

"Application of ODE's and PDE's

In Engineering"

The Differential equations have wide applications in various engineering and Science disciplines. In general, modeling of the variation of a physical quantity, such as temperature, pressure, displacement, velocity, stress, strain, current, voltage, or concentration of a pollutant, with the change of time or location, or both would result in differential equations. Similarly, studying the variation of some physical quantities on other physical quantities would also lead to differential equations. In fact, many engineering subjects, such as mechanical vibration or structure dynamics, heat transfer or theory of electric circuits, are founded on the theory of differential equations. It is practically important for engineers to be able to model physical problems using mathematical equations, and then solve these equations so that the behavior of the the systems concerned can be studied.

MOTIVATING Examples:

It is important for engineers to be able to model physical problems using mathematical equations, and then solve these equations so that the behaviour of the systems concerned can be studied. In this section, a few examples are presented to illustrate how practical problems are modeled mathematically and how differential equations arise in them.

Motivating example:

A tank contains a liquid of volume $V(t)$, which is polluted with a pollutant concentration in percentage of $c(t)$ at time t . To reduce the pollutant concentration, an inflow of rate Q_{in} is injected to the tank. Unfortunately, the inflow is also polluted but to a lesser degree with a pollutant concentration c_{in} . It is assumed that the inflow is perfectly mixed with the liquid in the tank instantaneously. An outflow of rate Q_{out} is removed from the tank. Suppose that, at time $t=0$, the volume of the liquid is V_0 with a pollutant concentration of c_0 . The equation governing the pollutant concentration

$c(t)$ is given by

$$- [V_0 + (Q_{in} - Q_{out})t] (dc(t)/dt) + Q_{in} c(t) = Q_{in} c_{in};$$

With initial condition $c(0) = c_0$. This is a first-order ordinary differential equation.

Difference between ODE and

PDE :

ODEs involve one or more functions of a single variable, with all derivatives ordinary ones relative to that variable.

PDEs allow functions of several variable and partial derivatives of the unknown functions with respect to those variables..... That's a second-order, linear, ordinary differential equation.