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SEMESTER : 4th

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SUBJECT : Differential Equation

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Q NO 1st

(i) $W = \sin(x+ct) + \cos(2x+2ct)$

Given

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2} \quad \text{---} (1)$$

NOW!

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial t}\left[\sin(x+ct) + \cos(2x+2ct)\right]$$

$$\frac{\partial}{\partial t}\left(\sin(x+ct)\right) + \frac{\partial}{\partial t}\left(\cos(2x+2ct)\right)$$

$$\frac{\partial w}{\partial t} = c\cos(x+ct) - 2c\sin(2x+2ct)$$

NOW

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left[c\cos(x+ct) - 2c\sin(2x+2ct)\right]$$

$$\boxed{\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)}$$

NOW!

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}\left[\sin(x+ct) + \cos(2x+2ct)\right]$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - 2\sin(2x+2ct)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left[\cos(x+ct) - 2\sin(2x+2ct)\right]$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

(1) $\longrightarrow$

$$-c^2\sin(x+ct) - 4c^2\cos(2x+2ct) = c^2\left[-\sin(x+ct) - 4\cos(2x+2ct)\right]$$

$$-c^2\cancel{\sin(x+ct)} - 4c^2\cancel{\cos(2x+2ct)} = -c\cancel{\sin(x+ct)} \cancel{-4c^2\cos(2x+2ct)}$$

$0 = 0$ (satisfied)

(ii) $w = \tan(2x+ct)$

Now $\frac{\partial w}{\partial t} = c\sec^2(2x+ct)$

& $\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left(c\sec^2(2x+ct)\right)$

$c_2^2 \cdot 2\sec^2(2x+ct)\tan(2x+ct)$

Now!

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 4\sec^2(2x+ct)\tan(2x+ct)$$

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(1) → $4c^2 \cancel{\sec^2 (2x+ct)} \tan(2x+ct) = 4c^2 \cancel{\sec^2 (2x+ct) \tan(2x+ct)}$

$0 = 0$ (Satisfied)

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Question No 2nd:

Given function is

$$f(x) = \begin{cases} x; & -\pi < x \leq 0. \\ 2x; & 0 \leq x \leq \pi. \end{cases}$$

We have to find the fourier coefficients a_0, a_n & b_n.

Now!

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left(\frac{x^2}{2}\right|_{0}^{\pi}$$

$$= \frac{1}{\cancel{\pi}}\left[0 - \frac{\pi^{\cancel{2}}}{2}\right] + \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\pi^{\cancel{2}}}{\cancel{2}} - 0\right]$$

$$\boxed{a_0 = -\pi/2 + \pi = \pi/2} \longleftarrow (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)\,dx$$

2-2

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right)-\left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}-\left(-\frac{\cos nx}{n^2}\right)\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2}-\frac{\cos n\pi}{n^2}\right]+\frac{2}{\pi}\left[\frac{\cos n\pi}{n^2}-\frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n+2(-1)^n-2}{n^2}\right] = \frac{(-1)^n-1}{\pi n^2}$$

So;

$$a_n \begin{cases} \frac{-2}{\pi n^2}; \text{ if } n \text{ is odd} \\ 0; \text{ if } n \text{ is even} \end{cases} \quad \text{—(2)}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx$$

$$\frac{1}{\pi}\left(x\left(-\frac{\cos nx}{n}\right)-\left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{2}{\pi}\left(x\left(-\frac{\cos nx}{n}\right)-\left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$\boxed{b_n = \frac{1}{\pi}\left[\left(\frac{-\pi\cos nx}{n}\right)+\frac{2}{\pi}\left(\frac{-\pi\cos nx}{n}\right) = \frac{-3\cos n\pi}{n}\right.}$$

$$= \frac{3(-1)^{n+1}}{n} \quad \text{--- } (3)$$

Sol! The required fourier Series is ; $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$

$$= \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{(2n-1)^2} + 3 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n}.$$

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3-1

Question NO 3rd:

Solve the initial value problem.

$y'' - 4y' + 13y = 8\sin 3x$,
$y(0) = 1$ and $y'(0) = 2$

Solution:

$$y'' - 4y' + 13y = 8\sin 3x \quad \text{---} \; (*)$$

$$y(0) = 01$$

$$y'(0) = 2$$

Associated Homgenous equation of (*) is $y'' - 4y' + 13y = 0$ —— (**)

Change (**) into Auxilary equation
pull $y = m$ in (**)

$$m^2 - 4m + 13 = 0$$

use Quardratic formula.

