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Assignment : Digital Signal processing

Q1(a):

Solution:

Consider the different equation

$y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$ — (1)

The homogenous equation of the system is

$y(n)-3y(n-1)-4y(n-2)=0$

The characteristic equation of the system is

$\lambda-3\lambda^{-1}4\lambda^{-2}=0$

$\lambda^{2}-3\lambda-4=0$

Determine the root of the characteristic equation

$\lambda^{-2}-4\lambda+\lambda-4=0$

$\lambda(\lambda-4)+1(\lambda-4)=0$

$(\lambda-4)(\lambda+1)=0$

$\lambda=-1, 4$

The homogenous solution is

$y_h(n)=C_1(-1)^n u)+C_2(4)^n u(n)$

Since 4 is a characteristic roots and the excitation is ②

$x(n) = 4^n u(n)$

We assume a particular solution of the form $y_p(n) = k n 4^n u(n)$

Then

$k n 4^n u(n) - 3k(n-1) 4^{n-1} u(n-1) - 4k(n-2) 4^{n-2} u(n-2)$

$= 4^n u(n) + 2(4)^{n-1} u(n-1)$

for $n = 2$

$k(32-12) = 4^2 + 8 = 24 \rightarrow k = 6/5$

The Total solution is

$y(n) = y_p(n) + y_h(n)$

$= \left[\frac{6}{5} n 4^n + C_1 4^n + C_2 (-1)^n\right] u(n)$

The solve for C_1 and C_2 we assume That

$y(-1) = y(-2) = 0$ Then

$y(0) = 1$ and

$y(1) = 3y(0) + 4 + 2 = 9$

Here

$c_1 + c_2 = 1$ and

$24/5 = 4c_1 - c_2 = 9$

$4c_1 - c_2 = \frac{21}{5}$

Therefore

$c_1 = \frac{26}{25}$ and $c_2 = \frac{-1}{25}$

The Total solution is

$y(n) = \left(\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} (-1)^n\right) u(n)$

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Q2 (b):-

Solution:-

consider the different equation

$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

$y(n) - 0.6y(n-1) + 0.08y(n-2) + x(n)$

To obtain the homogenous equation input

$x(n) = 0$

$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0$

Determine the solution to the homogenous equation (4)

$y_h(n) = \lambda^n$

Substitude the solution obtained in the homogenous equation

$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$

$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$(\lambda - 0.2)(\lambda - 0.4) = 0$

Therefore, the roots are

$\lambda_1 = 0.2, \quad \lambda_2 = 0.4$

The General form of the solution to the homogenous equation is

$y_h(n) = c_1(\lambda_1)^n + c_2(\lambda_2)^n$

$y(n) = c_1(0.2)^n + c_2(0.4^n) \longrightarrow (1)$

$\lambda = 0.2, \quad \lambda = 0.4$ Hence

$y_h(n) = c_1 \frac{1^n}{5} + c_2 \, 2^n/5$

with $x(n) = \delta(n)$ the initial condition are

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Q2(a):

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Solution:-

The Z-Transform is

$$X(Z) = \frac{1}{(1-2Z^{-1})(1-Z^{-1})^2}$$

The expression is written as

$$Y(Z) = \frac{1}{\left(1-\frac{2}{Z}\right)\left(1-\frac{1}{Z}\right)^2}$$

$$= \frac{1}{\left(\frac{Z-2}{Z}\right)\left(\frac{Z-1}{Z}\right)^2}$$

$$= \frac{1}{\frac{(Z-2)(Z-1)^2}{Z^3}}$$

$$= \frac{Z^3}{(Z-2)(Z-1)^2}$$

$y(0) = 1,$

$y(1) - 0.6y(0) = 0$

$y(1) = 0.6$

Hence $c_1 + c_2 = 1$ and

$\frac{1}{5} c_1 + 2/5 = 0.6$

$\Rightarrow c_1 = -1, \; c_2 = 3$

There fore $h(n) = [-(\frac{1}{5})^n + 2(2/5)^n] u(n)$

The step response is

$$S(n) = \sum_{k=0}^{n} h(n-k), \; n > 0$$

$$= \sum_{k=0}^{n} [2[2/5]^{n-k} - (1/5)^{n-k}]$$

$$= \left\{ \frac{1}{0.12} \left(\frac{2}{5}^{n+1} - 1 \right] - \frac{1}{0.16} \left((1/5)^{n-1} - 1 \right\} \right.$$

$u(n)$

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2 (a)

$x(z)$ has a simple pole at $p_1 = 2$ and a double $p_2 = p_3 = 1$ In such case the appropiate partial-fraction expansion is (7)

$$X(z) = \frac{z^3}{(z-2)(z-1)^2} = \frac{A_1}{z-2} + \frac{A_2}{z-1} + \frac{A_3}{(z-1)^2}$$

The problem is to determine the coefficient A_1, A_2 and A_3.

