①

Ans 1 a)

(a) $\frac{d}{dx}\left(\frac{2x^3-3x^2+5}{x^2+1}\right)$

Using Quotient rule, we have

$$= \frac{(x^2+1)\frac{d}{dx}(2x^3-3x^2+5)-(2x^3-3x^2+5)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(6x^2-6x)-(2x^3-3x^2+5)(2x)}{(x^2+1)^2}$$

$$= \frac{6x(x^2+1)(x-1)-(2x^3-3x^2+5)2x}{(x^2+1)^2}$$

$$= \frac{2x[3x(x^2+1)(x-1)-(2x^3-3x^2+5)]}{(x^2+1)^2}$$

(b) $\frac{(x^2+1)^2}{x^2-1}$

Again use Quotient rule

$$= \frac{(x^2-1)\frac{d}{dx}(x^2+1)^2-(x^2+1)^2\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{(x^2-1)2(x^2+1)(2x)-(x^2+1)^2(2x)}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)[2(x^2-1)-(x^2+1)]}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)[2x^2-2-x^2-1]}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)(x^2-3)}{(x^2-1)^2}$$

As

2(a) $y = (1+2\sqrt{x})^3 \cdot x^{2/3}$

$\frac{2}{3} - \frac{1}{2}$ $\frac{4-3}{6}$ $1/6$

Let $x = u$

$\Rightarrow y = (1+2\sqrt{u})^3 u^{2/3}$

$$\frac{dy}{du} = (1+2\sqrt{u})^3 \frac{2}{3} u^{\frac{2}{3}-1} + u^{2/3} 3(1+2\sqrt{u})^2 \cancel{2} \frac{1}{\cancel{2}\sqrt{u}}$$

$$\frac{dy}{dx} = (1+2\sqrt{u})^3 \frac{2}{3} u^{-1/3} + 3u^{1/6}(1+2\sqrt{u})^2$$

& $\frac{dx}{du} = 1$

using chain rule.

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{dy}{dx}$$

$$= (1+2\sqrt{u})^2 \left[(1+2\sqrt{u}) \frac{2}{3} u^{-1/3} + 3u^{1/6} \right] \times (1)$$

$$\Rightarrow \frac{dy}{dx} = (1+2\sqrt{u})^2 \left\{ \frac{2}{3}(1+2\sqrt{u}) u^{-1/3} + 3u^{1/6} \right\}$$

use $u = x$

$$\Rightarrow \frac{dy}{dx} = (1+2\sqrt{x})^2 \left\{ \frac{2}{3}(1+2\sqrt{x}) x^{-1/3} + 3x^{1/6} \right\}$$

2(b)

$$y = \sqrt{\frac{1-x}{1+x}}$$

Let $u = \frac{1-x}{1+x}$ — (1)

$y = \sqrt{u}$ (2)

1) $\Rightarrow$ $$\frac{du}{dx} = \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2}$$

$$\frac{du}{dx} = \frac{-1 - \cancel{x} - 1 + \cancel{x}}{(1+x)^2}$$

$$\frac{du}{dx} = \frac{-2}{(1+x)^2}$$

2) $\Rightarrow$ $$\frac{dy}{du} = \frac{1}{2\sqrt{u}}$$

us chain rule.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{u}} \times \frac{(-2)}{(1+x)^2}$$

$(x+1)^{2-\frac{1}{2}}$

use $u = \frac{1-x}{1+x}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\cancel{2}}\sqrt{\frac{1+x}{1-x}} \times \frac{(-\cancel{2})}{(1+x)^2}$$

$$\boxed{\frac{dy}{dx} = -\frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}}$$

3) a $\int \frac{1}{\sqrt{x^3}} dx$

$= \int \frac{1}{(x^3)^{1/2}} dx$

$= \int \frac{1}{x^{3/2}} dx$

$= \int x^{-3/2} dx$

use formul $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$= \frac{x^{-3/2+1}}{-3/2+1} + C$ $\quad \frac{-3+2}{2}$

$= \frac{x^{-1/2}}{-1/2} + C$

$\int \frac{1}{\sqrt{x^3}} dx = \frac{-2}{\sqrt{x}} + C$

3(b) $\int \frac{1}{(6x+7)^6} dx$

$= \int (6x+7)^{-6} dx$

using again above mention formula

$= \frac{1}{6} \int (6x+7)^{-6} (6) dx \rightarrow$ *) check $\frac{1}{6} \times 6 = 1$

$= \frac{1}{6} \frac{(6x+7)^{-6+1}}{-6+1} + C$

$\int \frac{1}{(6x+7)^6} dx = \frac{-1}{30(6x+7)^5} + C$