Course: Calculus and analytical geometry
Program: BS (SE, CS)
Instructor: Muhammad Abrar Khan
Examination: Final Paper
Total Marks: 50
Date: June. 27, 2020

Note: Attempt all questions. Use examples and diagrams where necessary.

Q.1

a) Differentiate $\frac{3x^4-2x^3+5}{x^3+1}$ with respect to x.

b) Differentiate $\frac{(x^3+1)^2}{x^3-1}$ with respect to x.

Q.2

a) Find the Integration of $\int \frac{1}{\sqrt{x^5}}$ dx.

b) Find the Integration of $\int \frac{1}{(8x+7)^8}$dx.

Q.3

a) Find the Integration of $\int \frac{-x+9}{2x^2-8x+6}$dx by Partial fractions.

b) Find the Integration of $\int \frac{4x^2+8x}{(x^{2+1})(x^2+2x+3)}$dx by Partial fractions.

Q.4

Solve each of the following matrix equations:

a) $X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$

b) $X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

c) $X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

Q.5

a) If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ Find A^2+BC

Page # 1

Prob: Differentiate w.r.t x

1) $\frac{3x^4 - 2x^3 + 5}{x^3 + 1}$

$\frac{d}{dx}\left(\frac{3x^4 - 2x^3 + 5}{x^3 + 1}\right)$

Patient Rule.

$= \frac{(x^3+1)\frac{d}{dx}(3x^4 + 2x^3 + 5) - (3x^4 + 2x^3 + 5)\frac{d}{dx}(x^3+1)}{(x^3+1)^2}$

$= \frac{(x^3+1)(12x^3 + 6x^2 + 0) - (3x^4 + 2x^3 + 5)(3x^2 + 0)}{(x^3+1)^2}$

$= \frac{(x^3+1)(12x^3 + 6x^2) - 3x^4 + 2x^3 + 5(3x^2)}{(x^3+1)^2}$

$= \frac{12x^6 + 6x^5 + 12x^3 + 6x^2 - 9x^6 + 6x^5 + 15x^2}{(x^3+1)^2}$

$= \frac{3x^6 + 12x^5 + 12x^3}{}$

$= \frac{3x^6 + 12x^3 - 9x^2}{(x^3+1)^2}$

$= \frac{3x^2(x^4 + 4x - 3)}{(x^3+1)^2}$ Ans.

Page # 2

Q1

(ii) $\frac{(x^3+1)^2}{x^3-1}$

$\frac{d}{dx}\left(\frac{(x^3+1)^2}{(x^3-1)}\right)$

$\Rightarrow \frac{(x^3-1)\frac{d}{dx}(x^3+1)^2 - (x^3+1)^2\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$

$\Rightarrow \frac{(x^3-1)\,2(x^3+1)\frac{d}{dx}(x^3+1) - (x^3+1)\,2x}{(x^3-1)^2}$

$\Rightarrow \frac{(x^3-1)\,2(x^3+1)\,3x^2 - (x^3+1)(2x)}{(x^3-1)^2}$

$\Rightarrow \frac{6x^2(x^3-1)(x^3+1) - (x^3+1)(2x)}{(x^3-1)^2}$

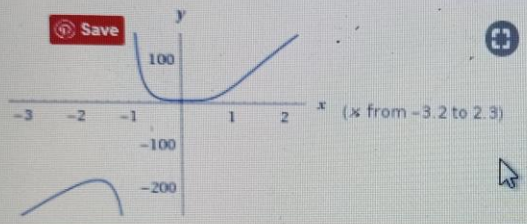
$\Rightarrow \frac{x^3+1\ [6x^2(x^3-1) - (2x)]}{(x^3-1)^2}$

$\Rightarrow \frac{(x^3+1)\left[6x^4 - 6x^2 - 2x\right]}{(x^3-1)^2}$

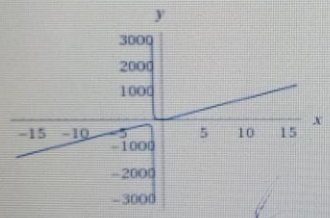
$\Rightarrow \frac{(x^3+1)(6x^4 - 6x^2 - 2x)}{(x^3-1)^2}$ Ans.

$$\frac{81x^4 - 8x^3 + 5}{x^3 + 1}$$

Plots:



(x from -3.2 to 2.3)



(x from -16.8 to 15.9)

Enlarge Customize

Step-by-step solution

Alternate forms:

$$\frac{81x^4 - 8x^3 + 5}{(x+1)(x^2 - x + 1)}$$

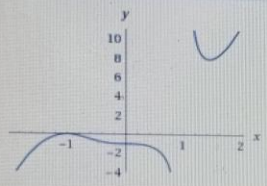
$$\frac{(81x - 8)x^3 + 5}{x^3 + 1}$$

Support for Latitud... Best Article Spinner... WhatsApp The Official Websit... Student Informatio... JPG to PDF - Conve... Use case tutorial ex...

