P(1)

Name :- Salman Aklam
ID :- 16359
Linear Algebra
BS (SE) II Section B

QNo-1

$$x_1 - 3x_2 + x_3 = 0$$
$$0x_1 + 2x_2 - 8x_3 = 8$$
$$5x_1 + 0x_2 - 5x_3 = 10$$

Sol

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 10 \end{bmatrix}$$

$$A x = B$$
$$x = A^{-1}B$$

find $|A|$
if $|A| \neq 0$ the equation will be Consistent.

$$|A| = \begin{vmatrix} 1 & -3 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{vmatrix} \quad C_3 + C_1$$

$$= \begin{vmatrix} 1 & -3 & 2 \\ 0 & 2 & -8 \\ 5 & 0 & 0 \end{vmatrix} = 5\begin{vmatrix} -3 & 2 \\ 2 & -8 \end{vmatrix}$$

$$= 5(24-4) \Rightarrow 5(20) = 100$$

Ans So The $|A| \neq 0$
The equation are Consistent

P(3)

$a_{21} = (-1)^{2+1} \begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = -1(28+10) = -38$

$a_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = (1)(21-25) = -4$

$a_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = (-1)(-4+5) = -1$

$a_{31} = (-1)^{3+1} \begin{vmatrix} 4 & 5 \\ -1 & 5 \end{vmatrix} = (1)(20+5) = 25$

$a_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 5 \\ 2 & 5 \end{vmatrix} = (-1)(15-10) = -5$

$a_{33} = (-1)^{3+1} \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = (1)(4+5) = 9$

So the adj of A is

$$A_{adj} = \begin{bmatrix} 3 & 11 & -1 \\ -38 & -4 & -1 \\ 25 & -5 & 9 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$A^{-1} = \frac{1}{58} \begin{bmatrix} 3 & 11 & -1 \\ 38 & -4 & -1 \\ 25 & -5 & 9 \end{bmatrix}$$

Q.No 2 Find A^{-1} by adjoint Method

Sol:

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 5 \\ 5 & -2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$

$$|A| = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 5 \\ 5 & -2 & 7 \end{vmatrix} \text{ R2}$$

$$|A| = 3\begin{vmatrix} -1 & 5 \\ -2 & 7 \end{vmatrix} - 4\begin{vmatrix} 2 & 5 \\ 5 & 7 \end{vmatrix} + 5\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$$= 3(-7+10) - 4(14-25) + 5(-4+5)$$

$$= 3(3) - 4(11) + 5(1)$$

$$= 9 + 44 + 5 \Rightarrow |A| = 58$$

So inverse can exist

$$A_{adj} = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 5 \\ 5 & -2 & 7 \end{bmatrix}$$

$$a_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 5 \\ -2 & 7 \end{vmatrix} = 1(-7+10) = 3$$

$$a_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 5 \\ 5 & 7 \end{vmatrix} = (-1)(14-25) = 11$$

$$a_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = (1)(-4+5) = -1$$

P(4)

$$A^{-1} = \begin{bmatrix} \frac{3}{58} & \frac{4}{58} & -\frac{1}{58} \\ \frac{38}{58} & -\frac{4}{58} & -\frac{1}{58} \\ \frac{25}{58} & -\frac{5}{58} & \frac{9}{58} \end{bmatrix}$$

$$\begin{array}{r} 27 \\ -14 \\ \hline 13 \end{array}$$

P(5)

QNO·3 Solve the following Systems of linear equations by Gauss-Jordan Method.

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Sol

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & -3 & 14 \end{bmatrix} \xrightarrow{R_1 = \frac{1}{2}R_1} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & -3 & 14 \end{bmatrix}$$

$$\xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -6 & 13 \end{bmatrix} \xrightarrow{R_2 = \frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -6 & -13 \end{bmatrix}$$

$$R_3 = R_3 + 2R_2 \quad \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -6 & -9 \end{bmatrix} \xrightarrow[R_3 = -\frac{1}{6}R_3]{R_1 = R_1 - R_2} \begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3/2 \end{bmatrix}$$

$$R_1 = R_1 - 2R_3 \quad \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3/2 \end{bmatrix}$$

$$x + 0y + 0z = 10$$
$$0x + y + 0z = 2$$
$$0x + 0y + z = -3/2$$

So

$$x = 10$$
$$y = 2$$
$$z = -3/2$$

P(6)

QNO.6

Reduce the matrix to Normal form and find its rank.

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 3 \end{bmatrix} \quad \begin{matrix} R_2 - 3R_1 \\ R_3 - R_1 \end{matrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & -6 \end{bmatrix} \quad R_2 \longleftrightarrow R_3$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 + R_2$$

This in Normal form.

So the non-zero Row are two
So the Rank are also (2)