Q 1

The function $g(t)$ is defined by

$g(t) = 0$, $t < 0$

t^2 $0 \le t \le 3$

$2t+3$ $3 < t \le 4$

12 $t > 4$

(a) State any point of discontinuity.

(b) Find if they exist

i $\lim_{t \to 3} g$

Sol

(a) To check possibility of the discontinuity of the function is at $t = 0$ & 4

=> first at $t = 0$

$g(t) = t^2$

$g(0) = 0^2 = 0$

for R.H.L

$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$

$= \lim_{h \to 0} 1 + h^2 + 2h$

Apply limits

$= 1 + 0^2 + 2(0)$

$= 1$

for L.H.L

$\lim_{h \to 0} g(1-h) = 2t+3$

$= \lim_{h \to 0} 2 - 2h + 3$

Apply limit

$= 2 - 2(0) + 3$

$= 5$

R.H.L $\neq$ L.H.L

=> Now at $t = 4$

$g(4) = 2(4)+3$

$= 8+3$

$= 11$

For R.H.L

$\lim_{h \to 0} g(4+h) = \lim_{h \to 0} 2(1+h)+3$

$= \lim_{h \to 0} 2+2h+3$

Apply limit

$2+2(0)+3 \Rightarrow 5$

For L.H.L

$\lim_{h \to 0} g(1-h) = 12$

$g(4) = R.H.L \neq L.H.L$

point of discontinuity is at $t = 4$

(b) Find if they exist

1) $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.L $\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + 2h$ Apply limits

$= 1 + 3^2 + 2(3) \Rightarrow 16$

L.H.L $\lim_{n\to 3} g(1-h) = \lim_{n\to 3} 2t+3$

$= \lim_{n\to 3} 2(1 - h)+3$

$= \lim 2 - 2h+3$

$= n \to 3$

Apply limit

$= 2-2(3)+3$

$= 2-6+3$

$= -1$

R.H.L $\neq$ L.H.L (do not exist since L.H.L is -ve)

Q 2

Sol) $y = x^2 + \sin x$

Maclarin Series

$f(x) = f(0) + f'(0) + f'' \frac{x^2}{2!} + f''' \frac{x^3}{3!} + \cdots \frac{f^{(k)}(0) x^k}{k!} \cdots$ (★)

$y(x) = x^2 + \sin x$

$y(0) = 0 + \sin 0$

$= 0 + 0$

$y(0) = 0$ ------ (1)

To find Derivatives

$y(x) = x^2 + \sin x$

$\frac{dy}{dx} = \frac{d}{dx}(x^2 + \sin x)$

$\frac{dy}{dx} = 2x + \cos x$

$y'(0) = 2(0) + \cos(0)$

$y'(0) = 1$ ------ (A)

$y'' = \frac{d}{dx}(2x + \cos x)$

$y'' = 2 - \sin x$

$y''(0) = 2 - \sin(0)$

$y''(0) = 2$ ------ (B)

$$y'''(x) = \frac{2}{2x}(2 - \sin x)$$

$$y'''(x) = 0 - \cos x$$

$$y''(x) = -\cos x$$

$$y'''(0) = -\cos(0)$$

$$y'''(0) = -1 \quad \text{------} \quad (i)$$

Put in eq (*)

$$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cdots$$

$$y(x) = x + \frac{2x^2}{2!} - \frac{x^3}{3!} + \cdots$$

$$y(x) = x + x^2 - \frac{x^3}{3!} + \cdots$$

Then proved.

Q. Find y by logarithmic differentiation

$y = x^3 (1+x)^9 e^{6x}$

Sol) $y = x^3 (1+x)^9 e^{6x}$

taking ln on b/s

$\ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$ ①

Using log property $\because \ln e^a = a$

$\ln y = 3\ln x + 9\ln(1+x) + 6x$

$\frac{1}{y} \cdot \frac{dy}{dx} = 3\frac{1}{x} + 9\frac{1}{x+1} + 6x$

$\frac{dy}{dx} = 3\frac{y}{x} + 9\frac{y}{x+1} + 6xy$

$\frac{dy}{dx} = \frac{3x^3(x+1)^9 e^{6x}}{\cancel{x}} + \frac{9x^3(3x+1)^{\cancel{9}\,8} e^{6x}}{\cancel{x+1}} + 6x \cdot x^3(x+1)^9 e^{6x}$

$= 3x^2(x+1)^9 e^{6x} + 9x^3(x+1)^8 e^{6x} + 6x^4(x+1)^9 e^{6x}$

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$1+xy = x^2+y^2$

Sol

$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$

$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2)$

$0 + \frac{x\,dy}{dx} + y\frac{dx}{dx} = 2x + 2y\frac{dy}{dx}$

$\frac{x\,dy}{dx} + y = 2x + 2y\frac{dy}{dx}$

$\frac{x\,dy}{dx} - \frac{2y\,dy}{dx} = 2x - y$

$\frac{dy}{dx}(x-2y) = 2x-y$

$\frac{dy}{dx} = \frac{2x-y}{x-2y}$ —— (1)

$y' = \frac{dy}{dx} = \frac{2x-y}{x-2y}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{2x-y}{x-2y}\right)$ Quotient Rule.

$\frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\,d/dx\,(x-2y)}{(x-2y)^2}$

$y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$

$y'' = \frac{2x - xy' - 4y + 2yy') - (2x - 4xy' - y + 2yy')}{(x-2y)^2}$

$$y' = \frac{4xy' - 3y - xy'}{(x-2y)^2}$$

$$y'' = \frac{4x\left(\frac{2x-y}{x-2y}\right) - 3y - x\left[\frac{2x-y}{x-2y}\right]}{(x-2y)^2}$$

$$y'' = \frac{4x(2x-y) - 3y(x-2y) - x(2x-y)}{(x-2y)(x-2y)^2}$$

$$y'' = \frac{6x^2 + 6y^2 - 6xy}{(x-2y)^3}$$

Solved