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Name : Syed. M. Zahoor   Assignment

ID : 12595   Electromagnetic field

Q1 $G = 2a_x - 3a_y + 4a_z$

Solution

ⓐ In the direction of a_ρ = The incremental work is give by $dw = -qE \cdot dL$ where in this case

$$dL = d\rho a_\rho = 6 \times 10^{-6} a_\rho \text{ Thus}$$

$$dw = (20 \times 10^{-6} C)(100 V/m)(6 \times 10^{-6} m)$$

$$= -12 \times 10^{-9} J$$

$$= -12 nJ$$

ⓑ In the direction of a_Φ = In this case $dL = 2 d\phi a_\phi = 6 \times 10^{-6}$ and so

$$dw = -(20 \times 10^{-6})(-200)(6 \times 10^{-6})$$

$$= 2.4 \times 10^{-8} J$$

$$= 24 nJ$$

ⓒ In the direction of a_z = Hence, dL

$$dL = dz a_z = 6 \times 10^{-6} a_z$$

$$dw = -(20 \times 10^{-6})(300)(6 \times 10^{-6})$$

$$-3.6 \times 10^{-8} J$$

$$= -36 nJ$$

(d) In The direction of E

$$a_E = \frac{100a_\rho - 200a_\phi + 300a_z}{[100^2 + 200^2 + 300^2]^{1/2}}$$

$$= 0.267a_\rho - 0.535a_\phi + 0.802a_z$$

Thus $dw = -(20\times10^{-6})(100a_\rho - 200a_\phi + 300a_z]\cdot(0.267a_\rho - 535a_\phi + 0.802a_z \times 6\times10^{-6}$

$= -44.9nJ$

(e) In The direction of $G = 2a_x - 3a_y + 4a_z$

$$a_G = \frac{2a_x - 3a_y + 4a_z}{[2^2 + 3^2 + 4^2]^{1/2}}$$

$$= 0.371a_x - 0.557a_y + 0.743a_z$$

Now

$dw = -(20\times10^{-6})(100a_\rho - 200a_\phi + 300a_z]\cdot(0.371a_x - 0.057a_y + 0.743a_z)$

$= -(20\times10^{-6})(37.1(a_\rho\cdot a_x(6\times10^{-6}) - 55(a_\rho\cdot a_y) - 74.2(a_\phi\cdot a_x) + (a_\phi\cdot a_y) + 222.9](6\times10^{-6})$ 111.4

where at P_1 $(a_\rho\cdot a_x) - (a_\phi\cdot a_y) = \cos(40^\circ) = 0.766$

$(a_\rho\cdot a_y) = \sin(40^\circ) = 0.643$

$(a_\phi\cdot a_x) = -\sin(40) = -0.643$

Now Substituting These result in

$dw = (20\times10^{-6})(28.4 - 35.8 + 47.7 + 85.3 + 222.9](6\times10^{-6})$

$= -41.8nJ$

(3)

(Q2) Let $B = 10\left[\sin \pi/6\right] a_x + 5 \sin \pi/6\, a_y + 10 \cos(\pi/6) a_z]$

<u>Solution</u>

(a) $E_p = -10\left(\sin(\pi/6) a_x + 5\sin(\pi/6) a_y + 100\cos(\pi/6) a_z\right)$

$= -[5a_x + 25a_y + 50\sqrt{3}a_z$

(b) $dW_x = -q\, E \cdot dL a_x$

$= -2\times10^{-9}(-5)(10^{-3})$

$= 10^{-11} J = 10 pJ$

(c) of $a_y = ?$

$dW_y = -q\, E \cdot dL a_y$

$= -2\times10^{-9}(-25)(-10^{-3})$

$= \cancel{50\times10^{-11}} = 50\times10^{-11} J = 50 pJ$

(d) of a_z -

$dW_z = -q\, E \cdot dL a_z$

$= -2\times10^{-9}(-50\sqrt{3})(10^{-3})$

$= 100\sqrt{3}\, pJ$

(e) of $(a_x + a_y + a_z)$

$dW_{xyz} = -q E \cdot dL \dfrac{(a_x + a_y + a_z)}{\sqrt{3}}$

$= \dfrac{10 + 50 + 100\sqrt{3}}{\sqrt{3}}$

$= 135 pJ$

(Q No 3)

Solution:

(a) $P(1,2,3)$ toward $Q(2,14)$
The vector along this direction will be
$Q-P=(1,-1,1)$ from which $a_{PQ}=[a_x-a_y+a_z]/\sqrt{3}$

$$dw = -qE\cdot dL$$

$$= -(50\times10^{-6})\left(120\, a_\rho \left(\frac{a_x-a_y+a_z}{\sqrt{3}}\right)\right](2\times10^{-3})$$

$$= -(50\times10^{-6})(120)((a_x\cdot a_\rho)-(a_\rho\cdot a_y)]\frac{1}{\sqrt{3}}(2\times10^{-3})$$

At P, $\phi = \tan^{-1}(2/1) = 63.4°$ Thus $(a_\rho\, a_x)=$
$\cos(63.4°) = 0.447$
$(a_\rho\cdot a_y) = \sin(63.4°) = 0.894$
Substituting these we obtain
$dw = 3.1\mu J$.

(b) $Q=(2,1,4)$ toward $P(1,2,3)$ A little through is in order here: Note that the field has only a radical component and does not depend on ϕ or z and P and Q are at the same radius $(\sqrt{5})$ from z axis. Thus the answer is $dN = 3.1\mu J$ as in part a. This is also found by going through the procedure as in part a but with the direction (Roles of P & Q) reversed

(Q4) Compute the Value $\int_A^P G \cdot dL$

Solution

(a) Straight line Segment (A(1, -1, 2) To B(1, 1, 2) To P(2, 1, 2): In general we should have

$$\int_A^P G \cdot dL = \int_A^P 2y\,dx$$

The change in x occurs when moving b/w B & P during which $y = 1$

$$\int_A^P G \cdot dL = \int_B^P 2y\,dx$$

$$\int_1^2 2(1)\,dx$$

$$= 2$$

(b) Straight Line Segment A(1, -1, 2) To (2, 1, 2) To P(2, 1, 2), In this Case Change in occurs when moving from A to C during which $y = -1$

$$= \int_A^P G \cdot dL = \int_C^P 2y\,dx$$

$$= \int_1^2 2(-1)\,dx$$

$$= -2$$

(47)

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QNo (5) Let $G = 3xy^3 a_x + 2z a_y$. find.

Solution:

Let $G = 3xy^3 a_x + 2z a_y$

(a) Straight Line $y = x - 1$, $z = 1$

$$= \int G\,dL = \int_2^4 3xy^2 + \int_1^3 2z\,dy$$

$$= \int_2^4 3x(x-1)^2\,dx + \int_1^3 2(1)\,dy$$

$$= 90$$

(b) Parabola $6y = x^2 + 2$, $z = 1$

$$= \int G\,dL = \int_2^4 3xy^2 + \int_1^3 2z\,dy$$

$$= \int_2^4 \frac{1}{12} x(x^2+2)^2 + \int_1^3 2(1)\,dy$$

$$= 82$$