# Linear Algebra


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| Name | Muhammad Ishfaq |
| Student Id | 16002 |
| Department | CS2 |
| Instructor | Mansoor Qadir |

Iqra National University Peshawar, Pakistan

1

Question 1:-

Solution:- My ID is 16002.

$$V_1 = \begin{bmatrix} 1 \\ 6 \\ 0 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}, \quad V_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

They will be linearly dependent if determinant equal to 0

So

$$\begin{bmatrix} 1 & 6 & 0 \\ 6 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\det V = 1 \begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} - 6 \begin{vmatrix} 6 & 0 \\ 0 & 2 \end{vmatrix} + 0 \begin{vmatrix} 6 & 0 \\ 0 & 0 \end{vmatrix}$$

$$= 1(0-0) - 6(12-0) + 0$$

$$\det V = -72 \neq 0$$

So it is not linearly dependent.

Question 3:-

Solution:-

Four things to define vector space.

* Comutative group under (+)
* Distributive Property

$$f(v_1+v_2) = f v_1 + f v_2$$

* Associative Property

$$f_1(f_2+v) = (f_1 \times f_2) + fv$$

Scaling by 1:-

$$1.V = V$$

(a)

V= 2×2 matrices with entrices in R

$$K \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} Ka & d \\ Kc & d \end{pmatrix} \text{ for } K \in R.$$

Now
clearly V= 2×2 matrices with ~~entires~~ entires in R is Commutative group.

Now
let $f$ $v_1 = \begin{pmatrix} ka_1 & d_1 \\ kc_1 & d_1 \end{pmatrix}$ $v_2 = \begin{pmatrix} ka_2 & b_2 \\ kc_2 & d_2 \end{pmatrix}$

$\in R$

Then $f(v_1 + v_2) =$

$$\Rightarrow f\left[\begin{pmatrix} ka_1 & b_1 \\ kc_1 & d_1 \end{pmatrix} + \begin{pmatrix} ka_2 & b_2 \\ kc_2 & d_2 \end{pmatrix}\right]$$

$$\Rightarrow f\begin{pmatrix} ka_1 & b_1 \\ kc_1 & d_1 \end{pmatrix} + f\begin{pmatrix} ka_2 & b_2 \\ kc_2 & d_2 \end{pmatrix}$$

$$\Rightarrow fv_1 + fv_2$$

So $f(v_1 + v_2) = fv_1 + fv_2$

(b)

let $f_1, f_2$ $v = \begin{pmatrix} ka & b \\ kc & d \end{pmatrix} \in R$

Then.

$$f_1(f_2 + v)$$

$$f_1 = \left(f_2 + \begin{pmatrix} ka & b \\ kc & d \end{pmatrix}\right)$$

4

$$\Rightarrow f_1 \times f_2 + f_1 \begin{pmatrix} ka & b \\ kc & d \end{pmatrix}$$

$$f_1 \times f_2 + f_1 V$$

$$\Rightarrow (f_2 \times f_1) + V f_1$$

$$\Rightarrow [f_2 + V] f_1$$

5

Question 4:-

Solution:-

(a) if $\det M \neq 0$ then M have an inverse.

because we know that

$$M^{-1} = \frac{\text{Adj } M}{\det M}$$

So the only possibility which

$\det M \neq 0$ does M have an inverse.

(b)

All possible $2\times 2$ bit matrices, with determinant 1 is

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

(c) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$.

(d) My ID is 16002

Solution:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 0 & 6 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\det = 1\begin{vmatrix} 0 & 6 \\ 1 & 2 \end{vmatrix} - 6\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + 0\begin{vmatrix} 1 & 1 \\ 0 & 6 \end{vmatrix}$$

$$= 1(0-6) - 6(2-1) + 0$$

$$= -6 - 6 + 0$$

$$\boxed{\det A = -12}$$

Question (2)

(a)

for Product X

$$\text{Total Cost}_x = 1000 \times (450+250+150)$$

For product ~~B~~.

$$TC_Y = 500 \times (400+350+450)$$

Total cost A and B $= TC_X + TC_Y$.