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Semester :- 12<sup>th</sup>

Subject :- Structure Analysis I

Section :- B

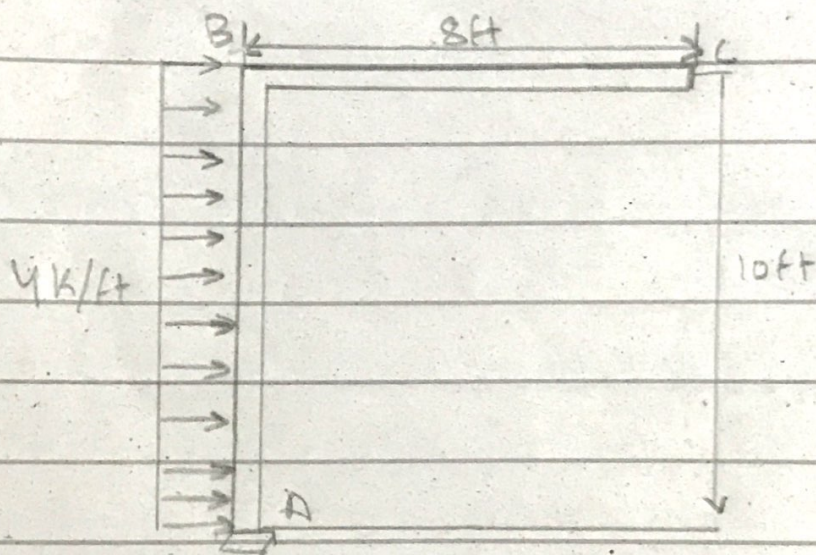
Submitted to :- Engr. Amjad Islam.

Final Term

①

Qno1 :-

Determine the vertical displacement of free end point C on the frame shown in figure. Take  $E = 29(10^3)$  ksi and  $I = 600 \text{ in}^4$  for both members. Use method of Virtual work.



⇒ Given Data:-

Uniform load = 4 k/ft

$E = 29(10)^3 \text{ ksi}$

$I = 600 \text{ in}^4$

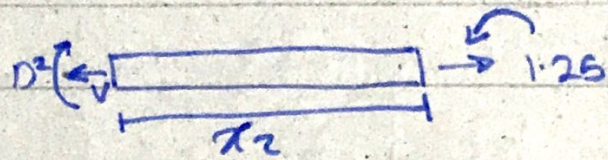
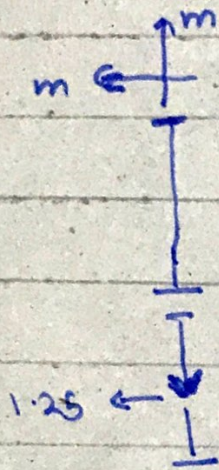
⇒ Required :-  $\Delta_{vert}$

Vertical displacement.

②

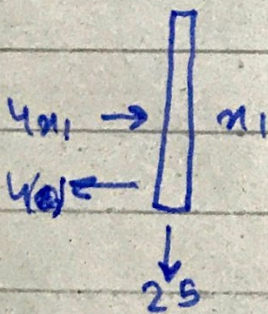
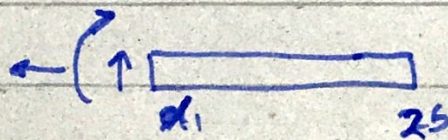
Solution:

Now virtual moment



$$m_2 = 1.25x$$

real moment



$$m_2 = 25\alpha_2$$

$$m'' = \frac{40\alpha_1 - \frac{1}{2}\alpha_1(x_2)}{40\alpha_1 - 2\alpha_1^2}$$

Now by virtual work Equation

$$\Delta DC = \int_0^L \frac{m M dx}{E}$$

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$$\Delta C = \int_0^{10} \frac{(1x_1) (40x_2 - 2x_2^2)}{E} dx_1 + \int_0^8 \frac{(1.25x_2)(25x_2)}{E_1}$$

$$\Delta C = \frac{1}{EI} \left| \frac{40x^3}{3} - \frac{2x^3}{4} \right|_0^{10} + \left| \frac{31.25x^3}{3} \right|_0^8$$

