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I'D : 7769

Sec : A

Subject : Intro to structural dynamic and earthquicke engg

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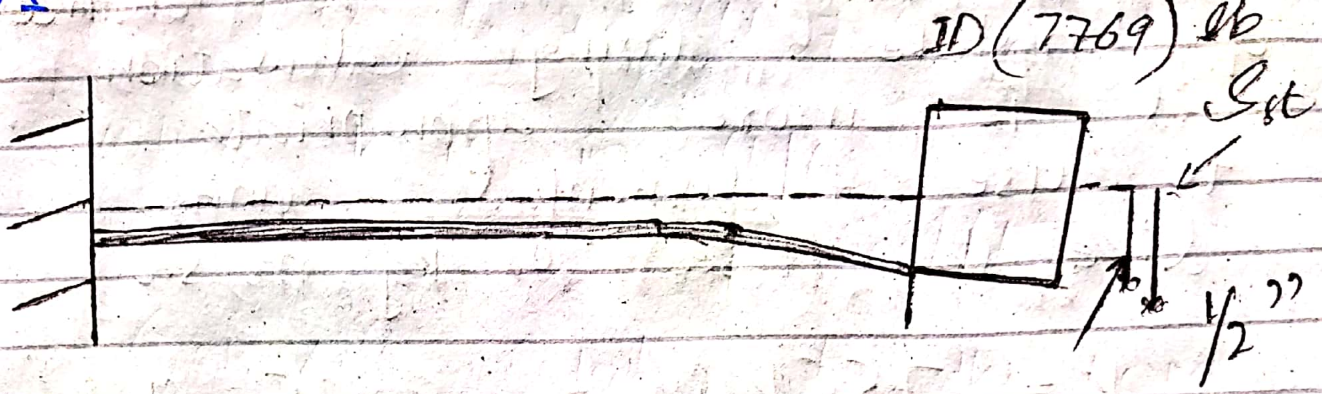
Date : 29. 06. 020

Q No 1



Sol ::

The general E.O.M For SDOF system is

$$Ku + C\dot{u} + m\ddot{u} = P(t)$$

In our case system is undamped $(C=0)$ undergoing free vibration $(p(t)=0)$

Hence general EOM becomes

$$Ku + m\ddot{u} = 0$$

$$K = 3EI/L^3$$

$$K = \frac{3 \times 29000 \ k/in^2 \times 150 \ in^4}{(10 \times 12 \ in)^3}$$

$$K = 7.55 \ k/in$$

In order to eliminate the chances of mistake during calculation it is more appropriate to use fundamental units like lb, ft sec or kg, m, sec

