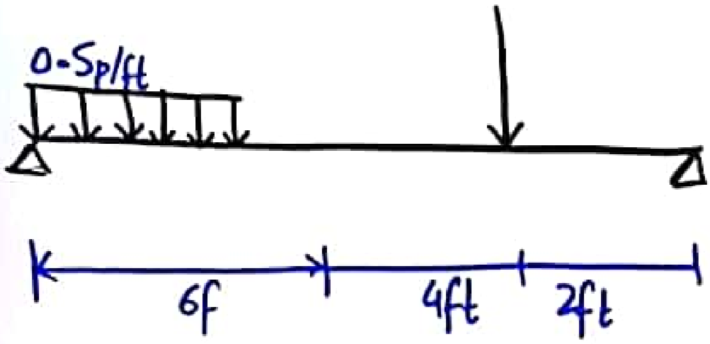
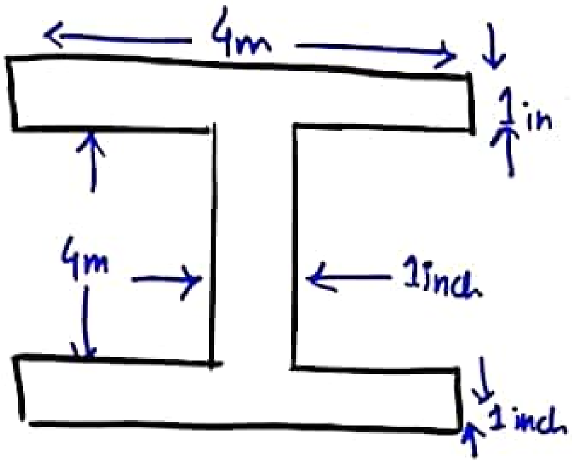
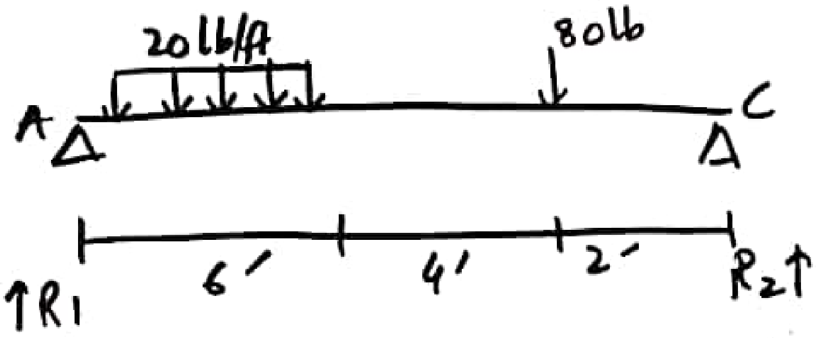
QUESTION ①

Given Beam:





Note: Put the value of $P = 40$ so we have



First to find unknown reaction at the support apply equilibrium equation.

$\Sigma f_x = 0$ i.e. $R_2 = 0$

$\Sigma f_y = 0$ $\oplus\uparrow \downarrow\ominus$

$R_1 + R_2 = (20 \times 6) lb + 80 lb$

$R_1 + R_2 = 120 Lb + 80 lb$

$\boxed{R_1 + R_2 = 200 Lb}$ — (1)

