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Section : B

Subject : DIFFERENTIAL EQUATION

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①

Q No. 1:  $f(t) = 1 + t \quad -\pi \leq t \leq \pi$

Sol,

Here we use the formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \rightarrow (i)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t) dt \Rightarrow a_0 = \frac{1}{2\pi} \left[ t + \frac{t^2}{2} \right]_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left( \pi - (-\pi) + \frac{\pi}{2} - \left( -\frac{\pi}{2} \right) \right)$$

$$a_0 = \frac{1}{2\pi} \left( 2\pi + \frac{2\pi^2}{2} \right)$$

$$a_0 = \frac{1}{2\pi} (2\pi + \pi^2)$$

(2)

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t) (\cos nt) dt$$

$$a_n = \frac{1}{\pi} (1+t) \frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \left( \frac{\sin nt}{n} \frac{d}{dt} (1+t) \right)$$

$$a_n = \frac{-1}{\pi} \left( (1+t) \frac{\sin nt}{n} - \frac{\cos nt}{n^2} \Big|_{-\pi}^{\pi} \right)$$

$$a_n = \frac{-1}{n^2 \pi} \left( \cos n\pi - \cos n(-\pi) \right)$$

$$a_n = \frac{-1}{n^2 \pi} (-1 - (-1))$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t) \sin nt dt$$

$$b_n = \frac{1}{\pi} \left( (1+t) \int_{-\pi}^{\pi} \sin nt - \int_{-\pi}^{\pi} (\sin nt \frac{d}{dt} (1+t) dt) \right)$$

$$b_n = \frac{1}{\pi} \left( (1+t) \frac{(-\cos nt)}{n} \right)$$

(3)

$$b_n = \frac{1}{\pi} \left( \frac{(1+t)(-\cos n\pi t)}{n} \Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{-\cos nt}{n} dt \right) \quad (1)$$

$$b_n = \frac{1}{\pi} \left( \frac{-(1+t)(\cos nt)}{n} \Big|_{-\pi}^{\pi} + \left( \frac{\sin nt}{n^2} \Big|_{-\pi}^{\pi} \right) \right)$$

$$b_n = \frac{-1}{n\pi} \left( (1+\pi)(\cos n\pi) - (1+(-\pi))(\cos n(-\pi)) \right)$$

$$b_n = \frac{-1}{n\pi} (\cos n\pi + \pi \cos n\pi - \cos n\pi + \pi \cos n\pi)$$

$$b_n = \frac{-1}{n\pi} (2\pi \cos n\pi)$$

~~Here constant =~~

Here  $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$b_n = \frac{2}{n} (-1)^{n+1}$$

So eqn become

$$f(x) = \frac{1}{2\pi} (2\pi + \pi) + 0 + \sum_{n=1}^{\infty} \frac{2}{n} (-1)^{n+1} \sin t$$

Q.No.2

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen values = ?

Sol:

Step # 01

We have;

$$(A - \lambda I)x = 0 \quad A = \text{Given matrix}$$

Step # 02

We have; The characteristic equation  
and given by

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} = -\lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

Step # 03

$$\lambda^3 - \left| \begin{matrix} \text{Sum of} \\ \text{diagonal Element} \end{matrix} \right| \lambda^2 + \left| \begin{matrix} \text{Sum of} \\ \text{diagonal matrix} \end{matrix} \right| \lambda - |A| = 0 \quad \text{--- (B)}$$

Sum of Diagonal element =  $1+1+2=4$

Sum of // matrix =  $\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$$= (-6) + (2) + (1)$$

$$= -3$$

By putting value in eq (B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{--- (C)}$$

$$\begin{aligned}
 |A| &= \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0 \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} \\
 &= 1(2-8) - 0 + 1(6-0) \\
 &= -6 + 6 \\
 &= 0
 \end{aligned}$$