~~10.64 mm~~

$$\Delta C = 10.64 \text{ mm}$$

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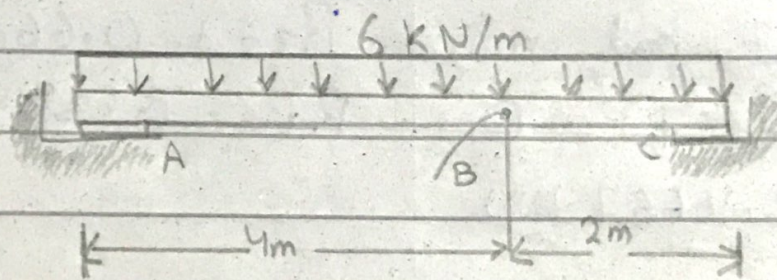
Qno 2

Determine the slope and displacement at point B. Assume the support at A is a pin and C is a roller.

Take  $E = 200 \text{ GPa}$ ,  $I = 60 (10)^6 \text{ mm}^4$ .

Use Castigliano's Theorem.

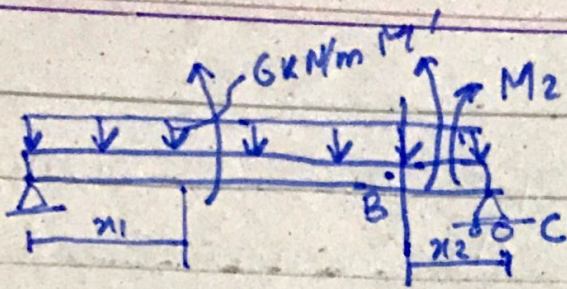
Given Data:-



Required:-

Slope and displacement at  
Point B

=> Solution:-



$$18 \text{ kN} + 0.1667 \quad 18 - 0.1667$$

(a)

$$R_1 + R_2 = 0 \quad \text{--- (1)}$$

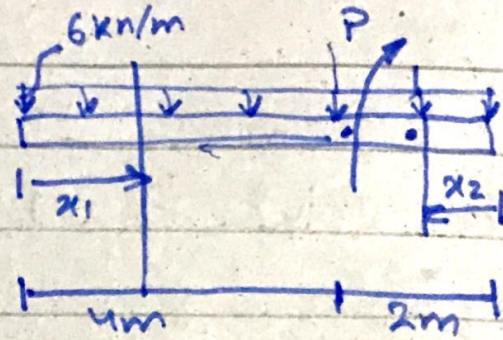
$$\sum M_A = 0 \quad \hookrightarrow +$$

$$1 + R_2(6) = 0$$

$$\Rightarrow 0.16667 \text{ put in (1)}$$

$$R_1 + (0.1667) = 0$$

$$R_1 = 0.16667 \text{ kN}$$



(b)

