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Subject = Multi varaite calculas

Sesional Assignment

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Q. $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = ?$

Solution We claim that $\frac{xy}{x^2+y^2}$ has no limit or limit does not exist at $(x,y)\to(0,0)$. Because if we try to reach to point $(0,0)$ through the path $y=mx$, then

$$\frac{xy}{x^2+y^2} = \frac{x(mx)}{x^2+(mx)^2}$$

$$= \frac{mx^2}{x^2+m^2x^2}$$

$$= \frac{mx^2}{x^2(1+m^2)} = \frac{m}{1+m^2}.$$

Taking limit $(x,y)\to(0,0)$, then

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{x\to 0} \frac{x^2m}{x^2(1+m^2)} = \frac{m}{1+m^2}$$

→ For each value of m limit value is different but limit value should be unique. Thus limit does not exist.

Q2 $f(x,y) = \begin{cases} \dfrac{\sin(x^2+y^2)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$

Solution. (1) Since we are given

$f(0,0) = 1 \longrightarrow (i)$

(2) We have to find $\lim_{(x,y)\to(0,0)} f(x,y)$

We choose the path $y = mx$, then

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to 0} f(x,mx)$$

$$= \lim_{x\to 0} \frac{\sin(x^2+(mx)^2)}{x^2+(mx)^2}$$

$$= \lim_{x\to 0} \frac{\sin(x^2(1+m^2))}{x^2(1+m^2)}$$

$$= 1 \quad \text{by } \lim_{x\to 0} \frac{\sin x}{x} =$$

Hence $\lim_{(x,y)\to(0,0)} f(x,y) = 1 \longrightarrow (ii)$

From (i) and (ii), we see that

$\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$.

$\Rightarrow f(x,y) = \begin{cases} \dfrac{\sin(x^2+y^2)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$

is continuous at $(x,y) = (0,0)$.

Q3 $f(x,y) = e^x \sin y + e^y \cos x$

Solution: $f(x,y) = e^x \sin y + e^y \cos x$

$$\Rightarrow \frac{\partial f}{\partial x} = e^x \sin y - e^y \sin x \longrightarrow (i)$$

$$\Rightarrow \frac{\partial f}{\partial y} = e^x \cos y + e^y \cos x \longrightarrow (ii)$$

Taking derivative of the (i) w.r.t x again

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left[e^x \sin y - e^y \sin x\right]$$

$$\Rightarrow \frac{\partial^2 f}{\partial x^2} = e^x \sin y - e^y \cos x \longrightarrow (iii)$$

Taking partial derivative of the (ii) w.r.t y

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left[e^x \cos y + e^y \cos x\right]$$

$$\Rightarrow \frac{\partial^2 f}{\partial y^2} = -e^x \sin y + e^y \cos x. \longrightarrow (iv)$$

Taking partial derivative of (i) w.r.t y

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left[e^x \sin y - e^y \sin x\right]$$

$$\Rightarrow \frac{\partial^2 f}{\partial y \partial x} = e^x \cos y - e^y \sin x \longrightarrow (v)$$

(iii), (iv) and (v) are the 2^{nd} order partial derivative of given function.

$\underline{\underline{Q_4}}$: $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = -2\hat{i} + 3\hat{k}$, $\vec{c} = 7\hat{j} - 4\hat{k}$

Solution

$$a \cdot \vec{b} \times \vec{c} = \begin{vmatrix} 1 & 2 & -1 \\ -2 & 0 & 3 \\ 0 & 7 & -4 \end{vmatrix}$$

Expanding R_1, we get

$$= 1(0-21) - 2(8-0) - 1(-14-0)$$

$$= -21 - 16 + 14$$

$$= -21 - 2 = -23$$

$$\Rightarrow \quad \vec{a} \cdot \vec{b} \times \vec{c} = -23.$$

Sol:-

$$4x^2 - y^2 + 3z^2 = 10$$

and $P(2, -3, 1)$

Now

let $F(x,y,z) = 4x^2 - y^2 + 3z^2 - 10$

$F_x = 8x \Rightarrow F_x(2,-3,1) = 8(2) = \boxed{16}$

also, $F_y = -2y \Rightarrow F_y(2,-3,1) = -2(-3) = \boxed{6}$

also $F_z = 6z \Rightarrow F_z(2,-3,1) = 6(1) = \boxed{6}$

we know that

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 10$$

So, $16(x-2) + 6(y+3) + 6(z-1) = 10$

$\Rightarrow 16x - 32 + 6y + 18 + 6z - 6 = 10$

$\Rightarrow 16x + 6y + 6z - 20 - 10 = 0$

$\Rightarrow 16x + 6y + 6z - 30 = 0$

$\Rightarrow 16x + 6y + 6z = 30$

$\Rightarrow 6z = -16x - 6y + 30$

$\Rightarrow \boxed{z = -\frac{8}{3}x - y + 5}$

which is the required eq of tangent plane.

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