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Assignment

Question #1

Determine the Response $y(n), n \geq 0$ of the System described by the Second-order difference equation

$$y[n] - 3y(n-1) - 4y(n-2) = x(x) + 2x(n-1)$$

to the input $x[n] = 4^n u(n)$

Solution:

$$y(n) - 3y(n-1) - 4y(n-2) = x[n] + 2x(n-1) \quad — (1)$$

The homogenous equation of the System:

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

The charastic equation of the System

$$\lambda - 3\lambda^{-1} - 4\lambda^{-2} = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda^2 - 4\lambda + \lambda - 4 = 0$$

$$\lambda(\lambda-4) + 1(\lambda-4) = 0$$

$$(\lambda-4)(\lambda+1) = 0$$

$$\lambda = 4, \lambda = -1$$

So,

$$y_h(n) = C_1[-1]^n u[n] + C_2(4)^n u[n]$$

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Since 4 is a characteristic root and the excitation is

$$x[n] = 4^n u[n]$$

assume a particular solution

$$y_p[n] = Kn4^n u[n]$$

Then

$$Kn4^n u[n] - 3K[n-1]4^{n-1}\mu[n-1] - 4K[n-2]4^{n-2} u[n-2] = 4^n \mu[n] + 2(4)^{n-1}\mu(n-1)$$

For $n=2$:

$$K(32-12) = 4^2+8 = 24$$

$$K = \frac{6}{5}$$

The total solution is

$$y[n] = y_p(n) + y_h(n)$$

$$= \left[\frac{6}{5}n4^n + c_1 4^n + c_2[-1]^n\right]u[n]$$

To solve for c_1 & c_2 we assume that

$$y(-1) = y(-2) = 0$$

$y(0) = 1$ & $y(1) = 3y(0) + 4 + 2 = 9$

Hence

$$c_1 + c_2 = 1$$

$$\frac{24}{5} + 4c_1 - c_2 = 9$$

$$4c_1 - c_2 = \frac{21}{5}$$

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$C_1 = \frac{26}{25}$

$C_2 = \frac{-1}{25}$

The total Solution is:

$$y[n] = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4n - \frac{1}{25}(-1)^n\right] u(n)$$

Question# 02

(b): Determine the impulse response and unit step response of the system describe by difference equation

$y(n) = 0.6y[n-1] - 0.08y[n-2] + x[n]$

Solution:

$y[n] = 0.6y[n-1] - 0.08y[n-2] + x[n]$

$y[n] - 0.6y[n-1] + 0.08y[n-2] = x[n]$

homogenous equ:

$x[n] = 0$

$y[n] = 0.6y(n-1) + 0.08y(n-2) = 0$

$y_h[n] = \lambda^n$

So.

$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0.$

$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0.$

$\lambda^2 - 0.6\lambda + 0.08 = 0.$

$(\lambda - 0.2)(\lambda - 0.8) = 0.$

$\lambda_1 = 0.2, \ \lambda_2 = 0.4$

Thus, the general form of the solution to the homogenous equation is

⑤

$y_h[n] = c_1(\lambda_1)^n + c_2(\lambda_2)^n$

$y(n) = c_1(0.2)^n + c_2(0.4)^n$ — ①

$y_h(n) = c_1 \frac{1}{5}^n + c_2\left(\frac{2}{5}\right)^n$

$y(0) = 1$, $y(1) - 0.6y(0) = 0.$

$y(1) = 0.6$

$\frac{1}{5}c_1 + \frac{2}{5} = 0.6$

$\Rightarrow$ $c_1 = -1$, $c_2 = 3.$

therefore

$h[n] = [-(\frac{1}{5})^n + 2(\frac{2}{5})^n]u(n)$

The step response:

$y(n) = \sum_{k=0}^{n} h(n-k)$, $n > 0$

$= \sum_{k=0}^{n} [2(\frac{2}{5})^{n-k} - (\frac{1}{5})^{n-k}]$

$= (\frac{1}{0.12}(\frac{2}{5}^{n+1} - 1) - \frac{1}{0.16}(\frac{1}{5}^{n+1} - 1] \; u(n).$

Question# 02

Part (a) Determine the Causal Signal $(x(n))$ having z-transform.