Next

$\Sigma M_A = 0$ $\oplus\ \ominus$

$R_2 \times 12 - 10 \times 80 - (20 \times 6) \times 3 = 0$

$12R_2 = 800 + 360$

$12R_2 = 1160$

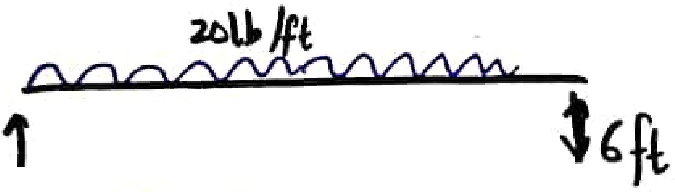
$R_2 = 96.66$

$\Rightarrow R_1 + R_2 = 200$

$\Rightarrow R_1 = 200 - 96.66$

$R_1 = 103\,lb$

Now shear force at change point of Beam



Shear force at 6ft from support

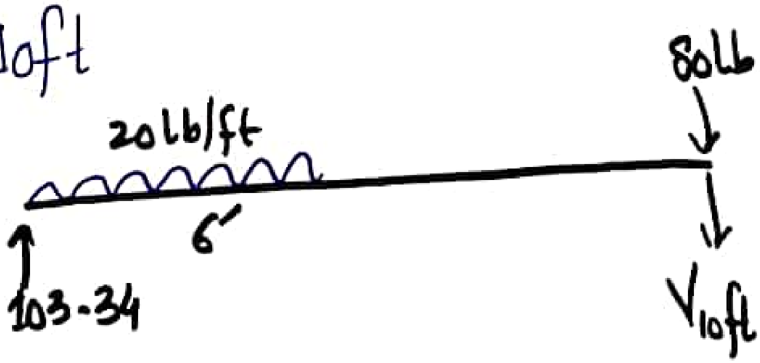
$\Sigma f_y = 0$ ↑ ⊕ ↓ ⊖

$103.34 - 20 \times 6 - V_{6ft} = 0$

$V_{6ft} = -16.66$

$\Rightarrow$ Now shear force at 10ft

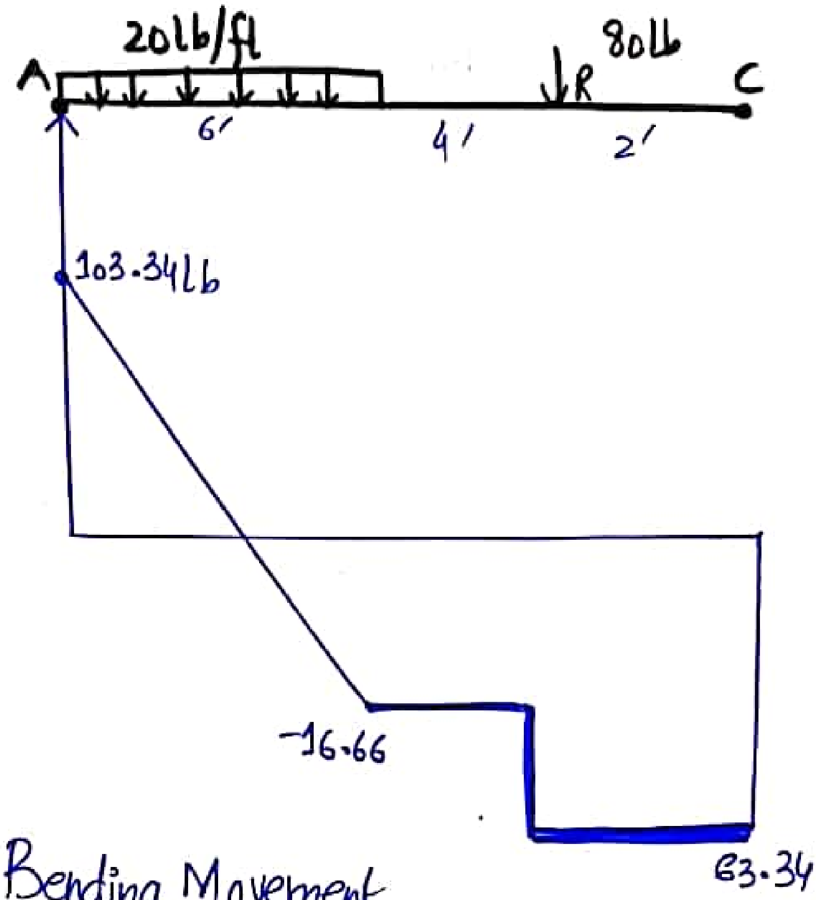
$\Sigma f_y = 0$ ⊕↑ ↓⊖



$103.34 - 20 \times 6 - 80 - V_{10ft} = 0$

$V_{10ft} = 63.34\,lb$

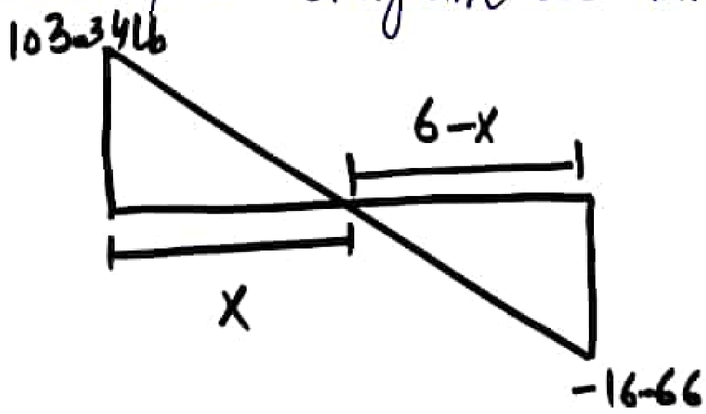
Now draw shear force diagram ③



⇒ Point of Maximum Bending Movement

As we know that the point where shear force is ~~max~~ minimum the bending moment it maximum so from point of zero shear corresponding point will have maximum bending moment.

