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7911

Section:

A

Subject:

Differential Equations

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(1)

Solution:

1:2:1, 2:1:1, 2:0:2

40

50

P	E
A	E

P	P
A	E

P	P
A	A

B₁

B₂

B₃

Let x, y and z be the Cost/Kg Pak, Egyptian, American Cotton respectively then according to the given conditions

$$\frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z = 40$$

$$\frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z = 50$$

$$\frac{2}{4}x + \frac{2}{4}z = 60$$

⇒ A

$$1x + 2y + 1z = 160$$

$$2x + 1y + 1z = 200$$

$$1x + 1z = 120$$

→ (B)

(2)

In matrix form we can write as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$AX = B$$

$$\Rightarrow A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

3)

$$\text{First } |A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} \quad \text{Expand by } R_1$$

$$= 1 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(1-0) - 2(2-1) + 1(0-1)$$

$$= 1 - 2 + 1(-1)$$

$$= -2$$

Now determinant of A_1

$$|A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix} \quad \text{Expand by } R_1$$

$$= 160 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1 \begin{vmatrix} 200 & 1 \\ 120 & 0 \end{vmatrix}$$

$$|A_1| = -120$$

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Similarly

$$|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix} \quad \text{Expand by } R_1$$

$$= 1 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} - 160 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix}$$

$$|A_2| = -40$$

$$|A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix} \quad \text{Expand by } R_1$$

$$= 1 \begin{vmatrix} 1 & 200 \\ 0 & 120 \end{vmatrix} - 2 \begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix} + 160 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(120 - 0) - 2(240 - 200) + 160(0 - 1)$$

$$|A_3| = -120$$

According to Cramer's rule

(5)

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = 60$$

$$(x, y, z) = (60, 20, 60)$$

Pakistan = 60

Egyptian = 20

American = 60