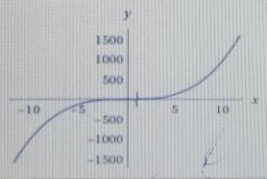
Input:

$$\frac{(x^3 + 1)^2}{x^3 - 1}$$

Plots:



(x from -1.9 to 2)



(x from -11.6 to 11.7)

Enlarge Customize

Alternate forms:

$$\frac{(x+1)^2(x^2 - x + 1)^2}{(x-1)(x^2 + x + 1)}$$

$$\frac{(x^3 + 2)x^3 + 1}{x^3 - 1}$$

$$x^3 - \frac{4(x+2)}{3(x^2 + x + 1)} + \frac{4}{3(x-1)} + 3$$

Step-by-Step Solutions for...

- Calculus
- Algebra
- Trigonometry
- Equation Solving
- Chemistry

Student pricing

Q2 (a) Find Integration.

$$\int \frac{1}{\sqrt{x^5}}\,dx$$

$$\int \frac{1}{\sqrt{x^5}}\,dx$$

$$\int \frac{1}{(x^5)^{1/2}}\,dx$$

$$\int (x^5)^{-1/2}\,dx$$

$$\frac{1}{2}\frac{(x^5)^{1/2}}{1/2} + c.$$

$$\frac{1}{2} \times 2 (x^5)^{1/2} + c.$$

$$(x^5)^{1/2} + c \text{ Ans.}$$

Q2 Page # 4

(b) $\int \frac{1}{(8x+7)^8}\,dx$

Let Suppose $8x+7 = u$

$\frac{du}{dx} = \frac{d}{dx}\,8x+7$

$\frac{du}{dx} = 8$

$\frac{1}{8}\,du = dx$

$\int \frac{1}{(u)^8}\left(\frac{1}{8}\,du\right)$

$\int \frac{1}{8}\,u^{-8}\,du$

$\Rightarrow \frac{1}{8}\,\frac{u^{-7}}{-7} + C$

$\Rightarrow \frac{-1}{56}\,u^{-7} + C$

$\Rightarrow \frac{-1}{56u^7} + C$

$\Rightarrow \frac{-1}{56(8x+7)^7} + C$

$\Rightarrow \frac{-1}{56}(8x+7)^{-7} + C.$

Q3 Find Integration by partial fraction

(a)

$$\int \frac{-x+9}{2x^2-3x+6}\,dx$$

$$\int \frac{-x+9}{2x^2-2x-6x+6}\,dx$$

$$\int \frac{-x+9}{(2x^2-2x)-(6x-6)}\,dx.$$

$$\int \frac{-x+9}{2x(x-1)-6(x-1)}\,dx.$$

$$\int \frac{-x+9}{(2x-6)(x-1)}\,dx.$$

By Partial fraction.