From shear force diagram we have



we know that

$$\frac{103.34\,lb}{x} = -\frac{16.66}{6-x}$$

$$\Rightarrow (6-x)(103.34) = (x)(-16.66)$$

$$\Rightarrow 620.04 - 103.34x = -16.66x$$

$$\Rightarrow 620.04 = -16.66x + 103.34x$$

$$\Rightarrow 620.04 = 86.68x$$

$$\Rightarrow x = 7.15\,ft$$

Now determine the value of moment at 7.15ft

$$M_{7.15ft} - 103.34 \times 7.15 + (20 \times 7.15) \times \left(\frac{7.15}{2}\right)$$

$$M_{7.15} - 103.34 \times 7.15 + (20 \times 7.15) * \left(\frac{7.15}{2}\right) = 0$$

$$M_{7.15} - 738.8 + 143 * 3.57 = 0$$

$$M_{7.15} \cancel{*} \cancel{\$} - 738.8 + 510.51 = 0$$

$$M_{7.15} - 228.8 = 0$$

$$M_{7.15} = 228.8$$

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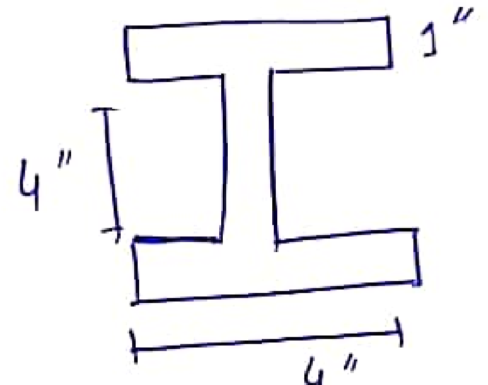
For shear stress we have: (6)

$$\tau = \frac{VQ}{Ib}$$

So first we determine moment of Inertia I f Q the give section of Beam



As the given figure is symmetrical along both the axis

So $\bar{x} = 4/2 = 2$ inch $\quad \bar{Y} = 6/2 = 3$ inch

i.e. $(\bar{x}, \bar{y}) = (2, 3)$

[Centre of gravity]

extreme left and Bottom

Area of point (1) = $4 \times 1 = 4 \text{ inch}^2$

Area of point (2) = $4 \times 1 = 4 \text{ inch}^2$

Area of point (3) = $4 \times 1 = 4 \text{ inch}^2$

Moment of inertia about X-axis (centroid I) I_{xx}

Determine distance b/w C.G of the whole section and corr. parts

Let G_1, G_2, G_3 be in the centre of gravity of point ① ② ③ and k_1, k_2 and k_3 be the distance b/w $\bar{y}$, and y_1, y_2, y_3 respectively.

6

So

$$k_1 = \bar{y} - y_1 \Rightarrow 3 - 0.5 \Rightarrow 2.5 \text{ inch}$$

$$k_2 = \bar{y} - y_2 \Rightarrow 3 - 3 \Rightarrow 0 \text{ inch}$$

$$k_3 = \bar{y} - y_3 \Rightarrow 3 - 0.5 \Rightarrow 2.5 \text{ inch}$$

So

$$I_{xx} = \frac{b_1 h_1^3}{12} + a_1 k_1^2 + \frac{b_2 h_2^3}{12} + a_2 k_2^2 + \frac{b_3 h_3^3}{12} + a_3 k_3^3$$

$$I_{xx} = \frac{(4)(1)^3}{12} + 4(2.5)^2 + \frac{(1)(4)^3}{12} + a_2(0) + \frac{4(1)^3}{12} + 4(1.5)$$

