ID 14226

AKMAL AFRIDI

Q1 a)

Non Homogenous equation

$$y'' + p(t)\,y' + q(t)\,y = g^t$$

If $g^t = 0$ then

Homogenous equation.

$$y'' + p(t)\,y' + q^{(t)}y = 0$$

Example:-

i) $y'' + 4y' + 5y = 0$

Sol:- $k^2 - 4k + 5 = 0$

$\Rightarrow D = 16 - 4.5 = -4$

$\Rightarrow k_{1,2} = \frac{4 \pm \sqrt{-4}}{2}$

$\Rightarrow k_{1,2} = 4 \pm \sqrt{-4}/2$

$\Rightarrow k_{1,2} = 2 \pm i$

Thus

$y(x) = e^{2x}[c_1 \cos x + c_2 \sin x)$

c_1 & c_2 are artibury constant

Example 2 $y'' + 9y = 2x^2 - 5$

Sol $k^2 + 9 = 0 \Rightarrow k^2 = -9$

$k_{1,2} = \pm 3i$

$y_0(x) = c_1 \cos 3x + c_2 \sin 3x$

$y_1' = Ax^2 + Bx + C$

$y_1' = 2Ax, \quad y_1'' = 2A$

$2A + 9(Ax^2 + Bx + C) = 2x^2 - 5$

$\Rightarrow 2A + 9Ax^2 + 9Bx + 9C = 2x^2 + 0 - 5$

$$\begin{cases} 9A = 2 \\ 9B = 0 \\ 2A + 9C = -5 \end{cases} \Rightarrow \begin{cases} A = 9/2 \\ B = 0 \\ C = -49/81 \end{cases}$$

$$y_1 = \frac{2}{9}x^2 - \frac{49}{81}$$

$$y = y_0 + y_1 = c_1 \cos 3x + c_2 \sin 3x + \frac{2x^2}{9} - \frac{49}{81}$$

Ans

AKMAL AFRIDI

Q1B(i) $16y'' + 24y' + 9y = 0$

Solution The characteristics equation & root one given as

$16y^2 + 24y' + 9y = 0$

Second order linear homogenous differential equation with Constant Co-efficient.

$ay'' + by' = 0$, assume the solution form e^{yt}

$16((e^{yt})'' + 24((e^{yt}))' + 9e^{yt} = 0$

Simplify $16((e^{yt}))'' + 24((e^{yt}))' + 9e^{yt} = 0$:

$e^{yt}(16y^2 + 24y + 9) = 0$

$e^{yt}(16y^2 + 24y - 9) = 0$

Solve $e^{yt}(16y^2 + 24y + 9) = 0$: $y = -3/4$ with multiplicity

$y = -3/4$ with $\times 2$

For one of the value y, the general Solution takes the form $y = C_1 e^{ft} - C_2 t e^{ft}$

$C_1 e^{-3/4t} - C_2 t e^{-3/2t}$

Refine

$y = C_1 e^{-3/4t} - C_2 e^{-3t/4t}$

Graph

Plotting $C_1 e^{-3/4} - C_2 e^{-3t/4}$

assuming $C_1 = 1$ $C_2 = 2$

equation
The characteristic $y - 4y - 12 = 0$
the equestion of
the given differentil equation
is $m^2 - 4m - 12 = 0$.
Evaluate the roots of the
equation as fallows.
$m^2 - 4m - 12 = 0$
$m^2 - 6m + 2m - 12 = 0$
$(m-6)(m+2) = 0$
$m = 6$ $m_2 = -2$
Therefore the roots the eqaation
$m = 6$ $m_2 = 2$
Since the roots of the
characteristic eaqtion are real
and distinc the solution
must be in from of $y = c$
Therefore the solution of
the homogeneous pare of
the equation is $C_1 e + C_2 e^{-2x}$

Page 5

To find the particular solution let $y = pe^{5x}$ be the solution.