$K = 7.55 \, k/in = 90625 \, lb/ft$

$m = \frac{7769 \, lb}{32.2 \, ft/sec^2}$

$$\boxed{m = 241.27 \text{ slug}}$$

$\omega_n = \sqrt{K/m}$

$\omega_n = \sqrt{\frac{90625}{241.27}}$

$\omega_n = 19.39 \text{ rad/sec}$

$T_n = 2\pi/\omega_n = 2\pi/19.39$

$$\boxed{T_n = 0.325 \text{ sec}}$$

Substituting the corresponding value in eq ①

$90625\, u + 241.27\, \ddot{u} = 0$

Where k is in lb/ft
and m is in lb · sec/ft^2

General solution to the EOM For undamped free vibration is

$u(t) = u(0) \cos(\omega_n t) + \dot{u}(0)/\omega_n \sin(\omega_n t)$

$u(0) = 1/2'' = 1/24$ ft in $\dot{u}(0) = 0$

$u(t) = ((1/24) \times \cos(19.39t) + 0)$

$u(t) = (1/24) \times \cos(19.39t)$

Equivalent static force at any time t is

$F_s(t) = k \cdot u(t)$

$F_s(t) \quad \dfrac{90625 \times \cos(19.39\, ft)}{24}$

$3561.86 \cos\ 19.39\, t)$

Amplitude of dynamic displacement ω for undamped free vibration

$$u_o = \sqrt{(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2}$$