$$R_1 + R_2 = 1$$

$$- (1)(4) + R_2(6) = 0$$

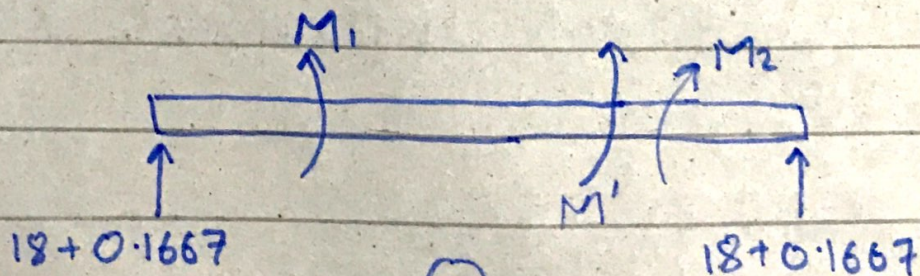
$$R_1 = 0.6667 \text{ kN}$$

$$R_2 = 1 - 0.6667 \text{ kN}$$

$$R_2 = 0.333 \text{ kN}$$

$$M_1 = (18 + 0.1667 M') x_1 - 2x_1^2$$

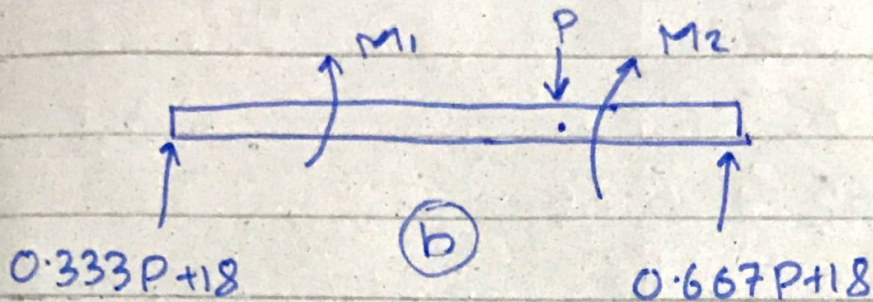
$$M_2 = (18 - 0.1667 M') x_2 - 2x_2^2$$



(a)

$$M_1 = (0.333 P + 18) x_1 - 2x_1^2$$

$$M_2 = (0.667 P + 18) x_2 - 2x_2^2$$



The displacement function shown in the figure 'a' above

$$\frac{\partial M_1}{\partial M'} = 0.1667 x_1 \quad \text{and} \quad \frac{\partial M_2}{\partial M'} = 0.1667 x_2$$

Set  $M' = 0$  then

$$\rightarrow M_1 = (18 + 0.1667(0))x_1 - 2x_1^2$$

$$\rightarrow M_1 = (18x_1 - 2x_1^2)$$

$$M_2 = (18x_2 - 2x_2^2)$$

$$\Delta_B = \int_0^L M \left( \frac{\partial M}{\partial M'} \right) \frac{dx}{EI}$$

$$= \int_0^{L/2} \frac{(18x_1 - 2x_1^2)(0.1667x_1)}{EI} dx_1 +$$

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$$\int_0^2 \frac{(18x_2 - 2x_2^2)(0.1667 x_2) dx_2}{Ei}$$

$$Q_B = \frac{42.65}{Ei} + \frac{6.66}{Ei}$$

$$Q_B = \frac{49.31}{Ei}$$

$$Q_B = \frac{49.31}{(200 \times 10^6 \text{ kPa})(0.00006)}$$

$$Q_B = 0.4411 \text{ rad}$$

For the displacement function  
are shown in figure 'b'

$$\frac{\partial M_1}{\partial p} = 0.333x_1 \text{ and } \frac{\partial M_2}{\partial p} = 0.6667x_2$$

$$\text{also } p = 0$$

then



$$M_1 = (18x_1 - 2x_1^2) \text{ kN-m}$$

$$M_2 = (18x_2 - 2x_2^2) \text{ kN-m}$$

thus

$$\Delta_B = \int_0^L M \left( \frac{dM}{dP} \right) \frac{dx}{EI}$$

$$\Delta_B = \int_0^4 \frac{(30x_1 - 2x_1^2)(0.333x_1)}{EI} dx + \int_0^2 \frac{(30x - 2x^2)(0.6667x)}{EI} dx$$