By putting values in (6),

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0$$

using Quadratic formula

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1$$

$$b = -4$$

$$c = -3$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

we have eigen values

$$\lambda = \left( 0, \frac{4 + \sqrt{28}}{2}, \frac{4 - \sqrt{28}}{2} \right)$$

required  
Solution



Q No. 3

Solve the..... linear eqs.

$$5x + 0 + 4z + 2m = 3$$

$$x - y + 2z + m = 1$$

$$4x + y + 2z + 0 = 1$$

$$x + y + z + m = 0$$

Sol:-

$$\left[ \begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right] \quad R_4 \quad R_2$$

$$\left[ \begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 + \frac{4}{5} & 1 \\ 0 & -1 & -\frac{4}{5} & \frac{4}{5} & \frac{3}{5} \\ 0 & 2 & -1 & 0 & -1 \end{array} \right] \quad -\frac{1}{5} \times R_3$$

$$\left[ \begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array} \right] \quad \textcircled{5} \quad \begin{array}{l} 5 \times R_2 \\ 5 \times R_2 \end{array}$$

$$\left[ \begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 6 & 1 \end{array} \right] \quad \begin{array}{l} 5 \times R_3 \\ 5 \times R_4 \end{array}$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array} \right] \quad R_2 \times 5$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array} \right] \quad R_3 - R_2$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right] \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/2 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right] \xrightarrow{R_2 \times -5}$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/2 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/4 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right] \xrightarrow{5/4 \times R_1}$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right]$$

$$\Rightarrow \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right]$$

$$(x, y, z, m) = \left( \frac{3}{4}, \frac{31}{21}, -\frac{11}{21}, \frac{1}{3} \right)$$

$$x = \frac{3}{4}$$

$$y = \frac{31}{21}$$

$$z = -\frac{11}{21}$$

$$m = \frac{1}{3}$$

Q No 4 verify that

$U(x, t) = \sin(x + 2t)$   
is a solution of the one-dime  
equation

Solution:

Given that  
 $U(x, t) = \sin(x + 2t)$

Differentiate w.r.t  $x$  Partially

$$\frac{\partial U}{\partial x} = \frac{\partial}{\partial x} \sin(x + 2t)$$

$$\frac{\partial U}{\partial x} = \cos(x + 2t) \frac{\partial}{\partial x} (x + 2t)$$

$$\frac{\partial U}{\partial x} = \cos(x + 2t) (1 + 0)$$

$$\frac{\partial U}{\partial x} = \cos(x + 2t)$$

$$\frac{\partial^2 U}{\partial x^2} = \frac{\partial}{\partial x} \cos(x + 2t)$$

$$\frac{\partial^2 y}{\partial x^2} = \sin(x+2t) \cdot \frac{\partial}{\partial x} (x+2t)$$

$$\frac{\partial^2 y}{\partial x^2} = -\sin(x+2t) (1+0)$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = -\sin(x+2t)}$$

and  $u(x,t) = \sin(x+2t)$

Differentiate w.r.t  
"t"

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial t} \sin(x+2t)$$

$$\frac{\partial}{\partial t} = \cos(x+2t) (0+2)$$

$$\frac{\partial^2 y}{\partial t} = 2 \cos(x+2t)$$

$$\frac{\partial^2 y}{\partial t^2} = (2) - \sin(x+2t) (0+2)$$

$$\frac{\partial^2 y}{dt^2} = -4 \sin(x + 2t)$$

We know that one-dimensional wave equation is

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

$$-4 \sin(x+2t) = c^2 [-\sin(x+2t)]$$

$$-4 \sin(x+2t) = -c^2 \sin(x+2t)$$

$$-4 \sin(x+2t) + c^2 \sin(x+2t) = 0$$

for the arbitrary constant  $c=2$

$$-4 \sin(x+2t) + (2)^2 \sin(x+2t) = 0$$

$$-4 \sin(x+2t) + 4 \sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for the arbitrary constant

$$c=2$$