$$I_{xx} = 4/12 + 25 + 64/12 + 4/12 + 25$$

$$I_{xx} = 4/12 + 25 + \frac{64}{12} + \frac{14}{12} + 25$$

$$I_{xx} = \frac{4 + (25)(12) + 64 + 4 + (25)(12)}{12}$$

$$I_{xx} = 56 \text{ inch}^4$$

Now

$$I_{yy} = \frac{b_1^3 h_1}{12} + \frac{b_2^3 h_2}{12} + \frac{b_3^3 h}{12}$$

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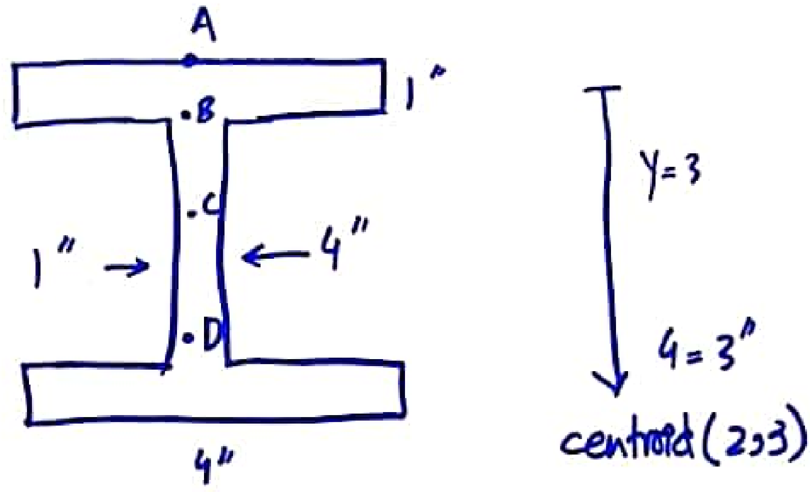
$$I_{yy} = \frac{(4)^3(1)}{12} + \frac{(1)^3(4)}{12} + \frac{(4)^3(1)}{12} \qquad I = I_{xx} + I_{yy}$$

$$I = 67 \text{ inch}^4$$

$$I_{yy} = \frac{64+4+64}{12} = 11 \text{ inch}^4$$

Next find the shear stresses at various point we have

$$\tau = \frac{V\phi}{Ib}$$



centroid (2,3)

(i) Shear stress at point "A" i-e. At the top fiber

$$\tau' = \frac{V\phi}{Ib} \qquad V = max = 96.66 \text{ lb}$$

$$I = 67 \text{ inch}^4$$

So, $$\tau = \frac{96.66(0)}{67(4)}$$

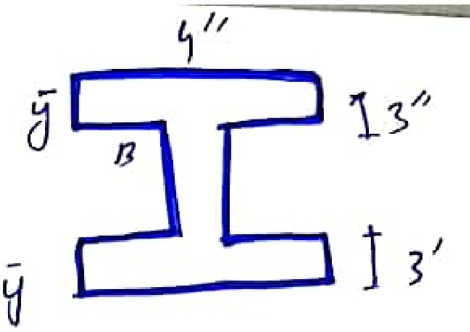
Here A=0 No area of the section exist above point A i-e. $\phi = A\bar{y} \Rightarrow 0$.

$$\tau = 0$$

(ii) Shear stress at point "B"

$$\tau = \frac{V\phi}{Ib} \Rightarrow \frac{96.66 \times (4 \times 1)(3-0.5)}{67 \times 4} \Rightarrow \frac{386.64 \times 2.5}{67 \times 4}$$

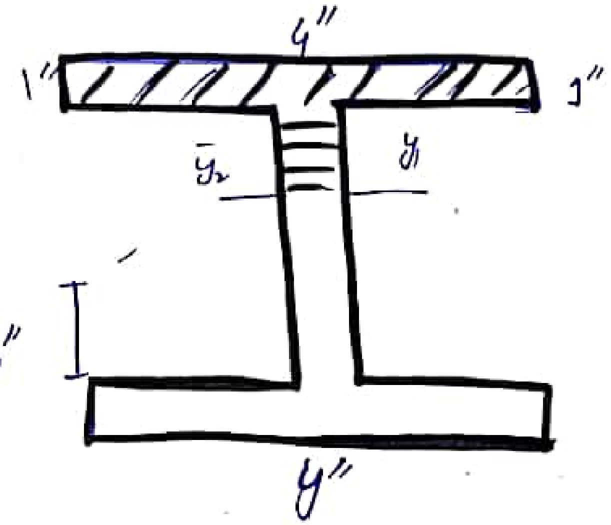
$\Rightarrow \dfrac{966.6}{67 \times 4}$



