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Subject : CALCULUS

(1)

Q#01)

a) differentiate $\frac{3x^3 - 5x^2 + 5}{(x^2 + 1)}$ w.r.t "x"

By using Quotient Rule

$$y = \frac{3x^3 - 5x^2 + 5}{(x^2 + 1)}$$

$$y = \frac{(x^2 + 1) \frac{d}{dx} (3x^3 - 5x^2 + 5) - (3x^3 - 5x^2 + 5) \frac{d}{dx} (x^2 + 1)}{(x^2 + 1)^2}$$

$$y = \frac{(x^2 + 1) \cdot 9x^2 - 10x - 3x^3 + 5x^2 - 5(2x)}{(x^2 + 1)^2}$$

$$y = \frac{9x^4 - 10x^3 + 9x^2 - 10x - 6x^4 + 10x^3 - 10x}{(x^2 + 1)^2}$$

$$y = \frac{9x^4 - 6x^4 - 10x^3 + 10x^3 + 9x^2 - 10x}{(x^2 + 1)^2}$$

$$y = \frac{3x^4 - 10x^3 + 10x^3 + 9x^2 - 10x}{(x^2 + 1)^2}$$

$$y = \frac{3x^4 + 9x^2 - 10x}{(x^2 + 1)^2} \quad \text{ANS}$$

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Q #01

b) differentiate $(x^2+1)^2$ w.r.t " x "

$$x^2-1$$

$$y = \frac{(x^2+1)^2}{(x^2-1)}$$

$$y = \frac{(x^2+1)(x^2-1)}{(x^2-1)}$$

$$(x^2-1)$$

$$y = \frac{(x^2+1)(x^2-1)}{(x^2-1)}$$

$$y = \frac{d}{dx} (x^2+1)$$

$$= 2x + 0$$

$$= \boxed{2x} \text{ ANS}$$

- (3)

Q#2

a)

$$y = (1 + 2\sqrt{x})^3 x^{2/3}$$

using chain Rule

$$y = (x + 2\sqrt{x})^3 x^{2/3}$$

$$y = (x + 2\sqrt{x x})^2$$

$$y = (x + 2x^{3/2})^2$$

Let $y = u^2$

where

$$u = x + 2x^{3/2}$$

$$\frac{du}{dx} = 1 + 3x^{1/2}$$

$$\boxed{du = (1 + 3\sqrt{x} dx)}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \quad \text{--- (A)}$$

chain Rule

(4)

$$\frac{dy}{dx} = (20) (1+3\sqrt{x} dx)$$

$$\frac{dy}{dx} = 2(x+2x^{3/2})^2 (1+3\sqrt{x})$$

ANS

(5)

Q#02

$$y = \sqrt{\frac{1-x}{1+x}}$$

$$y = \sqrt{(1-x)(1-x)}$$

$$(1-x)(1-x)$$

$$y = \frac{1-x}{\sqrt{1-x^2}} \quad (1)$$

$$\frac{dy}{dx} = \sqrt{1-x^2} \frac{d}{dx} (1-x) \cdot (1-x) \frac{d}{dx} (\sqrt{1-x^2})$$

$$\sqrt{1-x^2}^2$$

$$\frac{dy}{dx} = -\sqrt{1-x^2} + (1-x) \frac{x}{\sqrt{1-x^2}}$$

$$(1-x^2)$$

$$\frac{dy}{dx} = -\sqrt{1-x^2} + (1-x) \frac{x}{\sqrt{1-x^2}}$$

$$(1-x^2)$$

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$$\frac{dy}{dx} = \frac{-\sqrt{1-x^2} + (1-x) \cdot x}{\sqrt{1-x^2}}$$

$$(1-x^2)$$

$$(1-x^2) \frac{dy}{dx} = \frac{-1+x^2+x-x^2}{\sqrt{1-x^2}}$$

$$(1-x^2) \frac{dy}{dx} = \frac{-(1-x)}{\sqrt{1-x^2}}$$

$$(1-x^2) \frac{dy}{dx} = -y$$

$$\boxed{(1-x^2) \frac{dy}{dx} + y = 0} \quad \text{Ans}$$

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Q#03

a) Find Integration of $\int \frac{1}{\sqrt{x^3}} dx$

$$y = \int \frac{1}{\sqrt{x^3}} dx$$

$$= \int x^{-3/2} dx$$

$$= \int \frac{x^{-3/2+1}}{-3/2+1}$$

$$y = \frac{x^{-1/2}}{-1/2}$$

Hence

$$y = \frac{x^{-1/2}}{-1/2}$$

Ans

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Q#3

b)

$$\int \frac{1}{(6x+7)^6} dx \quad \text{--- (A)}$$

Soln:

let

$$6x+7 = y \quad \text{--- (1)}$$

$$1 dx = dy$$

putting (2)(1) in (A) we get

$$= \int \frac{1}{(y)^6} dy$$

$$= \int y^{-6} dy$$

$$= \frac{y^{-5}}{-5} + C$$

$$= -\frac{1}{5y^5} + C = \boxed{-\frac{1}{5} \left(\frac{1}{(6x+7)^5} \right) + C}$$

ANS