Substitute the solution $y = pe^{5x}$ in the equation as fol

$(Pe^{5x})'' - 4(Pe^{5x})' - 12(Pe^{5x}) = 3e^{5x}$

$25pe^{5x} - 20pe^{5x} - 12pe^{5x} = 3e^{5x}$

$-7pe^{5x} = 3e^{5x}$

$P = -\frac{3}{7}$

Thus the chosen solution becomes $y = -\frac{3}{7}e^{5x}$

Evalate the general solution is follows.

$y = y_h + y_p$

$y = c_1e + c_2e^{-2} - \frac{3}{7}e^{5x}$

Therefore the solution of the given differential equation is $y = c_1e^{6x} + c_2$

Part (1) As per bartlary guidelines for more then 1 question asked only first please upload other separatey.

Given that,

$2y'' + 5y' + 3y = 0$ with the initial condition $y(0) = 3, y'(0) = -4$

$2m^2 +$

The auaxiliary equation of the given equation is

$2m^2 + 5m + 3 = 0$

Soving the equation by the formula.

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

step 2

~~Part (2)~~

Therefore,

$$m = \frac{-5 \pm \sqrt{25 - 4 \times 6}}{4}$$

$$= \frac{-5 \pm \sqrt{1}}{4}$$

$$= \frac{-5 \pm 1}{4}$$

So, $m_1 = -1$ or $m_2 = \frac{-3}{2}$

Therefore the solution is
given by.

$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

That is $y = c_1 e^{-x} + c_2 e^{-\frac{3}{2}x}$

Part (b) step 3

Applying the intial conditions

$y(0) = 3,\ y(0) = 4$

$y'(x) = -c_1 e^{-x} - \frac{3}{2} c_2 e^{-\frac{3}{2}x}$

$y'(0) = 4 \Rightarrow -c_1 - \frac{3}{2} c_2 = 4 \quad (2)$

$y'' - 4y' + 9y = 0 \qquad y(0) = 0 \quad y($

Sol:-

$$y'' - 4y' + 9y = 0$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 9y = 0 \Rightarrow (D^2 - 4$$

$$m^2 - 4m + 9 = 0$$

$$m = 2 + \sqrt{5} \; ; \; m = 2 - \sqrt{5}$$

$$m = 2 + \sqrt{5}i, \; \cancel{m = 2 - \sqrt{5}}$$

$$y = e^{2x}(A\cos\sqrt{5}x + B\sin\sqrt{5}x$$

When $x = 0$, $y = 0$, $A = 0$;

Since: $y = e^{2x}(A\cos\sqrt{5}x + B$

$dy/dx = 2e^{2x}(A\cos\sqrt{5}x + B\sqrt{5}\sin\sqrt{5}$

$$g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$$

Solution

$$g(t) = 4\cos(4t) - 9\,\mathrm{Sin}(4t) + 2\cos(10t)$$

By using Laplace formula

$$g(t) = 4\left(\frac{3}{s^2+4^2}\right) - 9\left(\frac{4}{s^2+4^2}\right) + 2\left(\frac{2}{s^2+10^2}\right)$$

$$= \frac{4s}{s^2+16} - \cancel{65}\,\frac{36}{s^2+16} + \frac{2s}{s+100}$$

$$\longleftrightarrow$$

$$f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$$

10

Q3 $y(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$

Part3

$\mathcal{L}\{e^{3t} + \cos(6t) - e^{3t}\cos(6t)\}$

$= L\{e^{3t}\} + L\{\cos(6t)\} - L\{e^{3t}\cos 6t\}$

$= \dfrac{1}{s-3} + \dfrac{s}{s^2+36} + \dfrac{s-3}{(s-3)^2+36}$

$\Leftrightarrow$

Laplace transform:-

Laplace transform is an integral transformation that converts a function of a real variable often time to a function of a complex variables s. The transformation has many applications in science & engineering because it is a tool for solving differential equation. In particular it performs a transformation equation in to algebric equations & convolution in to multiplication

for example.

