 6 \times 10^{-6} a_z$ .

$$\begin{aligned} dW &= -(20 \times 10^{-6})(300)(6 \times 10^{-6}) \\ &= -3.6 \times 10^{-8} \text{J} \end{aligned}$$

$$= \boxed{-36 \text{ nJ}}$$

$\textcircled{d}$  In the direction of  $E$ .

$$a_E = \frac{100a_p - 200a_y + 300a_z}{[100^2 + 200^2 + 300^2]^{1/2}}$$

$$= 0.267a_p - 0.535a_y + 0.802a_z.$$

$\textcircled{1}$

Thus

$$dW = -(20 \times 10^6) [1000a_x - 2000a_y + 3000a_z] \cdot [0.267a_x - 535a_y + 0.802a_z] \times 6 \times 10^6$$

$$= \boxed{-44.9 \text{ mJ}}$$

© In the direction of  $G = 2a_x - 3a_y + 4a_z$ .

$$a_G = \frac{2a_x - 3a_y + 4a_z}{[2^2 + 3^2 + 4^2]^{1/2}}$$

$$= 0.371a_x - 0.557a_y + 0.743a_z$$

So Now

$$dW = -(20 \times 10^6) [1000a_x - 2000a_y + 3000a_z] \cdot [0.371a_x - 0.557a_y + 0.743a_z]$$

$$= -(20 \times 10^6) [37.1(a_x \cdot a_x) - 55.7(a_x \cdot a_y) - 74.2(a_y \cdot a_x) + 111.4(a_y \cdot a_y) + 222.9] (6 \times 10^6)$$

where at P,  $(a_x \cdot a_x) = (a_y \cdot a_y) = \cos(40^\circ) = 0.766$

$$(a_x \cdot a_y) = \sin(40^\circ) = 0.643$$

$$(a_y \cdot a_x) = -\sin(40^\circ) = -0.643$$

Now substituting these results in.

$$dW = -(20 \times 10^6) [28.4 - 35.8 + 47.7 + 85.3 + 222.9] (6 \times 10^6)$$

$$= \boxed{-41.8 \text{ mJ}}$$

Q No 2 Let  $E = 10 [\sin(\pi/6) a_x + 5 \sin(\pi/6) a_y + 10 \cos(\pi/6) a_z]$

Solution

(a)  $E_p = -10 [\sin(\pi/6) a_x + 5 \sin(\pi/6) a_y + 10 \cos(\pi/6) a_z]$   
 $= -[5 a_x + 25 a_y + 50 \sqrt{3} a_z]$

(b)  $dW_x = -q E \cdot dL a_x$   
 $= -2 \times 10^{-9} (-5) (10^{-3}) = 10^{-11} \text{ J}$   
 $= \boxed{10 \text{ pJ}}$

(c) of  $a_y$ ?  
 $dW_y = -q E \cdot dL a_y$   
 $= -2 \times 10^{-9} (-25) (10^{-3}) = 50^{-11} \text{ J}$   
 $= \boxed{50 \text{ pJ}}$

(d) of  $a_z$   
 $dW_z = -q E \cdot dL a_z$   
 $= -2 \times 10^{-9} (-50 \sqrt{3}) (10^{-3})$   
 $= \boxed{100 \sqrt{3} \text{ pJ}}$

(e) of  $(a_x + a_y + a_z)$ ?

$$dW_{xyz} = -q E \cdot dL \frac{(a_x + a_y + a_z)}{\sqrt{3}}$$

$$= \frac{10 + 50 + 100 \sqrt{3}}{\sqrt{3}}$$

$$= \boxed{135 \text{ pJ}}$$

(3)

QNO3

Solution:-

① P(1,2,3) toward Q(2,1,4)

The vector along this direction will be  $Q-P = (1, -1, 1)$   
from which  $a_{PQ} = [a_x - a_y + a_z] / \sqrt{3}$

$$\begin{aligned} dW &= -qE \cdot dL \\ &= -(50 \times 10^6) \left[ 120 a_p \cdot \frac{(a_x - a_y + a_z)}{\sqrt{3}} \right] (2 \times 10^{-3}) \\ &= -(50 \times 10^6) (120) \left[ (a_x \cdot a_p) - (a_p \cdot a_y) \right] \frac{1}{\sqrt{3}} (2 \times 10^{-3}) \end{aligned}$$

At P,  $\phi = \tan^{-1}(2/1) = 63.4^\circ$ , Thus  $(a_p \cdot a_x) = \cos(63.4^\circ)$   
 $= 0.447$

$(a_p \cdot a_y) = \sin(63.4^\circ) = 0.894$

Substituting these, we obtain

$dW = 3.1 \mu J$

② Q = (2, 1, 4) toward P(1, 2, 3) A little thought is in order here: Note that the field has only a radial component and does not depend on  $\phi$  or  $z$ . And P and Q are at the same radius  $(\sqrt{5})$  from  $z$  axis. Thus the answer is  $dW = 3.1 \mu J$  as in part a. This is also found by going through the procedure as in part a, but with the direction (roles of P & Q) reversed.

QNO4 Compute the value  $\int_A^P G \cdot dL$ .

Solution :-

(a) Straight line segment's  $A(1, -1, 2)$  to  $B(1, 1, 2)$  to  $P(2, 1, 2)$ : In general we would have.

$$\int_A^P G \cdot dL = \int_A^P 2y dx.$$

The change in  $x$  occurs when moving b/w  $B$  &  $P$ , during which  $y=1$ .

$$\int_A^P G \cdot dL = \int_B^P 2y dx$$

$$= \int_1^2 2(1) dx.$$

(b) Straight line segment  $A(1, -1, 2)$  to  $C(2, 1, 2)$

to  $P(2, 1, 2)$ : In this case change in  $x$  occurs when moving from  $A$  to  $C$  during which  $y=-1$

$$= \int_A^P G \cdot dL = \int_A^C 2y dx$$

$$= \int_1^2 2(-1) dx$$

$$= \underline{\underline{-2}}$$

(5).

QND 5 Let  $G = 3xy^3a_x + 2za_y$ . Find.

Solution:-

Let  $G = 3xy^3a_x + 2za_y$ .

(a) straight line  $y = x - 1, z = 1$

$$\begin{aligned} &= \int G \cdot dL = \int_2^4 3xy^2 + \int_1^3 2z dy \\ &= \int_2^4 3x(x-1)^2 dx + \int_1^3 2(1) dy \\ &= \underline{\underline{190}} \end{aligned}$$

(b) Parabola  $6y = x^2 + 2, z = 1$

$$\begin{aligned} &= \int G \cdot dL = \int_2^4 3xy^2 + \int_1^3 2z dy \\ &= \int_2^4 \frac{1}{12} x(x^2 + 2)^2 + \int_1^3 2(1) dy \\ &= \underline{\underline{82}} \end{aligned}$$

(c)