Submitted Dated 30/10/2020

Online Final – Term Examination Summer Semester 2020

DISCRETE STRUCTURE

Total Marks :50

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ID = 6844

BS (SE) Section B

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Question No. 1: **(10)**

a) Find the 36th term of the arithmetic sequence whose 3rd term is 7 and 8[th] term is 17.

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Q1 -

SOLUTION :→

Let "a" be The first term & "b" be The common difference of the arithmetic sequence.

$$a_n = a + (n-1) b \quad n \geq 1$$

$$\rightarrow \quad a_3 = a + (3-1) b$$

and = $a_8 = a + (8-1) b$

Given that

$$a_3 = 7 \quad \text{and} \quad a_8 = 17.$$

There fore

$$7 = a + 2b \quad \ldots\ldots\ldots (1)$$

$$17 = a + 7b \quad \ldots\ldots\ldots (2)$$

Subtracting (1) from (2), we get,

$$= 10 = 5b$$

$$= b = 10/5$$

$$= b = 2$$

Subtracting b=2 in (1) we have

$$7 = a + 2(2)$$

$$\frac{7}{4} = \frac{a4}{4} = a = 3$$

Thus

$$a_n = a + (n-1)b$$

$$a_n = 3 + (n-1) 2 \quad \text{(using values of a & b)}$$

Hence The value of 36th term is

$$a_{36} = 3 + (36-1) 2$$

$$= 3 + 70$$

$$\boxed{a_{36} = 73}$$ Answer

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Question No. 2: **(10)**

Find ***fog*(x)** and ***gof(x)*** of the functions f(x) = 2*x* + 3 and *g*(*x*) = –*x*² + 5

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Q 2 :→

SOLUTION :→

Given data

$f(x) = 2x + 3$

$g(x) = -x^2 + 5$

$fog(x) = ?$

$gof(x) = ?$

first we find $fog(x) = ?$

$g(x) = -x^2 + 5 \quad , \quad f(x) = 2x + 3$

$fog(x) = f(g(x))$

$fog(x) = f(-x^2 + 5)$

$= 2 . (-x^2 + 5) + 3$

$= -2x^2 + 10 + 3$

$= -2x^2 + 13$

$\boxed{fog(x) = -2x^2 + 13}$ —— $fog(x)$

Thus, we defined $gof(x)$ ..... ?

$f(x) = 2x + 3 , \quad g(x) = -x^2 + 5$

$gof(x) = g(f(x))$

$gof(x) = g(2x + 3)$

$= -(2x + 3)^2 + 5$

$= -(4x^2 + 12x + 9) + 5$

$= -4x^2 - 12x - 4$

so

$\boxed{gof(x) = -4x^2 - 12x - 4} \longrightarrow gof(x)$

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Question No. 3: **(10)**

Prove by mathematical induction that the statement is true for all integers n ≥ 1

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

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Q 3 :→

Proof :→

For $n=1$, The statement reduces to

$1^2 = \frac{1 \cdot 2 \cdot 3}{6}$ & is obviously true.

Assuming the statement is true for $n=k$:

$$1^2 + 2^2 + 3^2 + \ldots k^2 = \frac{k(k+1)(2k+1)}{6}, \longrightarrow (1)$$

we will Prove That The statement must be true for $n = k+1$:

$$1^2 + 2^2 + 3^2 + \ldots\ldots + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6} \longrightarrow (2)$$

The Left-hand side of (2) can be written as

$$1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2$$

In view of (1), This Simplifies to :→

$$(1^2 + 2^2 + 3^2 + \cdots k^2) + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}$$

$$= \frac{(k+1)(2k^2 + 7k + 6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

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Thus The Left-hand side (2) is equal to the right-hand side of (2).
This proves the inductive step.
There fore, by the principle of mathematical Induction, The given statement is true for every positive Integer n.

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Question No. 4: **(10)**

Discuss different types of relations with example in detail.

Answer:

Different Types of Relations

There are 8 main types of relations which include:

- Empty Relation
- Universal Relation
- Identity Relation
- Inverse Relation
- Reflexive Relation
- Symmetric Relation
- Transitive Relation
- Equivalence Relation

1. Empty Relation

An empty relation (or void relation) is one in which there is no relation between any elements of a set.