$\Rightarrow 3.606 \ lb/in^2$

$Q = A \cdot \bar{y}$

$\Rightarrow$ Shear stress at point "C" i.e. at NA



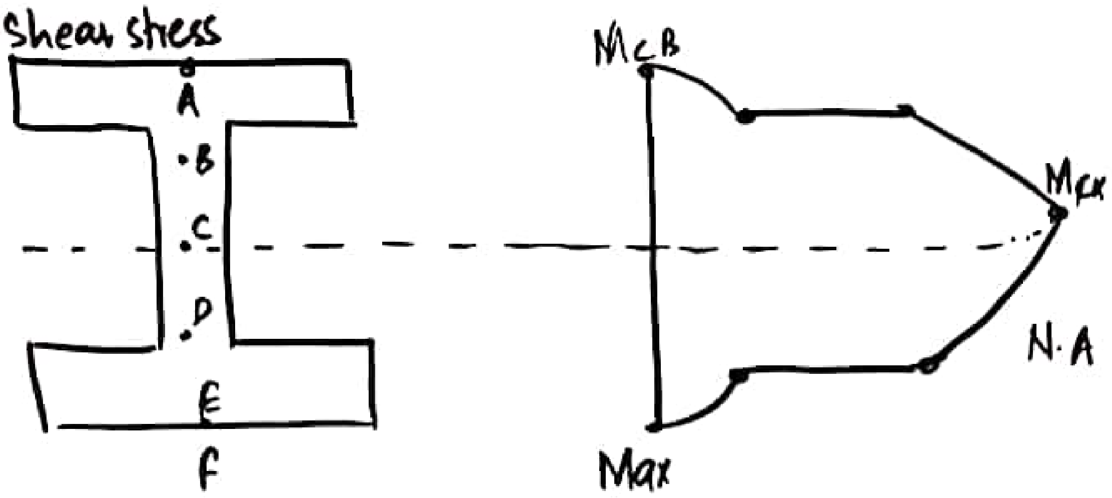
$$\tau = \frac{VQ}{It}$$

$$\tau = \frac{96.66 \times [4 \times 1 \times (3 + 0.5) + (1 \times 2)(2-1)]}{67 \times 1}$$

$$\tau = 15.86$$

iv. Shear stress at point D and E will be the same because of the symmetry.

Note :
The maximum shear stress value occurs at the Neutral axis and minimum at the top of the section.



Flexural stress determination

$$S = \frac{my}{I}$$

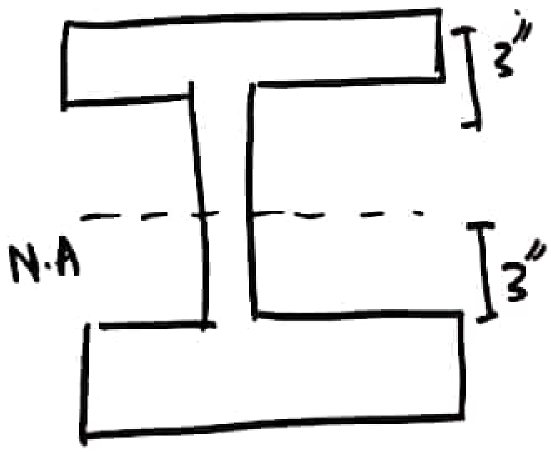
(i) Flexoral stress at the top fibre point A

$$S = \frac{My}{I}$$

$$S = \frac{228.8 \times (3)\cancel{-0.5}}{67}$$

$$S = 10.24 \ lb/inc^2$$

(ii) Flexural stress at point "B"

