Name: khalid Malik

ID : 14252

P#1

QNO1: An equation consisting of deravative of one more dependent varable with respect to one or more independent variable is sad to be D.E.

There are two types of DE

(I) O.D.E. Ordinary differential equation

Example: $\frac{dy}{dx} = 3xy$

(II) P.D.E. Partial differential Equation.

$$\frac{dy}{dx^4} - 2\left(\frac{dy}{dx}\right)^5 + 3y = 4x.$$

(b). A Separable DE is an equation in Same variable are on one side for example.

$$\frac{dy}{dx} = 3x \Rightarrow dy = 3x\,dx$$

Q1 · (I) $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$ $y(0) = -1$ P s#

Sol: $\dfrac{dy}{dx} = \dfrac{xy^3}{\sqrt{1+x^2}}$

$\Rightarrow \dfrac{dy}{xy^3} = \dfrac{dx}{\sqrt{1+x^2}}$

$\Rightarrow \dfrac{dy}{y^3} = \dfrac{x\,dx}{\sqrt{1+x^2}}$ by Separating the variable

Now taking integral.

$\Rightarrow \int \dfrac{dy}{y^3} = \int \dfrac{x\,dx}{\sqrt{1+x^2}}$

$\Rightarrow \int y^{-3} dy = \frac{1}{2} \int \dfrac{2x}{\sqrt{1+x^2}} dx$

$\dfrac{y^{-3+1}}{-3+1} = \frac{1}{2} \ln |x + \sqrt{x^2+1}| + c$

$\dfrac{y^{-2}}{-2} = \frac{1}{2} \ln |x + \sqrt{x^2+1}| + c$

$\dfrac{1}{y^2} = -\ln |x + \sqrt{x^2+1}| + c$ —— *

putting initial Condition

$\dfrac{1}{\cancel{y^2}} = -\ln |0 + \sqrt{0^2+1}| + c$

$\Rightarrow 1 = -\ln |1| + c$

$\Rightarrow 1 = -(0) + c \quad \Rightarrow \quad c = 1$

Putting $c = 1$ in eq $- *$

$$\Rightarrow \frac{1}{y^2} = -\ln|x + \sqrt{1+x^2}| + 1$$

$$\Rightarrow y^2 = \frac{1}{-\ln|x+\sqrt{1+x}|+1}$$

$$y^2 = \pm \frac{1}{\sqrt{1 - \ln|x + \sqrt{1+x^2}}}$$

(II) $\frac{dx}{dt} = \frac{t}{x}$

Sol : $\frac{dx}{dt} = \frac{t}{x}$

$\Rightarrow$ $xdx = tdt$ Now taking integration

$\Rightarrow \int x dx = \int t dt \Rightarrow x^2 + c_1 = t^2 + c_2$

$\Rightarrow x^2 - t^2 + c_1 - c_2 = 0$

$\Rightarrow x^2 - t^2 + c = 0$

$\Rightarrow x^2 = t^2 - c$

$\Rightarrow x = \pm \sqrt{t^2 - c}$

Q NO 2. (ii). $x'(x) + 2x = \sin t$

P#5

$$x'(x) = -2x + \sin(t)$$

$$(2x - \sin(t) dx + dx = 0$$

$$x'(x) = \sin(t) - 2x$$

$$x'(x) + 2x = \frac{1}{2} i e^{-1t} - \frac{1}{2} i e^{it}$$

$$x(x) = c_1 + x \sin(t) - x^2$$

PA

Q No : 3.

Part (a). $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0$

Sol: $(2xy - 9x^2)\,dx + (2y + x^2 + 1)\,dy = 0$.

$M\,dx + N\,dy = 0$

check for exactness

$\frac{\partial m}{\partial y} = \frac{d}{dy}(2xy \mp 9x^2)$

$= 2$

$\int m\,dx + \int N(y)\,dy + c = 0$

$\int (2xy - 9x^2)\,d + \int 2y\,dy + c = 0$

$\frac{2x^2}{2}y - \frac{9x^2}{3} + \frac{2y^2}{2} + x^2y - 3x^3 + y^2 + c = 0$

Now put $y(0) = -3$

$\frac{dN}{dx} = \frac{d}{dx}(2y + x^2 + 1) = 2x$

$\frac{dm}{dy} = \frac{dN}{dx} = 2x$ equaction is exect D.E

$y(0) = -3$

Put $X = 0$

and $y = -3$

$v^2(-3) - 3(0)^3 - (-3)^2 + C = 0$

$-9 = C$

$x^2 y - 3x^2 + y^2 - 9 = 0$

QNO 3: (ii) $\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1)y' = 0$

Sol: $\left(\frac{2ty}{t^2+1} - 2t\right)d + (m(t^2+1) - 2)\,dy = 0$

$Mdt + Ndy = 0$

Check exactness

$\frac{dm}{dy} = \frac{d}{dy}\left(\left(\frac{2t}{t^2+1}\right)y - 2t\right)$

$\frac{dm}{dy} = \frac{2t}{t^2+1}$

$\frac{dN}{dt} = -\frac{d}{dt}\left(\ln(t^2+1) - 2\right.$

$\frac{dN}{dt} = \frac{2t}{t^2+1}$

$\frac{dm}{dy} = -\frac{dN}{dt}$ so equation is exact solution is

$\left(\frac{2ty}{t^2+1}\,dt - (2t\,dt + \int 0\,dy + c = 0\right.$

$\ln(t^2+1)y - t^2 + C = 0 \rightarrow *$ Solution

Put $t = 5$, $y = 0$.

$\ln(t^2+1)y - t^2 + C = 0$ — * Solution

Put $t = 5,\ y = 0$

$\ln(6)(0) - 5^2 + C = 0$

$C = 5$ put in *

$\ln(t^2+1)y - t^2 + 5 = 0$

are required Solution.