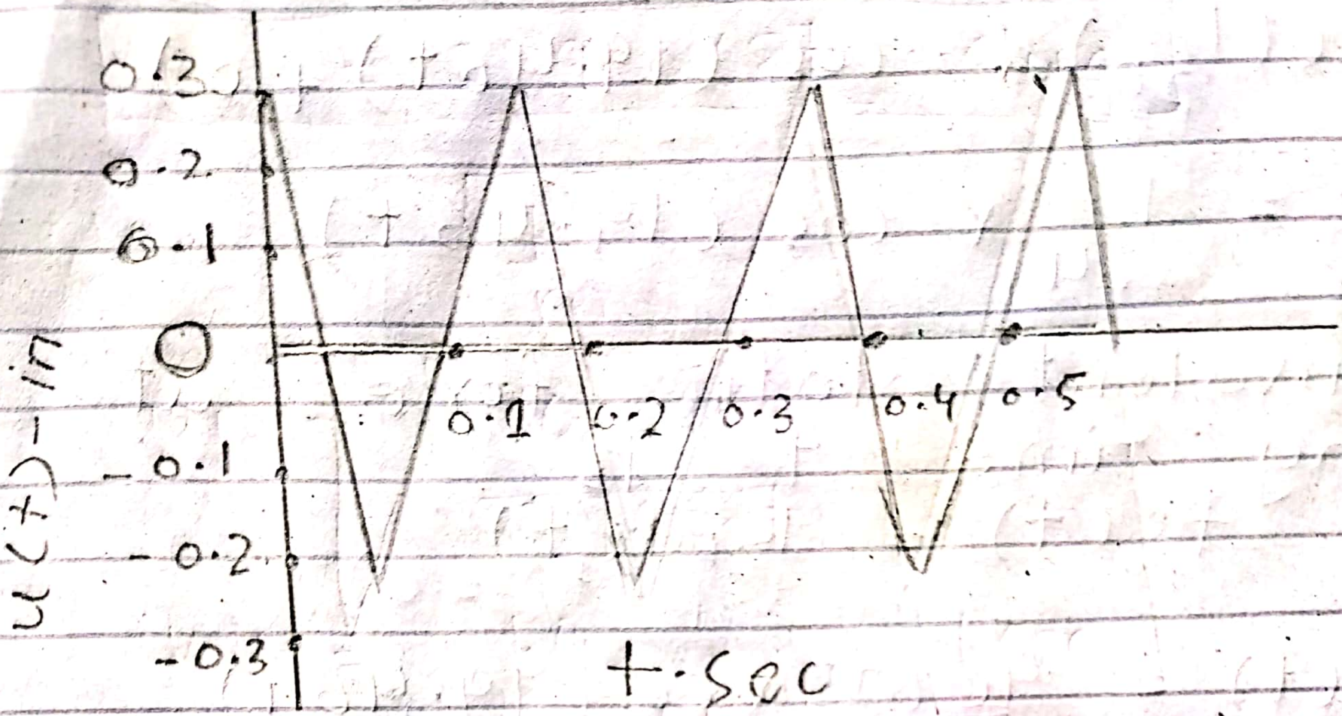
$$u_o = \sqrt{(1/24)^2 + 0}$$

$$u_o = 1/24 \text{ ft}$$

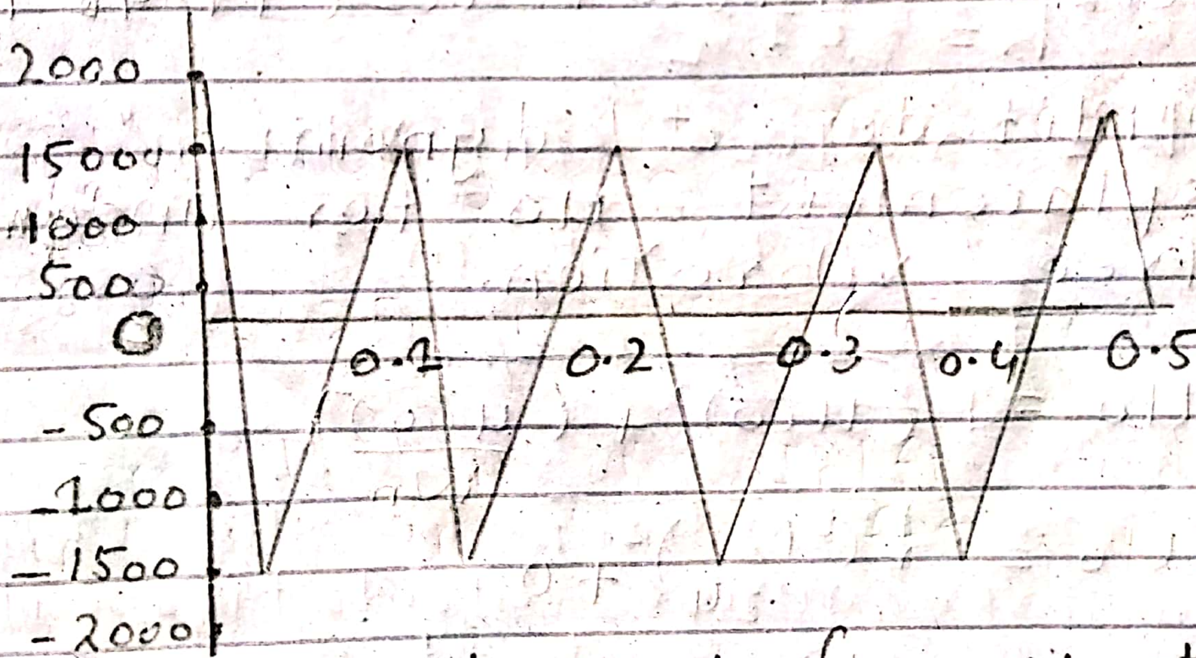
Aplitude of equivalent static force

$$k u_o = 90625 \times 1/24$$

$$k u_o = 3776.04 \text{ lb}$$

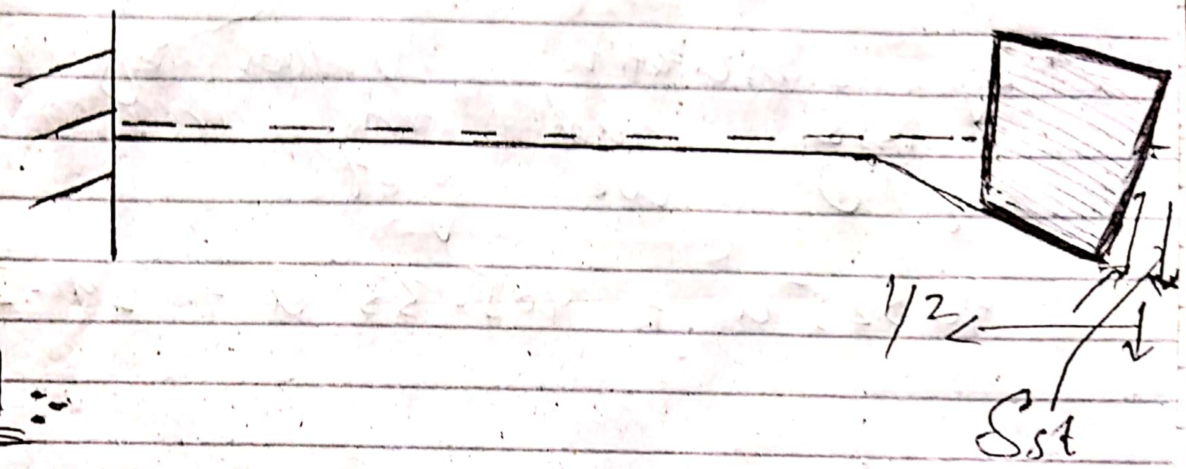


Undamped Dorce vibration



Undamped free vibration

Q no 2



Sol:-

E.O.M for damped free vibration is,

$ku + c\dot{u} + m\ddot{u} = 0$ ...... (i)

It is known from (qu 1)

$k = 90625$ lb/ft and
$m = 241.27$ slug
$\omega_n = 19.39$

$\Rightarrow$ $c = \zeta \times 2m\omega_n$

$= 2 \times 241.27 \times 19.39 \times \zeta$

$\zeta = 0.03 - 0.05$ with considerable cracking the damping ratio

$2 \times 241.27 \times 19.39 \times 0.05$

$C = 465.33$ lb sec/ft

* By substituting values of K, C and m in eq ① we get

$90.625 u + 465.33 \dot{u} + 241.27 \ddot{u} = 0$

- Solution to the Eqn for damped free vibration is;