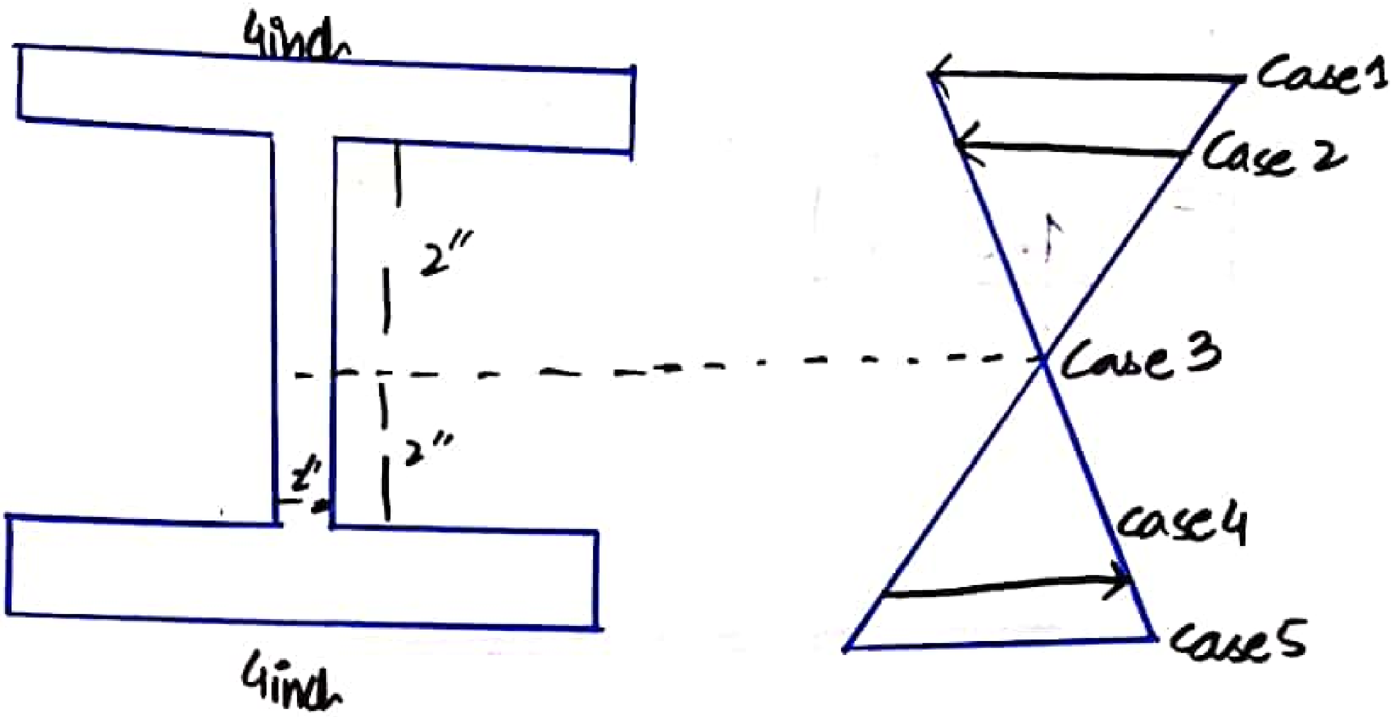


$$S = \frac{My}{I}$$

$$S = \frac{228.8 \times (3-0.5)}{67}$$

$$S = 8.53 \ lb/in^2$$

Bending Diagram

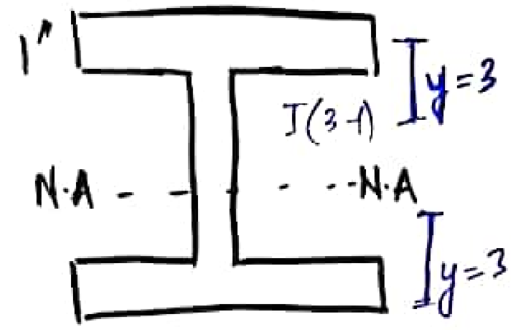


(iii) Flexural stress at point "C"

$$S = \frac{My}{I}$$

$$S = \frac{228.8 \times (3-1)}{67}$$

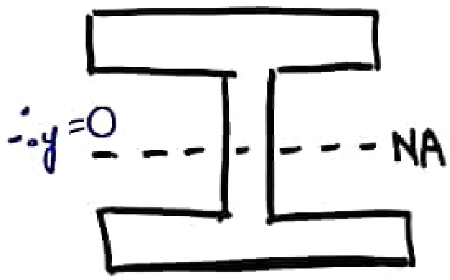
$$S = 6.82\ Lb/in$$



iv. Flexoral stress at neutral axis (N.A)

$$S = My/I$$

$$S = \frac{634.02 \times 0}{67}$$



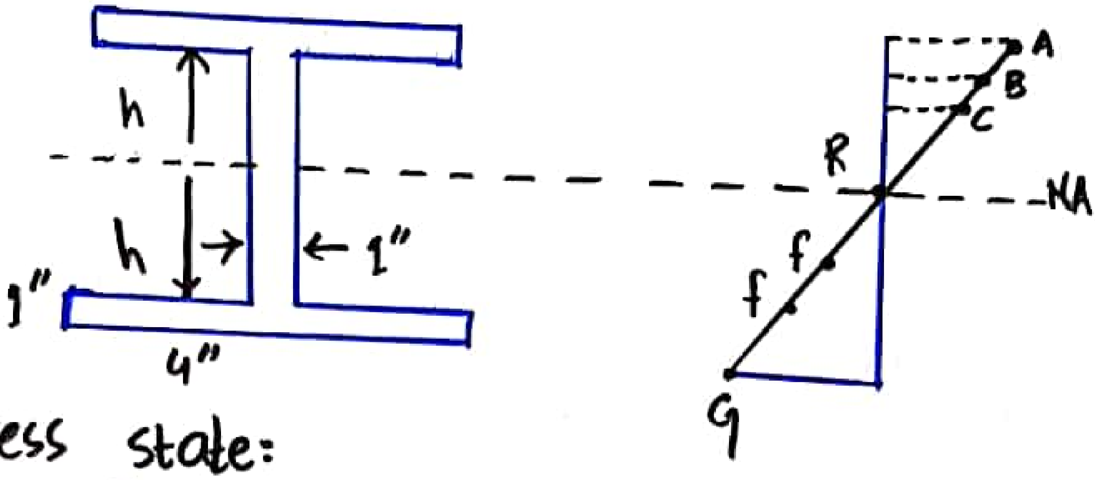
$$S = 0\ Lb/in$$

Flexural stress value at point E, F and G remain the same because of symetry. The upper portion above the N.A show tension and below the N.A show compression.