For example, if set A = {1, 2, 3} then, one of the void relations can be R = {x, y} where, $|x - y| = 8$. For empty relation,

$R = \phi \subset A \times A$

2. Universal Relation

A universal (or full relation) is a type of relation in which every element of a set is related to each other.

For example if a set A = {a, b, c}. Now one of the universal relations will be R = {x, y} where, $|x - y| \geq 0$. For universal relation, **$R = A \times A$**

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3. Identity Relation

In an identity relation, every element of a set is related to itself only.

For example, in a set A = {a, b, c}, the identity relation will be

I = {a, a}, {b, b}, {c, c}. For identity relation,

I = {(a, a), a ∈ A}

4. Inverse Relation

Inverse relation is seen when a set has elements which are inverse pairs of another set.

For example if set A = {(a, b), (c, d)}, then inverse relation will be

R-1 = {(b, a), (d, c)}. So, for an inverse relation,

R-1 = {(b, a): (a, b) ∈ R}

5. Reflexive Relation

In a reflexive relation, every element maps to itself.

For example, consider a set A = {1, 2,}. Now an example of reflexive relation will be

R = {(1, 1), (2, 2), (1, 2), (2, 1)}. The reflexive relation is given by- **(a, a) ∈ R**

6. Symmetric Relation

In a symmetric relation, if a=b is true then b=a is also true. In other words, a relation R is symmetric only if (b, a) ∈ R is true when (a,b) ∈ R.

An example of symmetric relation will be

R = {(1, 2), (2, 1)} for a set A = {1, 2}. So, for a symmetric relation,

aRb ⇒ bRa, ∀ a, b ∈ A

7. Transitive Relation

A relation in a set A is transitive if, (a, b) ∈ R, (b, c) ∈ R, then (a, c) ∈ R, for all a, b, c ∈ A

8. Equivalence Relation

A relation is said to be equivalence if and only if it is Reflexive, Symmetric, and Transitive.

For example, if we throw two dices A & B and note down all the possible outcome.

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Question No. 5 **(10)**

Suppose that an automobile license plate has three letters followed by three digits.

a. How many different license plates are possible?

b. How many license plates could begin with A and end on 0.

c. How many license plates begin with PQR

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Q 5 :→

SOLUTION :→

26 Letter (a, b, c, ---- z)

10 digits (0, 1, 2, - - ----9)

(A) Number of Possible licence Plates

License plates have three letters followed by three Digits.

$$= \underbrace{26 \times 26 \times 26}_{\text{At each Position any of The 26 letters Can be Placed.}} \times \underbrace{10 \times 10 \times 10}_{\text{At each of These Position any of The 10 digits Can be Placed.}}$$

Hence :– Number of Possible License Plates.

$$\boxed{= 17{,}576{,}000}$$

(b) Number of License plates begining with A end In 0. ~~which all the letters of digits are~~

$$\underset{A}{\overset{1}{1}} \times \overset{26}{26} \times \overset{26}{26} \times \overset{10}{10} \times \overset{10}{10} \times \underset{0}{\overset{1}{1}}$$

At first Position we Can place only A

At Sixth Position we Can Place only 0

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2nd Position can be filled by remaining 26 letters.
3rd Position can be filled by remain 26 letters.
4th Position can be filled by remain 10 digits.
5th Position can be filled by remaining 10 digits.

so
Number of Such License plates

= ~~13,200~~
= $\boxed{67,600}$

(c)
A license place contains 3 Letters followed by 3 digits.
while The Licence plate starts with PQR

First Letter : 1 way (needs to be P)
Second " : 1 way (" " " Q)
third Letter : 1 way (needs to be R)
First digit : 10 way (as There are 10 possible digits)
Second digit : 10 way = (")
Third digit : 10 way = (")

Use The multiplication rule:

$$\underset{P}{1} \times \underset{Q}{1} + \underset{R}{1} \times 10 \times 10 \times 10$$

$$= 10^3 = \boxed{1,000}$$

Thus
There are 1000 Possible licence Plates begining with PQR.

Completed.