$$\frac{A}{(2x-6)} + \frac{B}{(x-1)} = \frac{-x+9}{(2x-6)(x-1)}$$

$A(x-1) + B(2x-6) = 1$

put $x = 1$

$A(1-1) + B(2(1)-6) = 1$

$0 + B(2-6) = 1$

$B(-4) = 1$

$B = -\frac{1}{4}$

$x = 3$

$A(3-1) + B(2(3)-6) = 1$

$A(2) + B(0) = 1$

$2A = 1$

$A = 1/2$

$$\Rightarrow \frac{\frac{1}{2}}{2x-6} + \frac{-\frac{1}{4}}{x-1}$$

$$\Rightarrow \frac{1}{2(2x-6)} + \frac{1}{4(x-1)}$$

$$\therefore \int \frac{1}{4x-12} - \frac{1}{4x-4}$$

Now Integration

P= 7

Q 5

$$\int \frac{1}{4x-12}\,dx = \int \frac{1}{4x-4}\,dx$$

$u = 4x-12$

$\frac{du}{dx} = \frac{d}{dx}(4x-12)$

$\frac{du}{dx} = 4$

$du = 4\,dx$

$\frac{du}{4} = dx$

$u = 4x-4$

$\frac{du}{dx} = \frac{d}{dx}(4x-4)$

$\frac{du}{dx} = 4$

$\frac{du}{4} = dx$

$$\int \frac{1}{4}\frac{1}{\frac{u}{4}}\,du - \int \frac{1}{4}\frac{1}{u}\,du$$

$\frac{1}{4}\ln u + C - \frac{1}{4}\ln u + C$

$\frac{1}{4}\ln u - \ln u + C$

$\frac{1}{4}\ln\frac{u}{u} + C$

$\frac{1}{4}\ln\frac{4x-12}{4x-4} + C$ Ans

Q3 (b) $\int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)}\,dx$. $P=8$

$\int \frac{4x^2+8x}{(x^2+1)(x^2+3x-x+3)}\,dx$

$\int \frac{4x^2+8x}{(x^2+1)\,x(x+3)-1(x+3)}$ or

$\int \frac{4x^2+8x}{(x^2+1)(x-1)(x+3)}$

$\frac{1}{(x^2+1)(x-1)(x+3)} = \frac{Ax}{(x^2+1)} + \frac{B}{(x-1)} + \frac{C}{x+3}$

$1 = Ax(x-1)(x+3) + B(x^2+1)(x+3) + C(x^2+1)(x-1)$

$1 = Ax(x^2+3x-x-3) + B(x^3+3x^2+x+3) + C(x^3-x^2+x-1)$

$Ax(x^2+2x-3) + B(x^3+3x^2+x+3) + C(x^3-x^2+x-1)$

$A(1)(1+2-3) + B(1^3+3(1)+1+3) + C(1-1+1-1)$

$A(0) + B(8) + 0$

$B\,8 = 1$

$B = 1/8$

~~$A(-1)(-1-1)$~~

$A(-3)(-3-1)(-3+3) + B(3+1)(-3+3) + C(-3+1)(-3-1)$

$0 + 0 + C(9+1)(-4) = 1$

$C = \frac{1}{6}$ $\quad P = 9$

and $B = \frac{1}{4}$

$$\frac{8x}{x^2+1} + \frac{\frac{1}{4}}{x-1} + \frac{\frac{1}{6}}{x+3}$$

$$\frac{x}{8x^2+8} + \frac{1}{4x-4} + \frac{1}{6x+18}$$

$$\int \frac{x}{8x^2+8} + \int \frac{1}{4x-4} + \frac{1}{6x-18}$$

$$\int \frac{1}{16} \frac{1}{u}\, du + \int \frac{1}{4} \frac{1}{u}\, du + \int \frac{1}{6} \frac{1}{u}\, du$$

$$\frac{1}{16} \ln u + \frac{1}{4} \ln u + \frac{1}{6} \ln u + C$$

$$\frac{1}{16} \ln u^{1/16} + \ln u^{1/4} + \ln u^{1/6} + C$$

P = 10

Q4 Solve matrix equations

(a) $X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$

$$X = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix}$$

(Part (b)) $X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

P = 11

(b) $A + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2I$

$A = \begin{bmatrix} 3-2I & -1-2I \\ 1-2I & 2-2I \end{bmatrix}$

$A = \begin{bmatrix} -18 & -22 \\ 20 & -19 \end{bmatrix}$.

Q4 If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

Then find $A^2 + BC$.

$A^2 + BC = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$P = 2$

$$\Rightarrow \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} 1\times2+4\times2 & 1\times4+4\times1 \\ 2\times1+1\times2 & 2\times4+1\times1 \end{bmatrix} + \begin{bmatrix} -3\times1+2\times0 & -3\times0+2\times2 \\ 4\times1+0\times0 & 4\times0+0\times2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix} + \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0+0 \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} 10 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 12 \\ 8 & 9 \end{bmatrix}.$$

$$A^2 + BC = \begin{bmatrix} 7 & 12 \\ 8 & 9 \end{bmatrix}.$$

nput/?i=∫1%2F√%28x%5E5+%29+

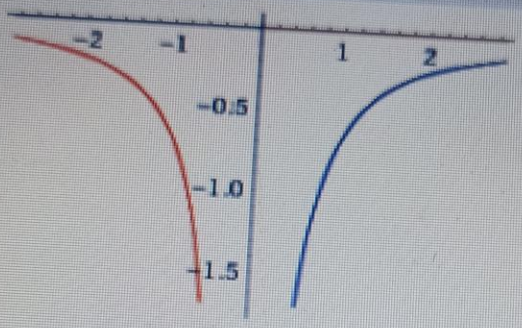
titud... Best Article Spinner... WhatsApp The Official Websit...