let $f(t)=1$ when $t \geq 0$

find $f($

Sol:-

$\mathcal{L}(1) = \mathcal{L}(1) \int e^{-st} dt = -\frac{1}{s} e^{-st}$

Such as integral is called as Improper integral & by definition is evaluated to the rule.

$$\int_0 e^{-st} f(t) dt) = \lim_{T \to 0} \int e^{-st} f(t)$$

Q49 i) $y'' - 4y' = e^{(3t)}$ $y(0) = 0,\ y'(0) = 0$

Solution :-

Take

$y(t) \to Y(s)$

$y'(t) \to sY(s) - y(0)$

$y''(t) \to s^2Y(s) - sy(0) - y'(0)$

Now taking the laplace equation for the $y'' - 4y' - e^{3t}$

$s^2Y(s) - sY(0) - Y'(0) - 4sY(s) + 4y(0) = \frac{1}{s-3}$

$s^2Y(s) - 4sY(s) = \frac{1}{s-3}$

$Y(s)(s^2 - 4s) = \frac{1}{s-3}$

$Y(s) = \frac{1}{s(s-4)(s-3)}$

Hence

$\frac{1}{s(s-4)(s-3)} = \frac{A}{s} + \frac{B}{s-4} + \frac{C}{s-3}$

Takes $s = 0$

$1 = A(-4)(-3)$

$A = 1/12$

Takes $s = 4$

$1 = B \times 4 \times 1$

$B = 1/4$

Takes $s = 3$

$1 = C \times 3 \times -1$

$C = -1/3$

Hence

$$Y(s) = 1/12 \frac{1}{s} + \frac{1}{4} \cdot \frac{1}{s-4} - \frac{1}{3} \cdot \frac{1}{s-3}$$

$$s^2 Y(s) + 3(sY(s)) + 2Y(s) = \frac{1}{s+1}$$

$$Y(s)[s^2 + 3s + 2] = \frac{1}{s+1}$$

$$Y(s) = \frac{1}{s+1\,[s^2+3s+2]}$$

$$= \frac{1}{(s+1)^2(s+2)}$$

Now find the Partial fraction of this fraction.

$$Y(s) = -\frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{s+2}$$

Since we have to terms of $s+1$. so take one with t multiple & find

$$y(t) = -e^{-1} + te^{-1} + e^{-2t}$$

Q4

Part B Solution:-

Consider the give eq & initial values

$$y'' + 3y' + 2y = e^{-t} \text{ \& } y(0) = 0, y'(0) = 0$$

Take the laplace transformation

$$L(y'') + 3L(y') + 2L(y) = L(e^{-t})$$

$$s^2 Y(s) - sy(0) - y'(0) + 3(sy(s) - y(0)) + 2y(s) = \frac{1}{s+1}$$

Put the initial value

further Simplify

$$s^2 Y(s) + 3(sY(s) + 2y(s) = \frac{1}{s+1}$$

$$Y(s) \cdot [s^2 + 3s + 2] = \frac{1}{s+1}$$

$$Y(s) = \frac{1}{s+1(s^2+3s+2)}$$

$$= \frac{1}{(s+1)^2 (s+2)}$$

Now find the Partial fraction of the transform

$$Y(s) = \frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{s+2}$$

Since we have to terms $s+1$

$$y(t) = -e^{-1} + te^{-1} + e^{-2t}$$

$$\frac{1}{s(s-4)(s-3)} = \frac{A}{s} + \frac{B}{s-4} + \frac{C}{s-3}$$

Takes $s=0$

$1 = A(-4)(-3)$

$A = 1/12$

Takes $s=4$

$1 = B \times 4 \times 1$

Takes $(s=3$

$1 = c \times 3 \times -1$

$c = -1/3$

Hence

$$y(s) = \frac{1}{12}\frac{1}{s} + \frac{1}{4}\frac{1}{s-4} - \frac{1}{3}\frac{1}{s-3}$$

IVP Solution.