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Solution: $$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

= By partial fraction method:

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1})}{(1-2z^{-1})(1-z^{-1})^2}$$

$$1 = A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1}) \quad (1)$$

Put $z = 1$

$$1 = A(1-1)^2 + B(1-2)(1-1) + C(1)(1-2)$$

$$1 = 0 + 0 - C$$

$$1 = -C$$

$$\boxed{C = -1}$$

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Put $z=2$ in eq (1)

$$1 = A\left(1-\frac{1}{2}\right)^2 + B\left(1-\frac{2}{2}\right)\left(1-\frac{1}{2}\right) + C\left(\frac{1}{2}\right)\left(1-\frac{2}{2}\right)$$

$$1 = A\left(\frac{1}{2}\right)^4 + B(1-1)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(1-1)$$

$$1 = \frac{A}{4} + B(0)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(0)$$

$$1 = \frac{A}{4} + 0 + 0$$

$$\boxed{A = 4}$$

Put $z=3$ in eq (1)

$$1 = A\left(1-\frac{1}{3}\right)^2 + B\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) + C\left(\frac{1}{3}\right)\left(1-\frac{2}{3}\right)$$

$$1 = A\left(\frac{4}{9}\right) + B(1/3)(2/3) + C(1/3)(1/3)$$

$$1 = \frac{40}{9}A + \frac{2}{9}B + \frac{1}{9}C$$

$$1 = \frac{40}{9}(4) + \frac{2}{9}B - \frac{1}{9}$$

$$1 + \frac{1}{9} - \frac{16}{9} = \frac{2}{9}B$$

$$-\frac{6}{9} \times \frac{9}{2} = B$$

$$\boxed{-3 = B}$$

Hence $x(n) = [4(2)^n - 3 - n]\,u(n).$

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Question # 02

(b) Determine the partial fraction expansion of the following

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

Sol:

Eliminate the -ve power by multiplying both numerator & de-nominator by z^2

$$X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A}{z-1} + \frac{B}{z-0.5}$$

$$z = (z-0.5)A + (z-1)B \quad \text{--- (1)} \quad \because \text{L.C.M}$$

Now set $z = p_1 = 1$ in eq (1), we elimate the term involving A. ~~B~~

$$1 = (1-0.5)A$$

$$\boxed{A = 2.}$$

Return to eq (1) $z = p_2 = 0.5$ thus eleminating the term involving A, So we have

$$0.5 = (0.5-1)B$$

$$\boxed{B = -1}$$

Question # 03

Part (a) A two-pole low pass filter has the System response

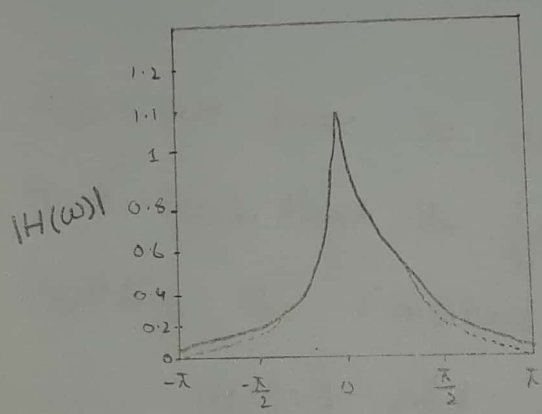
$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

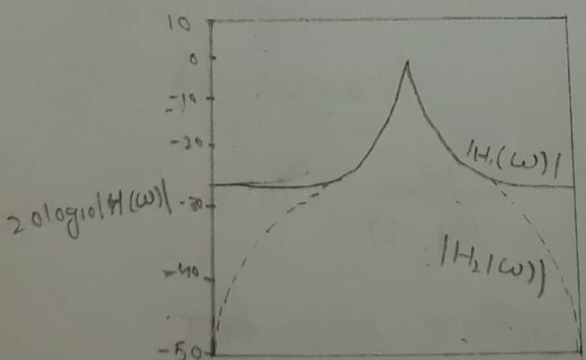
Determine the value of b_0 & P, Condition