We proceed as in the case of the distinct pole to determine A_1 we multiply both side of $3y(z-2)$ and evaluate the result

$z = -2$

$$(z-2)x(z) = A_1 + \frac{z-2}{z-1} A_2 + \frac{z-2}{(z-1)^2} A_3.$$

which we evaluate at $z = 2$

$$A_1 = \frac{(z-2)X(2)}{2} \Big|_{z=2}$$

$A_1 = 4$

$$A_2 = A_1 + \frac{z-2}{z-1}$$

$A_2 = -3$

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$$A_3 = \frac{A_1 + z - 2}{z - 1} A_2$$

$= -1$

$\therefore x(n) = [4(2)^n - 3 - 4] u(n)$

Q2(b)

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Solution:

first we eliminate the negative power by multiplying both numerator and de-nominater by Z^2. Thus

$$X(Z) = \frac{Z^2}{Z^2 - 1.5Z + 0.5}$$

The poles $X(Z)$ are $p_1 = 1$ and $p_2 = 0.5$

Consequently, the expansion of the form

$$\frac{X(Z)}{Z} = \frac{Z}{(Z-1)(Z-0.5)} = \frac{A_1}{Z-1} + \frac{A_2}{Z-0.5}$$

Simple method to determine A_1 and A_2 is to multiply the equation by the denomenation term $(Z-1)(Z-0.5)$ thus we obtain

$$Z = (Z-0.5)A_1 + (Z-1)A_2 \longrightarrow (1)$$

Now if we set $Z = p_1 = 1$ in eq (1) we eliminate the term involving A_2. Hence

$$1 = (1-0.5)A_1$$

Then we obtain the result $A_1 = 2$ Next we return eq ① and $z = p_2 = 0.5$ thus eliminating the term involving A_1 so we have. (9.1)

$$0.5 = (0.5 - 1) A_2.$$

and hence $A_2 = -1$, There fore the result of the partial fraction expansion is

$$\frac{X(z)}{z} = \frac{2}{z-1} = \frac{1}{z-0.5} \quad \text{Ans-}$$

$\Rightarrow X =$ x ——

QNO 3(a) :⇒

Solution :⇒

At $w =$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

hence

$$b_0 = (1-p)^2$$

At $w = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-p)^2}{(1-p^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4)+jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2}+jp\sqrt{2})^2}$$

Hence,

$$= \frac{(1-p)^4}{[(1-p/\sqrt{2})^2+p^2/2]} = 1/2.$$

or equivalently

$$\sqrt{2}\,(1-p)^2 = 1-p^2-\sqrt{2p}$$

The value of $p=0.32$ Satisfies This equation consequently the System function for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}.$$

Q 3(b):

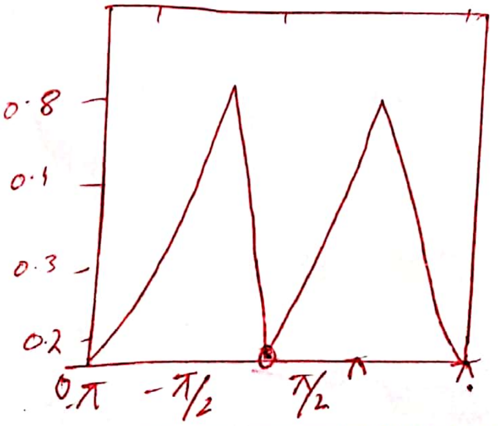
Answer:

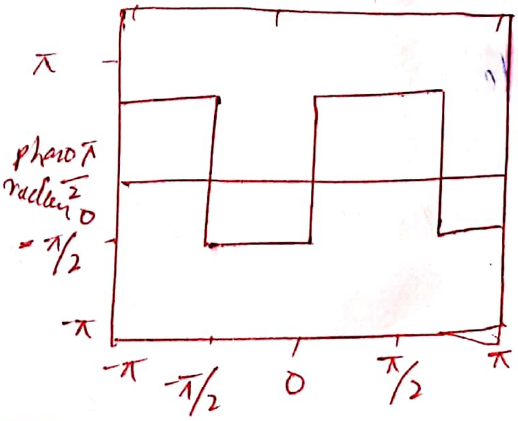
Clearly, The filter must have poles at $P_{1,2} = re^{\pm j\pi/2}$.

and Zeros at $z=1$ and $z=-1$, consequently the System function is.

$$H(z) = h\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$





The gain factor is determined by evaluating the frequency response $H(w)$ of the filter at $w = \pi/2$ Thus we have

$$H\left(\frac{\pi}{2}\right) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(w) = 4\pi/9$. Thus we have.