$$\Delta_B = \frac{218.5}{EI}$$

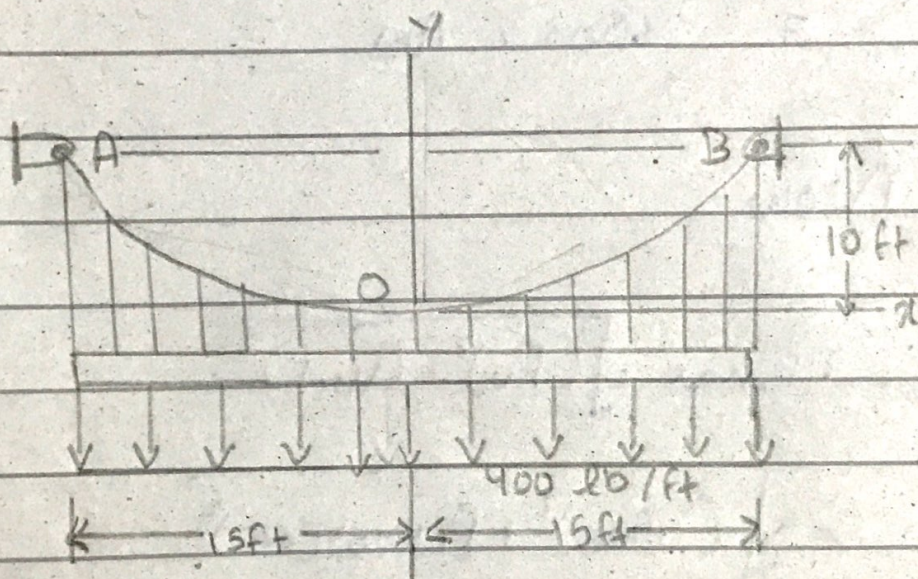
$$= \frac{218.5}{(200 \times 10^6)(0.00006)}$$

$$= \boxed{0.018 \text{ m or } 18 \text{ mm}}$$

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Qno 3

The cable is subjected to the uniform loading. If the slope of the cable at point O is zero, determine the equation of curve and the force in the cable at O and B.



~~Solution~~ ⇒ Required (i) Eq of curve?  
(ii) Force at O and B?

⇒ Solution :-

$$y = \frac{h}{L^2} x^2 = \frac{8}{(15)^2} x^2$$

$$y = 0.0356 x^2$$

Now we know that

$$T_0 = F_B = \frac{W_0 L^2}{2h}$$

$$= \frac{(400)(15)^2}{2(10)}$$

$$= 4500 \text{ lb}$$

$$= 4.500 \text{ k Ans}$$

Now

$$T_B = T_{\max} = \sqrt{(F_v)^2 + (W_0 L)^2}$$

$$= \sqrt{(4500)^2 + [(400)(15)]^2}$$

$$= 7500 \text{ lb}$$

$$= \boxed{7.5 \text{ k}} \text{ Ans}$$

Now

$$T_B = T_{\max} = W_0 L \sqrt{1 + \left(\frac{L}{2h}\right)^2}$$

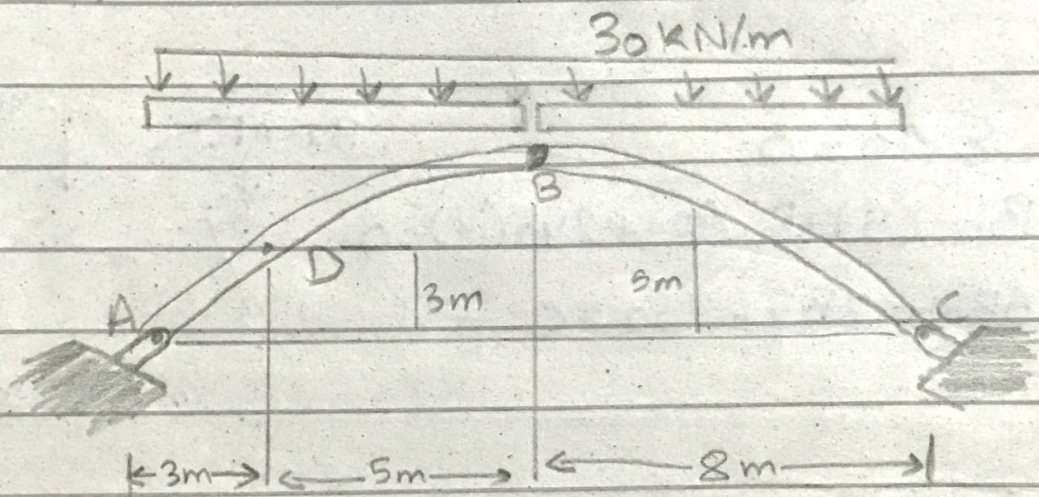
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$$400 (15) \sqrt{1 + \left(\frac{15}{2(10)}\right)^2}$$