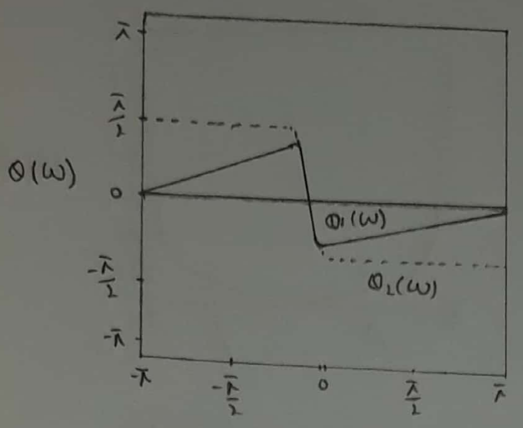
$H(0) = 1$ & $|H(\frac{\pi}{4})|^2 = \frac{1}{2}$

Solution:

Linear Time-Invariant Systems as Frequency-Selective filters







Magnitude of and phase response of (1) a sigle-pole filter and (2) a One-pole, one-zero filter:

$H_1(z) = (1-a)/(1-az^{-1})$

$H_2(z) = [(1-a)/2][1+z^{-1}/(1-az^{-1})]$

$a = 0.9$

Now we have to determine the value of b_0 & P. Such that the freequency response $H(\omega)$ Satifies the Conditions.

$H(0) = 1$ and $|H(\frac{\pi}{4})|^2 = \frac{1}{2}$

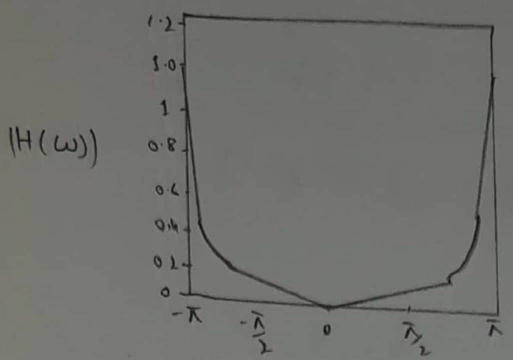
Sol At $\omega = 0$

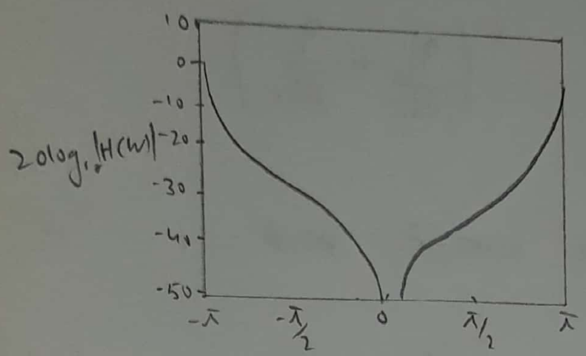
$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

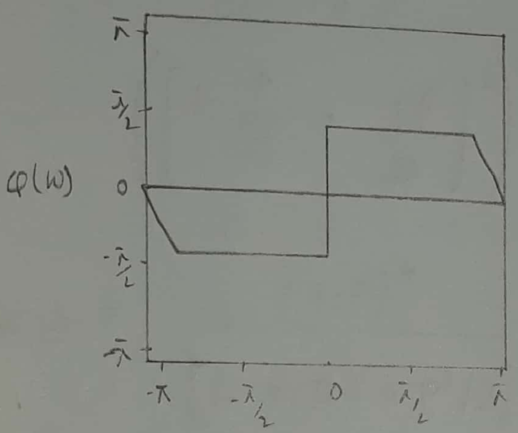
Both sides multiply by $(1-p)^2$

$$= \frac{b_0}{(1-p)^2} \times (1-p)^2 = 1 \times (1-p)^2$$

$$= b_0 = (1-p)^2$$







Magitude & phase response of a simple highpass filter;

$$H(z) = \left[\frac{1-a}{2}\right]\left[\frac{1-z^{-1}}{1+az^{-1}}\right]$$

$a = 0.9$

at $\omega = \frac{\pi}{4}$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

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$$= \frac{(1-p)^2}{(1-p\cos(\frac{\pi}{4}) + jp\sin(\frac{\pi}{4}))^2}$$