Note:

The flexoral stress value its maximum at extreme top ~~value~~ and bottom fibre at zero at N.A.

Flexural Stress diagram



Stress state:

Find stress state of a point element located 3ft from left support and 1 inch below from top fibre.

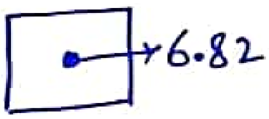
Fleural stress at point "C"

6=6.82

Shear stress at Point "C"

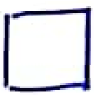
$\tau = 15.86$

Consider point "C" is a planner element

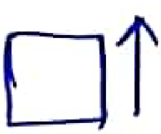


As the flexural stress is perpendicular to the cross section can be represented normal stress.

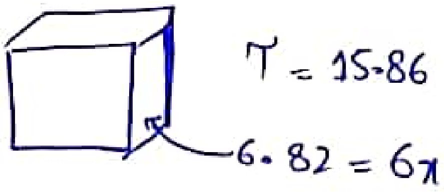
$\tau = 15.86$ psi is comprective because point "C" lies in compression zone of Beam cross section.



If point C lies below the centroid then stress would be tensile.



$\tau = 15.86$ psi



Combine stress on 2d element.

Find its principle stress

We have also find,

$6x = 6.82$

$6y = 0$

$T\!k^2 xy = 15.86$

Principle stress equation

$$6x,y = \frac{6x+6y}{2} \pm \sqrt{\left(\frac{6x-6y}{2}\right)^2 + Sx^2y}$$

$$= \frac{-6.82+0}{2} + \sqrt{\left(\frac{-6.82-0}{2}\right)^2 + (15.86)^2}$$

$$= -3.41 \pm \sqrt{-23.25+251.53}$$

$$= -3.41 \pm \sqrt{228.28}$$

$$= -3.41 \pm 15.10$$

$6x,y \Rightarrow -3.41 + 15.10 = 11.69$

$6x,y \Rightarrow -3.41 - 15.10 = -18.51$

Find $\phi P = ?$

$$\tan 2\phi P = \frac{6xy}{(6x-6y)/2} \Rightarrow \frac{\cancel{18}\ 15.86}{(-6.82-0)/2}$$

$$\Rightarrow \frac{15.86}{-3.41}$$

$$\Rightarrow -4.65$$

$\tan 2\phi P = -4.65$

$2\phi P = \tan^{-1}(-4.65)$

$\phi P = -38.93$

Put in general equation

$$\sigma_{max} = \frac{-6.82+0}{2} + \frac{-6.82+0}{2}$$

$$\cos 2(-38.93) + 15.86 \sin 2(-38.93)$$

$$\sigma_{p\,max} = -3.41 - 3.41$$

Max in plane shear stress in this case

$$\tan 2\phi_s = \frac{-(\sigma_x - \sigma_y)/2}{\sigma_{xy}}$$

$$= \frac{-(-6.82-0)/2}{15.86}$$

$$\tan 2\phi_s = +0.21$$

$$2\phi_s = \tan^{-1} +0.21$$

$$2\phi_s = +11.85$$

$$\phi_s = \frac{11.85}{2} \Rightarrow 5.92$$

Put in these general equation

$$\tau_{xy}' = -\left[\frac{\sigma_x - \sigma_y}{2}\right] \sin 2\phi + \tau_{xy} \cos 2\phi$$

$$= -\frac{(-6.82-0)}{2} \sin 2(5.92) + 15.86 \cos 2(5.92)$$

$$= 3.41 \times 5.38 + 15.86 \times -2.46$$

$$= 18.34 - 39.01$$

$$= 20.67$$

To draw Mohr's circle centre coordinate 15

$$(h,k) = \left(\frac{6x+6y}{2}, 0\right)$$

$$= \left(\frac{-6.82+0}{2}, 0\right)$$

$$\Rightarrow -3.41, 0$$

Radius of Mohr's circle

$$r = \sqrt{\left(\frac{6x-6y}{2}^2\right) + T^2xy} \Rightarrow \sqrt{\left(\frac{-6.82-0}{2}\right)^2 + (15.86)^2}$$

$$\Rightarrow 263.15$$

1 psi = 1cm

$\sigma_x = -6.82$, $\sigma_y = 0$

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$\tau_{xy} = 20.61\,psi$

Scale :