$u(t) = e^{-\zeta \omega n t} \left(u(0) \cos(\omega_D t) + \frac{1}{\omega_D} (\dot{u}(0) + u(0) \zeta \omega n) \sin(\omega_D t) \right)$

$\omega_D = 19.39$ rad/sec

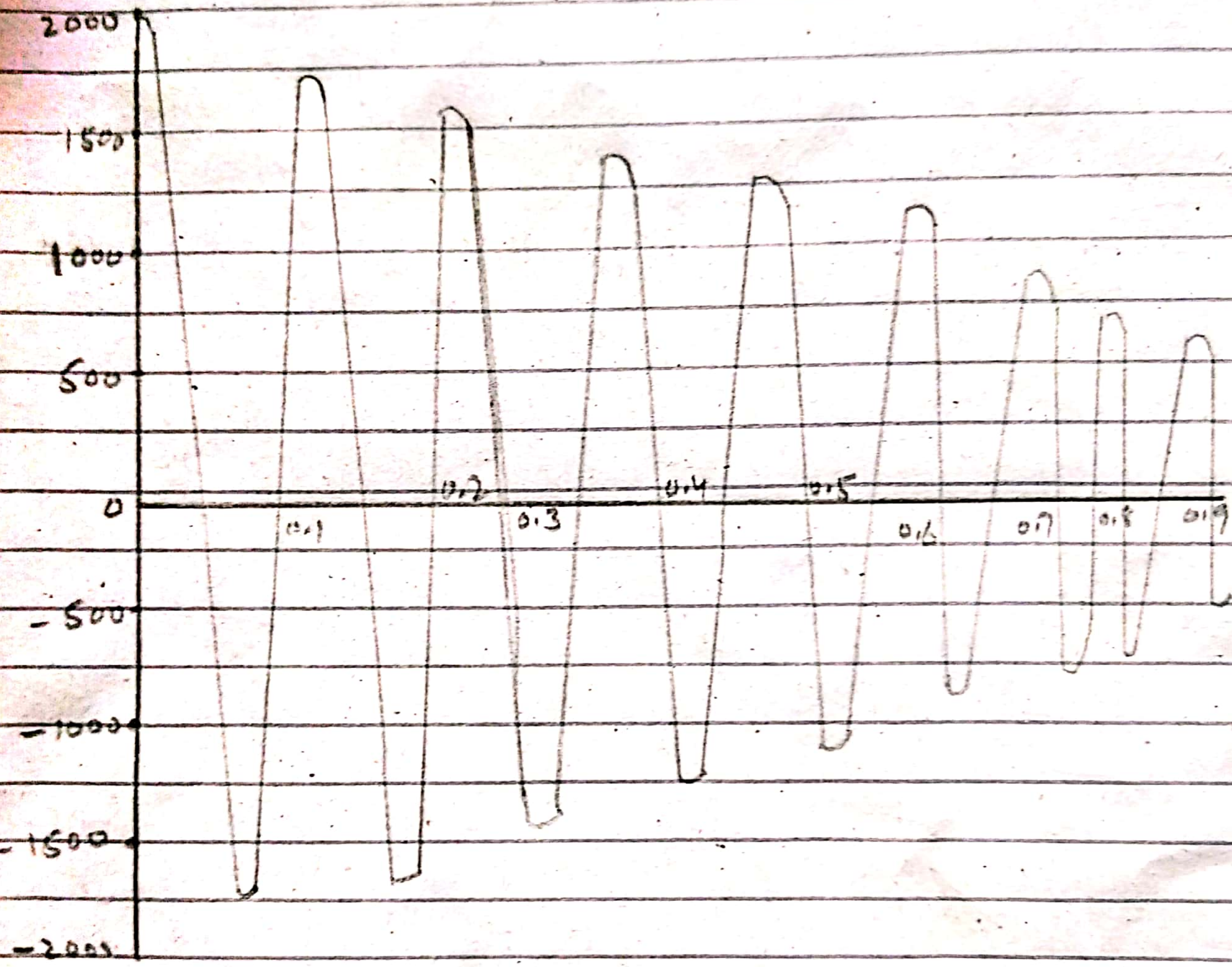
$u(t) = e^{-0.05 \times 19.395 t} \left(\frac{1}{24} \times \cos(19.39t) + \frac{1}{19.39} \times \left(0 + \frac{1}{24} \times 0.05 \times 19.39 \times \sin \times 19.39 t\right)\right)$