Extended Keyboard Upload

Indefinite integral:

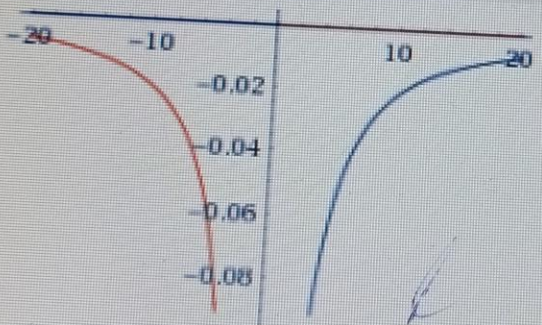
$$\int \frac{1}{\sqrt{x^5}}\,dx = -\frac{2x}{3\sqrt{x^5}} + \text{constant}$$

Plots of the integral:



(x from −2.9 to 2.9)

— real part
— imaginary part



(x from −20 to 20)

— real part
— imaginary part

Alternate form assuming x>0:

$$-\frac{2}{3x^{3/2}} + \text{constant}$$

Expanded form of the integral:

$$-\frac{2\sqrt{x^5}}{3x^4} + \text{constant}$$

Series expansion of the integral at x = 0:

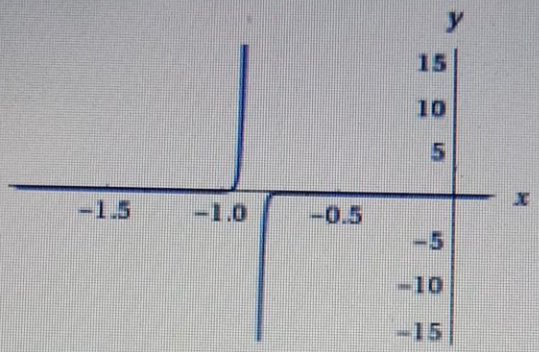
∫≡1/((⟦8x+7)⟧ ^8)

Extended Keyboard Upload

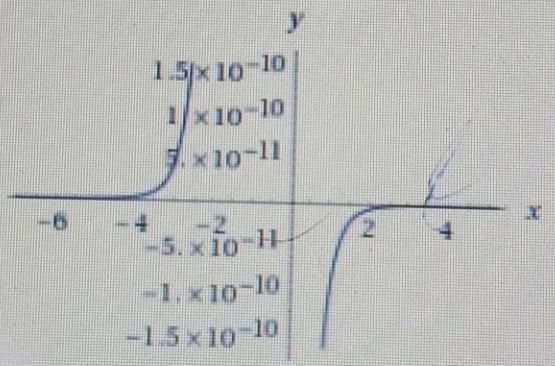
Indefinite integral:

$$\int \frac{1}{(8x+7)^8}\,dx = -\frac{1}{56(8x+7)^7} + \text{constant}$$

Plots of the integral:



(x from −1.9 to 0.1



(x from −7 to 5.2)

Alternate form of the integral:

$$-(1/(x(x(x(x(x(x(117\,440\,512\,x + 719\,323\,136) + 1$$
$$2\,409\,451\,520) + 1\,264\,962\,048) + 368\,947\,264) + 46$$

Expanded form of the integral:

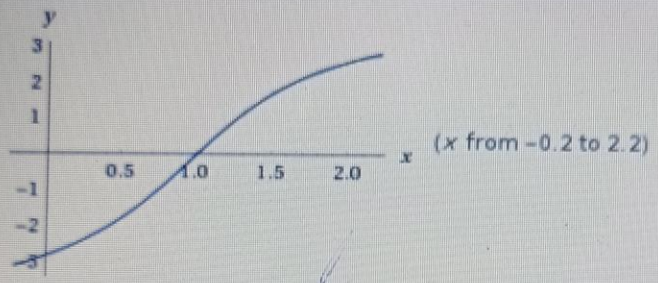
$$-\left(1/\left(117\,440\,512\,x^7 + 719\,323\,136\,x^6 + 1\,888\,223\,2\right.\right.$$

Extended Keyboard Upload

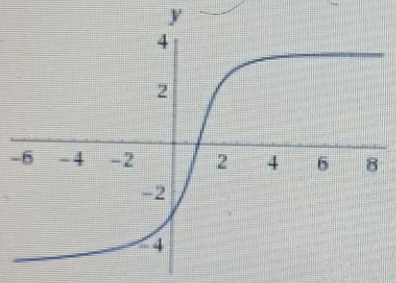
Indefinite integral: Appro

$$\int \frac{-x+9}{(2x)^2-8x+6}\,dx = 2\sqrt{2}\,\tan^{-1}\left(\sqrt{2}\,(x-1)\right) - \frac{1}{8}\log(2x^2-4x+3) + \text{const}$$

Plots of the integral:



(x from -0.2 to 2.2)



(x from -6.2 to 8.2)

Download Page