$$= 7500 \text{ lb} = \boxed{7.5 \text{ k Ans}}$$

Qno 4

The three-hinged spandrel arch is subjected to the uniform load of  $30 \text{ kN/m}$ . Determine the internal moment in the arch at point D.



Required:-

Internal moment in the arch at point D?

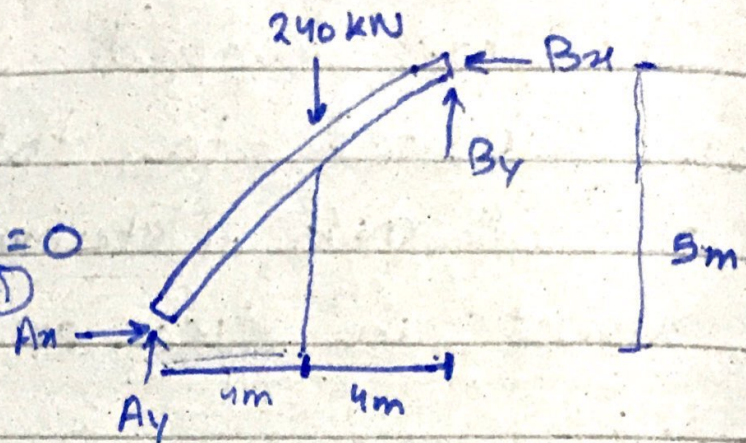
Solution:-

Member AB:

$$\sum M_A = 0$$

$$B_x(5) + B_y(8) - 240(4) = 0$$

$$\Rightarrow 5B_x + 8B_y - 960 = 0 \quad \text{--- (A)}$$

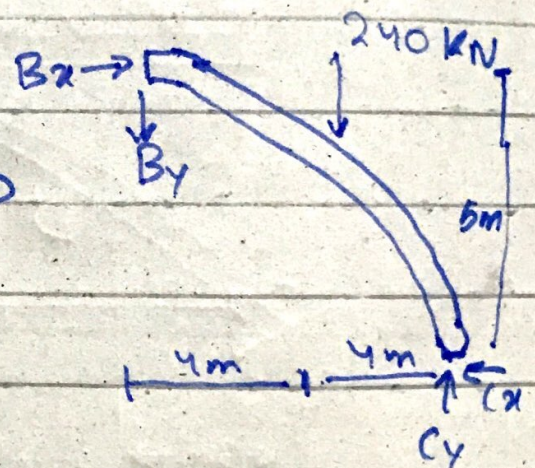


Member BC

$$\sum M_C = 0$$

$$-B_x(5) + B_y(8) + 240(4) = 0$$

$$\Rightarrow -5B_x + 8B_y + 960 = 0 \quad \text{--- (B)}$$



Solving Eq. A and B for  $B_x$  &  $B_y$

$$5B_x + 8B_y - 960 = 0$$

$$\underline{-5B_x + 8B_y + 960 = 0}$$

$$16B_y = 0$$

$$B_y = 0$$

Putting value in Eq A

$$5B_x + 8(0) - 960 = 0$$

$$5B_x = 960$$

$$B_x = 192 \text{ kN}$$

Segment DB

$$\sum M_D = 0$$

$$192(2) - 150(2.5) - M_D = 0$$

$$\Rightarrow 384 - 375 = M_D$$

$$M_D = 9 \text{ kN-m}$$

Internal moment at "D"