$u(t) = e^{-0.9695t} \left(0.0417 \times \cos(19.39t) + 0.052 \times (0.0404 \times \sin \times 19.39t) \right)$

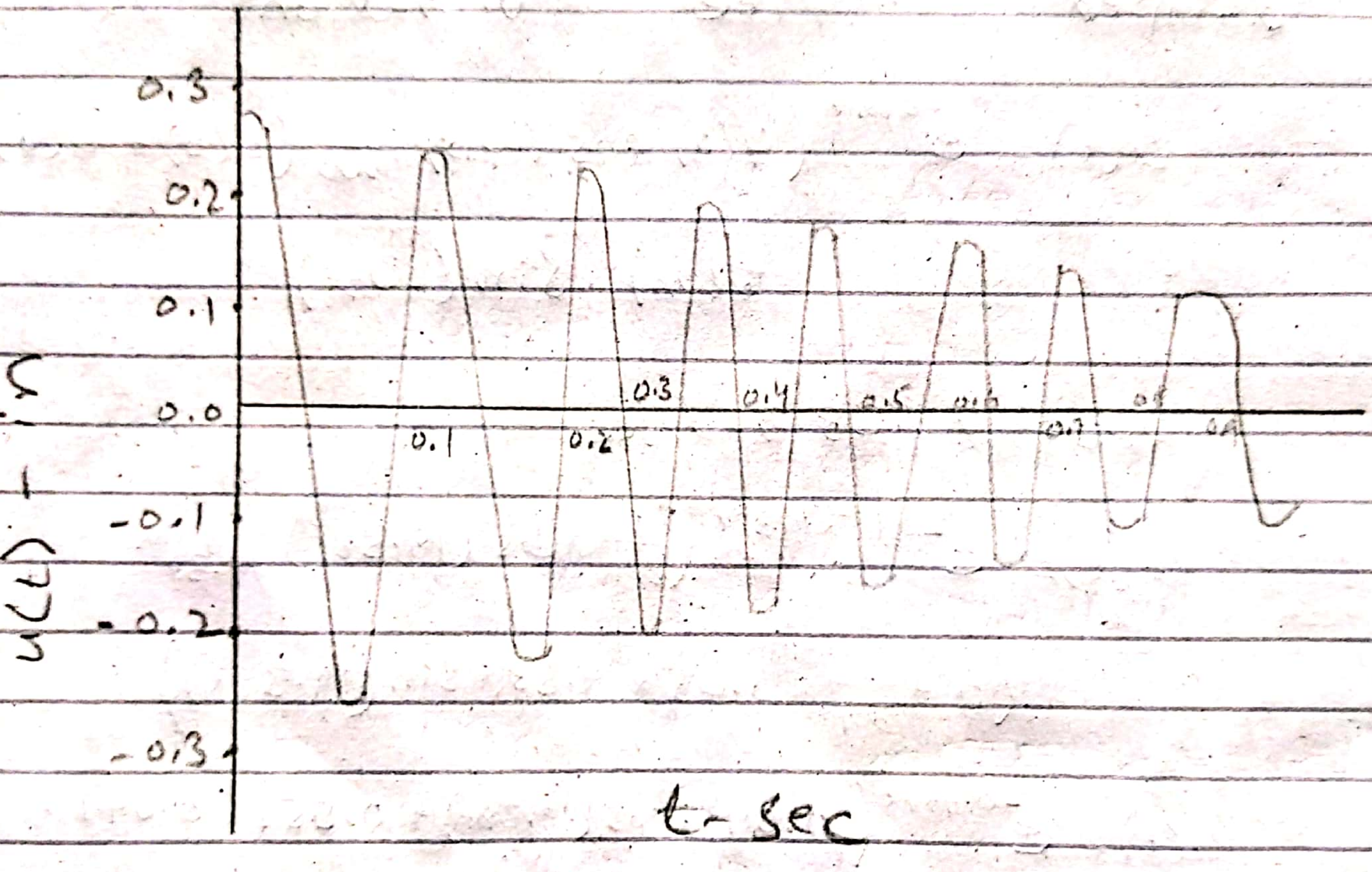
$$F_s(t) = k\,u(t) = 90625 \times u(t)$$

$$f_s(t) = e^{-0.9695t}\left(3779.1\cos(19.39t) + 190.4 \times \sin(19.39t)\right).$$

Damped Free vibration.