$a = 1$, $b = -4$, $c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$= \frac{4 \pm \sqrt{-36}\, i}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$= 2 \pm 3i$$

$$\boxed{\begin{aligned} m_1 &= 2 + 3i \\ m_2 &= 2 - 3i \end{aligned}}$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \text{ — (A)}$$

let

$$y_p = A \cos 3x + B \sin 3x \text{ —— (1)}$$

Diff w.r.t "x"

$$y'_p = -3A \sin 3x + 3B \cos 3x$$

$\Rightarrow$ Again Diff w.r.to "x"

$$y''_p = -9A \cos 3x - 9B \sin 3x$$

Putt in (*)

$$(-9A \cos 3x - 9B \sin 3x) - 4(-3A \sin 3x + 3B \cos 3x) + 13(A \cos 3x + B \sin 3x) = 8 \sin 3x$$

$$= -9A \cos 3x - 12B \cos 3x + 13A \cos 3x - 9B \sin 3x + 12A \sin 3x + 13B \sin 3x - 8 \sin 3x$$

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$= (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x = 8\sin 3x$

$= (4A - 12B)\cos 3x + (4B + 12A)\sin 3x + 8\sin 3x$

Comparing coeffients.

- $\underline{\sin 3x} \Rightarrow \quad 4B + 12A = 8$ — (a)
- $\underline{\cos 3x} \Rightarrow 4A - 12B = 0 \Rightarrow 4A = 12B$

$\boxed{A = 3B}$ — (b)

Putt (b) in (a)

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$B = 1/5 \longrightarrow$ (c)

Putt (C) in (b)

$A = 3/5 \longrightarrow$ (d)

putt c & d in (1)

$y_p = \frac{3}{5}\cos 3x + 1/5 \sin 3x \rightarrow$ (B)

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The General Solution is

$y = y_c + y_p$

$y' = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow$ (C)

NOW!

we needs to find the values of c_1 & c_2 for this.

put $x = 0$ & $y = 1$ in (C)

$1 = e^{3(2)}(c_1 \cos 3(0) + c_2 \sin 3(0)) \frac{3}{5}\cos 3(0) + \frac{1}{5}\sin 3(0)$

$1 = (c_1(1) + c_2(0)) + \frac{3}{5}(1) + \frac{1}{5}(0)$ —

$1 = c_1 + \frac{3}{5}$

$C1 = 2/5 \longrightarrow$ (D)

Diff (C) w.r.to "x"

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x$

(F)

~~Put~~

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Putt $y' = 2, x = 0$ in (F)

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x)$$
$$+ c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$$
$$-6/5 \sin 3x + 3/5 \cos 3x.$$

Putt $y' = 2, x = 0$

$$2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)$$
$$+ c_2(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0))$$
$$-6/5 \sin 3(0) + 3/5 \cos 3(0)$$

$$2 = c_1(2) + c_2(3) - 0 + 3/5$$

$$2 = 2c_1 + 3(2 + 3/5)$$

Putt $c_1 = 2/5$

$$2 = 4/5 + 3c_2 + 3/5$$

$$2 = 7/5 + 3c_2$$

$$3c_2 = 2 - 7/5$$

$$3c_2 = 3/5$$

$$c_2 = 3/15 \longrightarrow \text{(G)}$$

3.6

Putt D & G in C

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$$

$$\boxed{y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x}$$

Required General Solution.

Question No 4Th:

Solve:

$$(D^2 - DD')z = \cos x \cos 2y$$

Solution:

It is already in Symbolic form

$$(D^2 - DD')z = \cos x \cos 2y \quad \text{---} \; (*)$$

Putt AE $D^2 - DD' = 0$

- As we know that

$$\frac{D}{D'} = m \qquad \text{i-e } D = m, D' = 1$$

$$= m^2 - m = 0$$
$$m = 0, 1$$

Therefore $C.F = f_1(y) + f_2(y+x)$

from equation (*)

$$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y$$

$$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} 2\cos x \cos 2y$$

4-2

As:

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$C.F = f_1(y-x) + x f_2(y-x$

$P.I = \frac{1}{D^2 + DD' + D'^2}\{2(y-x) + \sin(x-y)\}$

By General method:

$m = -1, \; y - x = c$

$\Rightarrow \frac{1}{D+D'}\left[\int (2c + \sin(-c)\right] dx$

$\frac{1}{D+D'}\left[2cx - (\sin c)\, x\right]$

Replacing c by y-x

$\frac{1}{D+D'}\left[2x(y-x) - x\sin(y-x)\right]$

Again put $y-x=c$

$\int (2xc - x\sin c)\, dx$

$= cx^2 - \frac{x^2}{2} \sin c$

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Replacing c by $y-x$.

$$x^2(y-x) - \frac{x^2}{2}\sin(y-x) = x^2y - x^3 + \frac{x^2}{2}$$
$$\cdots\cdots \sin(x-y)$$

$$x^2(y-x) - \frac{x^2}{2}\sin(y-x) = x^2y - x^3 + \frac{x^2}{2}\sin(x-y)$$

Hence the required solution is

$$z = C.F + P.I = f_1(y-x) + x f_2(y-x) + x^2y - x^3$$
$$+ \frac{1}{2}x^2 \sin(x-y).$$

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