$$= \frac{(1-p)^2}{\left(1-\frac{p}{\sqrt{2}} + \frac{jp}{\sqrt{2}}\right)^2}$$

= take Square on above equation

$$= \frac{([1-p]^2)^2}{\left(\left[1-\frac{p}{\sqrt{2}} + \frac{jp}{\sqrt{2}}\right]^2\right)^2}$$

$$H(z) = \frac{[1-p]^4}{[(1-p/\sqrt{2})^2 + p^2/2]^2} = \frac{1}{2}$$

$$= \sqrt{2}(1-p)^2 = 1 + p^2 - \sqrt{2}p$$

the value of $p = 0.32$ satisfies this equestion.
desid. filter is:

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

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Question# 04 $x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & \text{otherwise} \end{cases}$

Part (a)

Solution: The fouries transform of this sequence

$$x(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The mgnitude & phase of $x(\omega)$ are illustrated for $L=10$. The N point DFT of $x(n)$ is simply $x(\omega)$ evaluted at the set of N equally spaced frequencies $\omega_k = \frac{2\pi k}{N}$

$k = 0, 1 \ldots\ldots N-1$

Hence

$$x(k) = \frac{1-e^{-j2\pi kL/N}}{1-e^{-j2\pi k/N}} \qquad k = 0, 1, N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$

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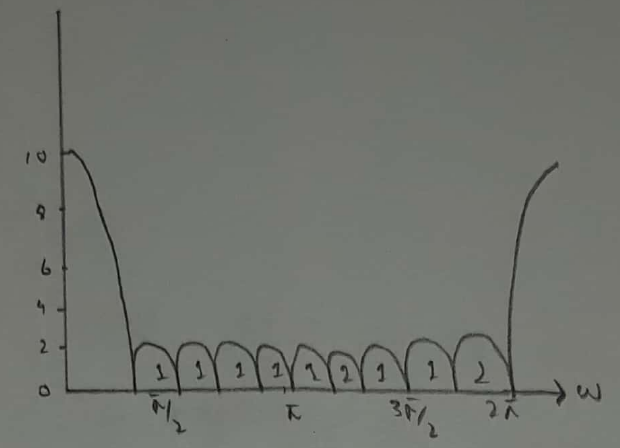

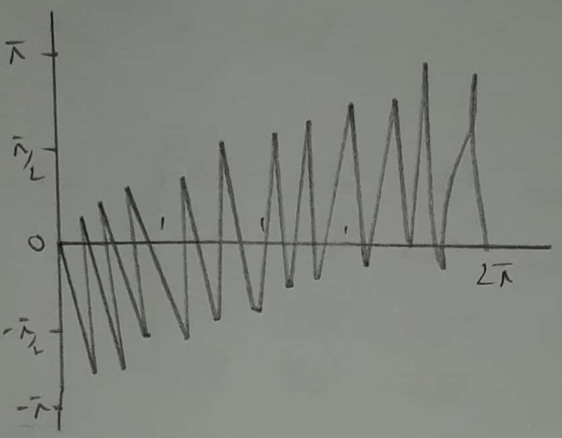

If N is Selected Such that N=L then the DFT becomes

$$x(K) = \begin{cases} L & K=0 \\ 0 & K=1,2,3 \ldots L-1 \end{cases}$$

Thus there is one non-zero value in DFT this is apparent from observation of $X(\omega)$. Since $X(\omega)=0$ at the frequency.

$\omega_K = \frac{2\pi K}{L}$ $K \neq 0$. The reader should verify that $x(n)$ can be recovered from $x(K)$ by Performing L-point IDFT.

Q # 04

(B)

Solution:

The first step is to determine the matrix W_4. By exploiting the periodicity property of W_4

$$W_N^{k+N/2} = -W_N^k$$

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then

$$y_4 = W_4 x_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The DFT of x_4 may be determined by conjugating the element in W_4 to obtain W_4^* & then applying the formula.