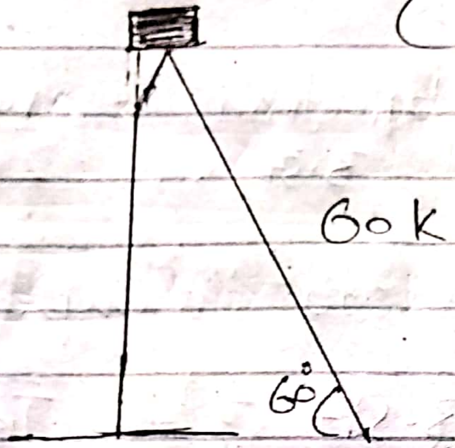
variation of equvalent static force with time.



variation of displacement with time.

QNo 3

$(7769/1000)$ in



Sol:

$u_1 = \frac{7769}{1000} = 7.769$ or $7.8''$

after $j = 7$, $u_{j+1} = u_8 = 2.286$ cm $= 0.9''$

a) ζ = Damping ratio

$j = \frac{1}{2\pi\zeta} \ln \left(\frac{u_1}{u_{j+1}}\right)$

$7 = \frac{1}{2\pi\zeta} \ln \left(\frac{7.8}{0.9}\right)$

$\boxed{\zeta = 0.049} = 4.9\%$

(b) $T_n = ?$

7 cycle of vibration are completed in 3.57 sec

Time required to complete one cycle

$= 3.57/7 = T_n$

$T_n = 0.51$ sec

Now

$\omega_0 = \omega_n \sqrt{(1-\zeta^2)}$

$2\pi/\omega_n = 2\pi / (\omega_n \sqrt{1-\zeta^2})$

$\Rightarrow T_n = \dfrac{T_n}{\sqrt{1-\zeta^2}}$

$T_n = T_0 \times \sqrt{1-\zeta^2}$

$T_n = 0.51 \times \sqrt{1-(0.048)^2}$

$T_n = 0.5094 = 0.51$ sec

(c) $K = ?$

$$k = \frac{60 \times \cos 60^\circ}{7.8} = 13.84 \, k/in$$

$$\boxed{K = 46080 \; lb/ft}$$

D ⟹ Weight of the tank $W = ?$

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{k}{(w/g)}} = \sqrt{\frac{k \times g}{w}}$$

$$\therefore \omega_n^2 = k \times g / w$$

$$w = k \times g / \omega_n^2$$

Also

$$\omega_n = 2\pi / T_n$$

$$w = kg \left(\frac{4\pi^2}{T_n^2} \right) = kg \times T_n^2 / 4\pi^2$$

$$W = \frac{46080 \times 32.2 \times (0.51)^2}{4\pi^2}$$

$$W = 9775.7 \; lb$$

$$\boxed{W = 9.77 \; k}$$

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e) $C = ?$

It is know that $\zeta = \frac{C}{2m\omega_n}$

$\Rightarrow C = \zeta \times 2m\omega_n = \zeta \times 2m \times (2\pi / T_n)$

$C = 0.049 \times 2 \times 2 \, (\pi / 0.51) \left(\frac{~~9478~~ 8.7}{32.2} \right)$

$\Rightarrow \boxed{C = 366.54}$ lb / sec ft

(f) Number of cycle to reduce displacement amplitude from 7.8 in to 0.5", $J = ?$

$j = \frac{1}{2\pi\zeta} \ln \left(\frac{u_1}{u_{j+1}} \right)$

$\Rightarrow j = \frac{1}{2 \times \pi (0.049)} \ln \left(\frac{7.8}{0.5} \right)$

$\boxed{J = 8.92 \text{ or } 9}$ cycle