1 psi = 2cm

Then

Radius, $r/_2 = \frac{263.15}{2} = 131.57\,cm$

$\theta = \tan^{-1}\left(\frac{20.61}{-6.82+15}\right)$

$\theta = \tan^{-1}(2.51) \neq \cancel{-3.90}$

$\theta = 68.27^\circ$

For $\sigma_x' = -18.51\,psi$

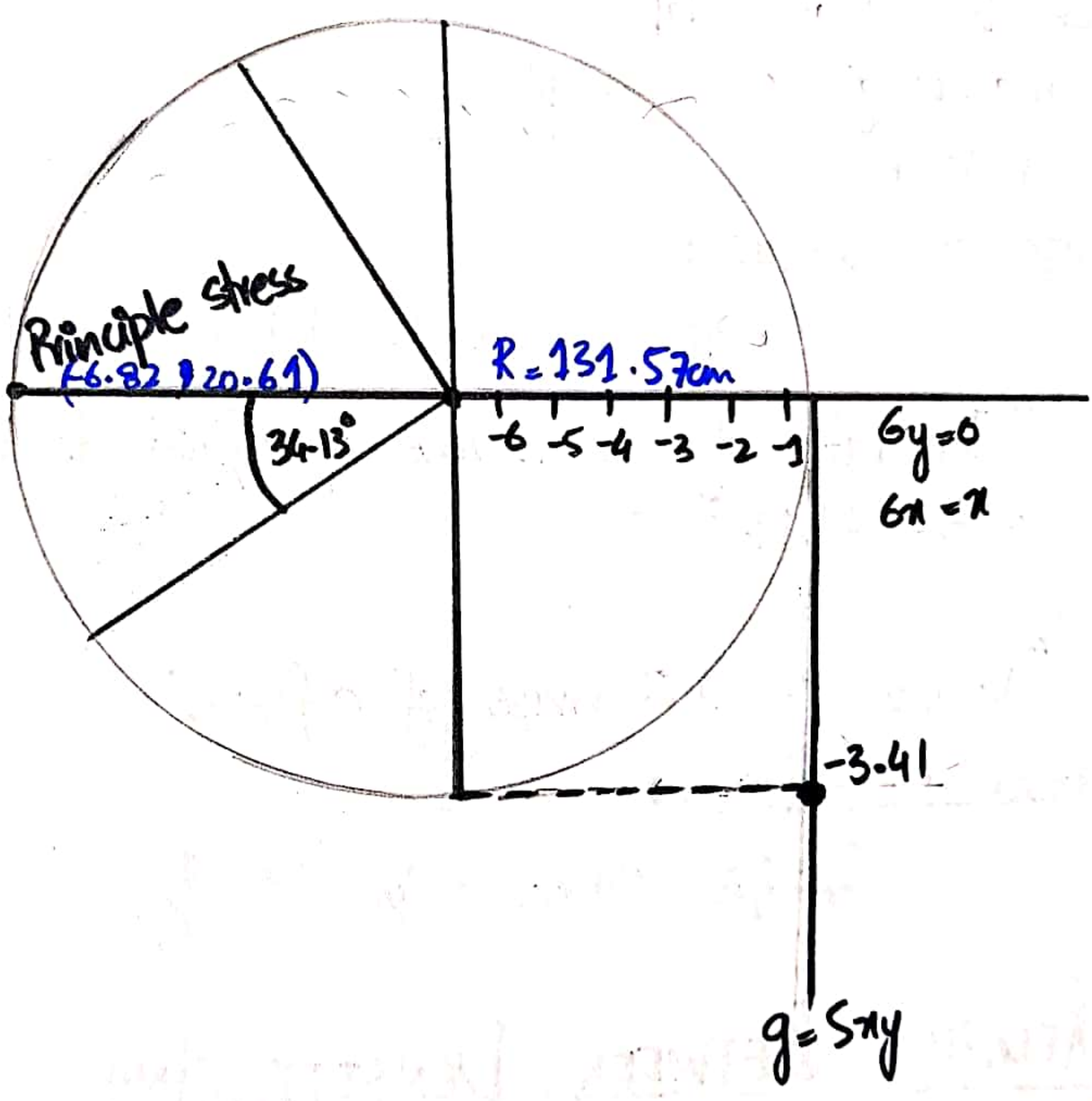
$\sigma_y' = 0$

Scale 1 psi = 2cm

Radius = $\frac{263.15}{2} = 131.575$

$\phi_p = \frac{68.27}{2}$

$\phi_p = 34.135^\circ$


Principle stress
(-6.82 , 20.61)
34.13°
R = 131.57cm
-6 -5 -4 -3 -2 -1
6y = 0
6x = x
-3.41
g = Sxy