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \quad \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(r\pi/9)} = \frac{1}{2}.$$

or equivalently.

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ Satisfies this equation Therefore, the system function for the desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

Q: 4(a)

Solution:

The fourier Transform of This sequence is

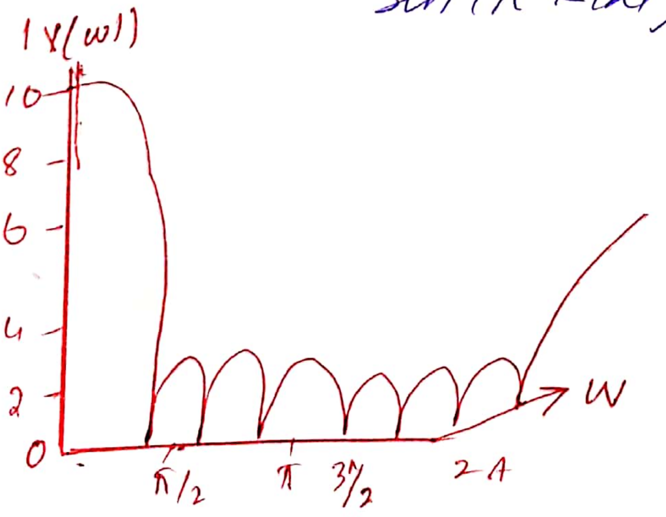
$$X(w) = \sum_{n=0}^{l-1} x(n) e^{-jwn}$$

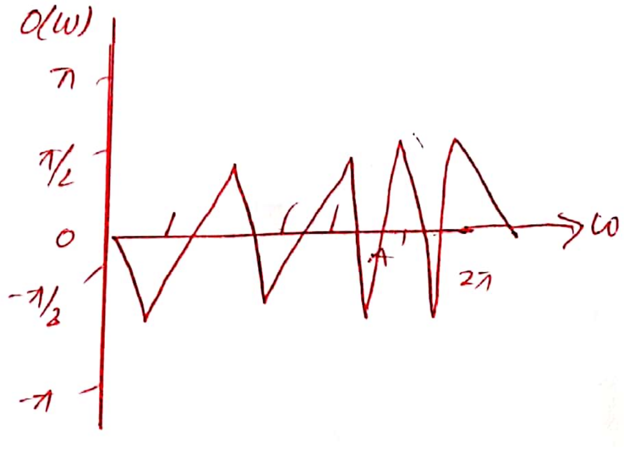
$$= \sum_{n=0}^{l-1} e^{-jwn} = \frac{1-e^{-jwL}}{1-e^{-jw}} = \frac{\sin(wL/2)}{\sin(w/2)} e^{-jw(l-1/2)}$$

The magnitude and phase of $X(w)$ are illustrain for $L = 10$ The N-point DFT of $x(n)$ is simply $X(w)$ evaluated at the set of N equally speed frequencies $w_k = 2\pi k/N$, $k = 0, 1 \cdots N-1$ Hence

$$X(k) = \frac{1-e^{-j2\pi kL/N}}{1-e^{-j2\pi k/N}}, \quad k = 0, 1, \ldots, N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-jk(l-1)N}$$





If N is selected such that N = L, then the DFT become

$$X(k) = \begin{cases} c_1 & k = 0 \\ 0 & k = 1, 2, \ldots, L-1 \end{cases}$$

Thus there is only one non zero value in DFT, This is apparent from observation of $x(\omega)$ since $X(\omega) = 0$ at the frequencies $\omega_k = 2\pi k/L$ $k \neq 0$ The reader should verify that $(x)(n)$ can be recovered from $X(k)$ by performing an L-point DFT.

Question 4 (b)

Solution:

We will determine the matrix W_4. By exploiting the periodicity property of W_4, and the symmetry property $W_N^{k+N/2} = -W_N^k$

matrix W_4 expressed as.

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^1 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then

$$Y_4 = W_4 y_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

DFT of X_4 is determined by determined by conjugating the element in W_4 to obtain W_4 and then applying the formula.

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